#3.40

Date Due

OCT 5 1949			
NOV 26 1951			
FEB 31 1952			
FEB 13 1952			
MAY 30 1952			
FEB 9 1953			
FEB 17 1954			
APR 5 1954			
OCT 22 1954			
~~AUG 22 1993~~			

zlw. pm

DESIGN OF CONCRETE STRUCTURES

DESIGN OF CONCRETE STRUCTURES

BY

LEONARD CHURCH URQUHART, C.E.
Professor in Charge of Structural Engineering,
Cornell University

AND

CHARLES EDWARD O'ROURKE, C.E.
Assistant Professor of Structural Engineering,
Cornell University

SECOND EDITION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1926

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE TO SECOND EDITION

In making this revision a large portion of the text has been entirely rewritten and a majority of the illustrative problems have been revised or new ones have been added to conform to the higher working stresses now frequently allowed in specifications. The changes and additions are so numerous that the book has been set throughout.

The chapter on plain concrete has been amplified by more complete treatment of the subject of scientific proportioning. Typical problems have been included in order to illustrate the methods of the selection of the mix for both large and small jobs. The chapter on columns has been entirely rewritten and enlarged. Additional tables have been included in this chapter as well as typical design problems. The chapter on continuous beams and building frames has also been entirely rewritten. The development of the theory of the rigid frame is given in detail and the application of the theory illustrated by typical problems.

References to the report of the Joint Committee are confined to the 1924 report, and when such references are given as recommendations the attempt has been made to set forth the reasons for such specifications.

Acknowledgement is made to many users of the book for helpful suggestions, and only lack of space has prevented the incorporation of more of these suggestions in this revision.

L. C. Urquhart.
C. E. O'Rourke.

Cornell University,
Ithaca, N. Y.
July, 1926.

PREFACE TO FIRST EDITION

The intent of this work is to provide a text on concrete and reinforced concrete which can be used for the elementary courses given in engineering schools. There has been no attempt to produce a handbook or to cover all the phases of concrete construction.

The chapters on the elementary theory of reinforced concrete are the development of the authors' notes made while teaching this subject during the past ten years. It is their aim to give sufficient development of the theory with illustrative problems to insure the beginner a thorough understanding of fundamentals.

Complete designs of the essential features of the more common concrete structures are given in order to furnish a vehicle for the bringing together of all of the fundamental theory involved. No chapter on miscellaneous structures has been attempted, as the variety of such structures is so great that nothing thorough could be given without greatly increasing the scope of the work.

Acknowledgment is made to Professor George A. Hool for permission to use the diagrams for rectangular beams with steel in top and bottom from "Concrete Engineer's Handbook" by Hool and Johnson, and to *Concrete* for permission to use a portion of the articles on Forms which appeared in that magazine during the year 1922.

The authors also wish to acknowledge their indebtedness to T. L. Collum, formerly instructor in Cornell University, for reviewing the manuscript and for helpful suggestions, and to D. H. Blakelock and W. G. G. Cobb, seniors in Cornell University, for checking the problems and for aid in making drawings.

L. C. Urquhart.
C. E. O'Rourke.

Cornell University,
Ithaca, N. Y.
September, 1923.

CONTENTS

CHAPTER IX

CHAPTER X

CHAPTER XI

CHAPTER XII

DESIGN OF CONCRETE STRUCTURES

CHAPTER I

PLAIN CONCRETE

1. Introductory. During the past twenty-five years concrete has taken its place as one of the most useful and important structural materials. Owing to the comparative ease with which it can be molded into any desired shape its structural uses are almost unlimited; so wherever Portland cement and suitable aggregates can be obtained it has, for certain classes of work, displaced older materials. This apparent ease with which concrete may be prepared has led to its being employed by anyone who feels that the material is suited to his particular purpose. In many instances, proper knowledge of the substance and skill in its manufacture are not available, so that the resultant concrete is little more than a bulky, heavy material, lacking the strength and other properties which it should have attained, and often failing to fulfil that purpose for which it was intended.

To the individuals who obtain such results, concrete is merely a shoveled-together mass of cement, sand, stone, and water, which in a short time attains a varying degree of hardness and an uncertain strength. To the engineer who is more or less familiar with the many factors and variables entering into its manufacture, the process of making concrete does not appear quite so elementary. Experience shows that the quantity and quality of cement, aggregates, and water, and the processes of mixing and curing are all involved in the production of concrete. Results are dependent upon all of these variables. It is, therefore, the problem of the engineer so to study and control these factors that a concrete of the desired quality may be obtained.

2. Concrete is a mixture of cement, sand, water, and aggregate, the latter being made up of more or less graded particles of such materials as sand, gravel, broken stone, cinders, or slag. The ingredients having been thoroughly mixed, the resultant more or less plastic mass is deposited in its final form before the hardening action caused by the cement takes place. For reinforced concrete work Portland cement should always be used, and other cements, such as natural, sand, and puzzolan cements will not be treated here.[1]

Portland cement is the product obtained by finely pulverizing the clinker obtained by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials with no additions subsequent to calcination except water and calcined or uncalcined gypsum. All Portland cement used in reinforced concrete construction should pass such standard specifications as those of the American Society for Testing Materials.[2] The color and rapidity of hardening of different brands of cement vary considerably and may be elements requiring special attention for the particular work involved.

Fine aggregate should consist of sand, stone screenings, or other inert materials with similar characteristics, or a combination thereof, having clean, hard, strong, durable, uncoated grains, and free from injurious amounts of dust, lumps, soft or flaky particles, shale, alkali, organic matter, loam, or other deleterious substances.

Coarse aggregate should consist of crushed stone, gravel, or other approved inert materials with similar characteristics, or combinations thereof, having clean, hard, strong, durable, uncoated particles free from injurious amounts of soft, friable, thin, elongated or laminated pieces, alkali, organic, or other deleterious matter.

Water used for concrete should be clean and free from oil, acid, alkali, organic matter, or other deleterious substances.

3. Fine Aggregate. In general, all particles passing a No. 4 sieve (4 meshes per linear inch) are considered as fine aggregate.

[1] For information about other cements and the chemistry of cements see "Concrete Plain and Reinforced," by Taylor and Thompson.

[2] Am. Soc. for Testing Materials Standards, p. 633, 1924.

Most specifications, however, allow some degree of latitude from this requirement. The report of the Joint Committee[3] recommends that not less than 85 per cent of the fine aggregate pass through a No. 4 sieve. Similarly it is generally advantageous that the fine aggregate be well graded from fine to coarse, and the report mentioned above recommends that not more than 30 nor less than 10 per cent of the fine aggregate pass through a No. 50 sieve. Extremely fine particles, if present in any great amount, are not beneficial to the strength of the resultant concrete, since they furnish so great an excess of surface area to be covered by the cement. Specifications vary as to the amount of these allowed, but in general not more than 6 per cent of the fine aggregate should pass through a No. 100 sieve.

Sand. All sands are derived from rocks which have broken down or disintegrated through the operation of physical agencies. In some cases, in addition to disintegration, there has been more or less decomposition involving the formation of new compounds. The principal disintegrating agencies are temperature changes and abrasion. The former cause cracking and a spalling off of particles of the constituent rock, because of the unequal expansion and contraction of the component minerals. The latter may be caused by the flow of water, wind, or glacial action. Chemical decomposition is brought about through the solvent power of water, aided often by the presence of various chemically active substances such as acids, which are carried by the water.

Quartz is the mineral which makes up the bulk of the particles of most sands. This is due to the fact that only the harder constituents of rocks survive disintegration and decomposition, and quartz is a common constituent of most rocks, and capable of resisting these destructive agencies. All quartz sands,

[3] The Joint Committee on Standard Specifications for Concrete and Reinforced Concrete consists of five representatives from each of the following: American Society of Civil Engineers, American Society for Testing Materials, American Railway Engineering Association, American Concrete Institute, Portland Cement Association. The Committee was organized in 1904, and presented progress reports in 1909 and 1912, and a final report in 1916, and was discharged. A new Committee of the same title and representing the same societies was organized in 1919, and presented a progress report in 1921 and a final report in 1924.

however, do not make suitable concrete aggregates, for comparatively small amounts of organic impurities will render the sand unfit for use. Sandstone is a common source of quartz sand. Here the character of the binder of the original rock will determine the quality of the sand, since the individual particles of the sand are made up of still smaller particles of quartz, bound together by silica, iron oxide, lime carbonate, or clay.

Pyroxene and *hornblende* are complex silicates possessing a degree of hardness, strength, and durability slightly inferior to that of quartz. Hornblende has inferior weathering qualities. *Feldspars* are considerably less strong and durable than quartz. *Mica* is a very objectionable constituent of sand, being soft, low in strength, and because of its laminated structure, offering opportunity for the percolation of water.

Sand deposits being usually the result of stream or glacial action, and also of such character that the percolation of surface water through the beds is very easy, are often contaminated by matter of organic origin carried in suspension by the water. Thus the coating of the grains by such substances as tannic acid,[4] sewage, manure, sugar, etc. is frequently encountered. Such a coating on the sand grains appears not only to prevent the cement from adhering, but also affects it chemically. Thus the effect of such substances is extremely detrimental, but at the same time their presence is hard to detect, a fact which increases the importance of carefully testing concrete sands.

Two tests may be made in the selection of a sand which may go far toward determining its suitability as an aggregate. It often happens that a sieve analysis of a dry sand will show a comparatively small percentage passing the No. 100 sieve, while in reality there are numberless other small particles loosely cemented together which by themselves would easily pass through the sieve. These particles usually consist of silt, loam, or clay, and are soluble or partially soluble in water. By taking a thoroughly dried sample of sand of known weight, adding sufficient water to cover the sample, and mixing thoroughly, a part of these fine particles is dissolved by the water. After

[4] See *Proceedings*, Am. Soc. for Testing Materials, vol. 20, part 1, p. 309. for the "Effect of Tannic Acid on the Strength of Concrete."

being allowed to settle for a few seconds the water may be poured off and the process repeated until the wash water is clear. The washed sand may then be thoroughly dried and weighed, and the percentage of silt, loam, and clay in the original sample calculated. There are several exact specifications for making this decantation test (see Appendix D), and a similar test for field use, but it is usually specified that a sand showing more than 3 per cent of silt, clay, and loam by this test is not suitable for use as a concrete aggregate. Organic impurities in sand may be approximately detected by the colorimetric test (see Appendix D).

By far the best method of selection of a sand for concrete is the actual test of mortar briquettes made with the selected sand at the same time and under the same conditions as other briquettes in which standard Ottawa sand[5] is used. The report of the Joint Committee referred to above recommends that a sand shall preferably not be used as a fine aggregate unless such briquettes show at ages of 7 and 28 days a tensile and compressive strength at least equal to those made with standard Ottawa sand. Such a test as this often eliminates a sand which appears to be clean, well graded, and entirely suitable for a concrete aggregate, but which, due to the presence of organic matter in its constituent grains, or due to a coating of the grains with tannic acid, is actually a very poor sand.

Screenings. Limestone screenings or crusher dust are sometimes used as fine aggregates, but the concrete made therefrom is usually inferior in quality to that made with an average sand, and is apt to be or become more permeable.

4. Coarse Aggregate. *Crushed Stone.* The quality of an aggregate of this type obviously depends upon the character of the original rock. The principal classes of rocks from which aggregates are derived are granites, trap rocks, limestones, and sandstones. Granite is an igneous rock, whose principal mineral constituents are quartz and feldspar, with varying amounts of

[5] Standard Ottawa sand is a natural sand obtained at Ottawa, Illinois passing a screen having 20 meshes and retained on a screen having 30 meshes per linear inch, prepared and furnished by the Ottawa Silica Company, Ottawa, Ill.

mica, hornblende, and other materials. The structural qualities of granite vary greatly, but granites as a class rank among the hardest, strongest, and most durable stones. Trap rock includes basalt, diabase, and a number of other igneous rocks possessing similar chemical and physical properties. The principal mineral constituents of most of these rocks are pyroxene and feldspar. They are generally rather fine grained, hard, tough, and durable. Limestone is a sedimentary rock which contains carbonate of lime, calcite, or carbonate of lime together with a double carbonate of lime and magnesia, dolomite, as the essential constituent. Sand and clay are common impurities, some varieties of which contain large amounts of shells and other fossils. Limestones vary greatly in structure, strength, hardness, and durability, and, although there are some limestones which have superior structural qualities, the average limestone is inferior to average granites and trap rocks as a concrete aggregate. Sandstones consist of grains of varying sizes, chiefly quartz, bound together by various cementing agencies or binders. A silicious binder produces a sandstone of the greatest structural strength, while an iron oxide or lime carbonate binder is much less efficient. A sandstone whose binder is clay is the least valuable of all as a concrete aggregate.

The maximum size of coarse aggregate advisable depends upon the character of the work. Since the stone is one of the strongest constituents of concrete, it is desirable to have as many and as large particles as possible. The greater the size of the particles, the less surface area there is to be coated, and the smaller amount of cement required for a concrete of given strength. When, however, the maximum size is comparatively large, it is very important that it be well graded down to the minimum size in order to make a dense, compact mass. In small reinforced concrete members the maximum size advisable is as small as ¾ in. in diameter, and rarely in any reinforced work is a diameter greater than 1½ in. advisable. On the other hand, for large, massive work, with no structural reinforcement, much larger sizes may be used to advantage. The specification for minimum size recommended by the Joint Committee is that not more than

10 per cent by weight pass a No. 4 sieve, nor more than 5 per cent by weight a No. 8 sieve.

Gravel. Gravel of good quality makes a suitable concrete aggregate. Gravel is nothing more than pieces of natural rock broken away from the parent ledges and worn down by stream or glacial action. Its strength as an aggregate depends upon the rock from which it came, provided it has not become decayed or coated with objectionable organic matter. Too much emphasis cannot be given to the necessity of determining the cleanness of the gravel. A clayey coating is easily detectable, but the transparent organic coatings which prevent adhesion are not so easily discerned, without chemical analysis, so that many weak and inferior concretes result from the use of an apparently clean, but really "dirty" gravel as an aggregate.

Natural gravel may have a large proportion of particles so small as to be classed as "fine aggregate." These may be screened out before using, or a sieve analysis of the natural gravel being made, the proper amount of additional fine aggregate (if any) to add to obtain the desired proportions may be determined.

Slag. Slag from blast furnaces is a hard though porous material of high compressive strength, which in some localities can be obtained much more cheaply than stone of good quality. It offers a very rough surface for the adhesion of the cement, and, provided the sulphur content is low, it may make an excellent aggregate for massive concrete construction. Generally it should not be used in thin sections exposed to any action from water on account of its porosity.

Cinders. Cinders as an aggregate have the advantage of making a resultant concrete considerably lighter in weight than that made from stone or gravel. Formerly it was thought that well-burned cinders made a more fire-resisting concrete than other aggregate, but more recent experiences have shown that cinder concrete is little, if any, better in this respect. Cinder concrete is inferior in strength to other concrete, and on account of the danger from the probable sulphur content, it is not used where any great structural strength is required. Its principal use occurs

for filling where no great strength is necessary. When used, cinders should be free from unburnt coal or fine ashes.

5. Proportioning of Ingredients. Haphazard and careless proportioning of the ingredients of concrete was formerly the rule rather than the exception, but the number of resultant failures is surprisingly small. In the early years of concrete construction little attention was paid to any of the ingredients except the cement. Specifications might require a clean, sharp sand, and a coarse aggregate of a specified crushed stone, but there was small emphasis on the grading of the aggregates or on the amount of water to be used. On the other hand, on almost all work of any magnitude, frequent and exhaustive tests of the cement were required. Under those conditions the practice grew of specifying certain arbitrary proportions for the mix, such as 1:2:4, or 1:3:6. When a concrete of high structural strength was desired, a rich mix was specified, and mixtures as rich as 1:1:2 have occasionally been used, while for less important or more massive work, mixtures as lean as 1:4:8 have been specified. Unfortunately this practice of arbitrary proportioning has persisted to a large extent to the present day, although it is now generally recognized that the relative consistency (amount of water used) has some relation to the strength of the resultant concrete.

Practically all measuring of quantities of the constituent materials for concrete is done by volume,[6] the cubic foot being the unit of measure usually specified. For example, a 1:2:4 mixture indicates 1 cu. ft. of cement, 2 cu. ft. of fine aggregate, and 4 cu. ft. of coarse aggregate. Since the volume occupied by a given quantity of cement varies as much as 30 per cent for different degrees of compactness, it is necessary that the degree of compactness when measured be constant. For this reason, it is usual to specify that one bag of cement weighing 94 lb. shall be considered as 1 cu. ft. of cement. Cement is sold by the bag or barrel, there being four bags per barrel.

The space occupied by a given number of sand grains varies considerably with the moisture content. Increases in volume of

[6] The Iowa State Highway Commission specifies measuring by weight. For an account of this method see, "Weighing Concrete Aggregates for Highway Pavements," *Proceedings*, American Concrete Institute, 1924.

25 per cent or more, caused by the addition of water to dry sand, are not uncommon.[7] Natural sand as it comes from the bank ordinarily contains from 2 to 4 per cent moisture by weight. Sand used in the laboratory is often entirely free from moisture, and sand used on the work may have either more or less moisture content than natural sand. For accurate proportioning, the moisture content must be taken into consideration, but only within the last few years has any attention been paid to this factor.

Many attempts have been made in the last 25 years to proportion concrete scientifically. Most of the methods developed, while producing more or less satisfactory results in the laboratory, were difficult of application in the field and were but seldom used on actual work. Some of the more widely known of these earlier methods are briefly described below.

Proportioning by Void Determinations. In this method the volume of the voids or air spaces existing in each of the aggregates is first determined and the attempt made to select the proportions of the ingredients so that the sand or fine aggregate just fills the voids in the coarse aggregate. No allowance is made for the expansion in the sand due to the water used in the mixing, nor for the space the water itself occupies. Sometimes an arbitrary allowance of additional cement is made to provide for the coating of the sand grains and coarse aggregate, but this coating, together with the water and its effect on the sand, changes the original conditions, and the theoretically solid mass is not obtained.

Proportioning by Mechanical Analysis. In this method of proportioning, a sieve analysis of each of the aggregates to be used is made. The sizes of the screen openings are plotted as abscissae, and the percentage passing each sieve as ordinates. A more or less regular curve may be obtained for each aggregate analyzed. Various proportions of each aggregate may now be used to form a curve for the combined mixture, the endeavor being to make this resultant curve coincide with the curve of maximum density,[8] according to the belief that the more dense

[7] See *Proceedings*, Am. Soc. for Testing Materials, vol. 20, part II, p. 147.

[8] See "Concrete Plain and Reinforced," by TAYLOR and THOMPSON, for Wm. F. Fuller's curves of maximum density.

the mix the stronger the concrete. A mixture proportioned by this method is apt to be extremely harsh and difficult to work on account of the large proportion of stone, so that in practice more sand is usually added, although some strength is sacrificed in so doing.

Proportioning for Maximum Density. Carrying the foregoing method still further, tests for maximum density, involving actual conditions, may be made. The proportions required by mechanical analysis and the maximum density curve having been determined, a small quantity of the materials is mixed with the requisite amount of water, and tamped into a tube of small diameter, and the volume occupied is measured. Other trials are made, slightly varying the proportions of cement, sand, and stone, but keeping the total amount of solid matter and amount of water constant. In this way the mixture of maximum density for the given aggregates may be ascertained. As in the foregoing method, such a mixture will be found harsh and difficult to work, so more sand is usually added.

6. Modern Developments in the Theory of Proportioning. During the last 10 years research and investigation conducted at the Structural Materials Research Laboratory under the direction of Duff A. Abrams have developed methods of proportioning based upon the ratio of the amount of water to the amount of cement and upon the grading of the aggregates. The tests made in this laboratory, as well as similar tests made in other laboratories, have been so great in number and have covered such a wide range of aggregates and such a variety of conditions that the data obtained are of great value and general in application. The water ratio theory and the grading of aggregates will be discussed in the following articles.

7. The Water Ratio Theory. The *water ratio theory* is that, for structurally sound and clean aggregates and fixed conditions of manipulation, the strength of the concrete is determined by the ratio of the volume of the mixing water to the volume of the cement, so long as *workable* mixtures are obtained. This infers that with a definite quantity of cement and a definite quantity of water, the strength of the resultant concrete is fixed no matter what additions in the form of aggregates are subsequently made.

Any change in the ratio of the amount of water to the amount of cement, however, changes the strength. The logic of this deduction may be appreciated if the cement and water are considered to be a paste or glue binding the aggregates together. The thicker the glue the greater the strength and *vice versa*.

The only limitations on this theory are that the aggregates used are clean and structurally sound and that the resultant mixture is workable. The term "workable" may be explained as follows: With a water cement ratio of 1, that is, equal volumes of cement and water, enough aggregates must be added to hold the water, as the cement itself can be hydrated with about one-third of this amount. On the other hand, if too much aggregate is added, the concrete becomes so dry that the water cement paste cannot hold it together. In between these two extremes will be found the workable mix. The degree of *workability* depends also upon the grading of the aggregates, and upon the relative proportions of fine and coarse aggregates. This factor will be considered in Arts. 9 and 11.

The amount of cement in each batch of concrete is easily determined but the amount of water actually used is not so readily ascertained. In addition to the free water added in mixing, the aggregates as used in the field contain a variable quantity of moisture, allowance for which must be made in the application of the water ratio theory.

8. Consistency. The degree of wetness of a concrete is known as the *consistency*. With the amount of the cement, the quantity, proportions, and character of the aggregates constant, the amount of water added determines both the consistency and the degree of workability of the resultant concrete. Since by the application of the water ratio theory the amount of water added also determines the strength of the concrete, it follows that under certain conditions the consistency may be used as a measure of the strength.

When this method is used as a criterion of strength it is not necessary to predetermine the amount of moisture in the aggregate, for this water as well as the free water added will affect the consistency.

In order to obtain the same amount of water in successive batches of concrete containing fixed quantities of cement and

aggregate, several methods have been devised for measuring the consistency or degree of workability. The two most widely used are the *slump test* and the *flow test*. The former is simple of application in the field as well as in the laboratory and has been adopted as a tentative specification for measuring the workability or consistency of concrete for pavements by the American Society for Testing Materials.[9] It is described in the Proceedings of the Society substantially as follows.

The test is made with a galvanized metal form in the shape of a frustum of a cone. The base and top are both open and parallel to each other and perpendicular to the axis of the cone. The diameter of the base is 8 in. and that of the top 4 in., and the altitude of the frustum 12 in. The mold or form is placed on a flat, non-absorbent surface and filled with the concrete to about one-fourth of its height, which is then puddled for 20 to 30 strokes of a ½-in. rod pointed at the lower end. The filling is completed in three successive similar layers, and the top struck off so that the mold is exactly filled. Three minutes after being filled, the mold is removed by raising vertically; and, after the concrete has ceased to subside, the height of the specimen is measured. The slump is the difference between 12 in. and the height of the specimen.

The flow test is made with a metal-covered table with the top so arranged that it can be raised and lowered ½ in. by means of a cam. The concrete is molded in a truncated cone 5 in. high, with a top diameter of 6¾ in. and a bottom diameter of 10 in. The concrete is lightly tamped into the mold on the table, the top struck off, and the mold immediately withdrawn. The table is then raised and dropped fifteen times in about 10 seconds. The diameter of the base divided by the original diameter, 10 in., is the "flow," generally expressed as a percentage. While this measure of the consistency gives more uniform results under all conditions, it is not so simple of application as the slump test and is rarely used outside of the laboratory.

The term "normal consistency" has been applied to a degree of workability represented by a slump of ½ to 1 in. A concrete

[9] See *Proceedings*, Am. Soc. for Testing Materials, D 62–20T, vol. 20, part I, and D 138–22T, vol. 22, part I.

having a normal consistency has about the maximum strength that can be obtained with definite proportions of cement and aggregate, but such a consistency is somewhat drier than will allow the concrete to be thoroughly worked around the reinforcement without excessive tamping. In a large proportion of reinforced concrete work, the stiffest consistency which allows of economical placing of the concrete is measured by a slump of from 6 to 7 in. Such a concrete requires from 20 to 25 per cent more water than a concrete of a consistency giving the maximum strength for the given amounts of cement and aggregate, with a corresponding reduction in strength, but the practical considerations mentioned above require the wetter concrete. Stiffer consistencies may be used where little or no reinforcement is present or where the economy of the increased strength is not overbalanced by the increased difficulty and labor of placing.

9. Size and Grading of Aggregates. The fact that the size and grading of the aggregates bear an important relation to the workability and strength of a concrete has long been recognized and various attempts have been made to evaluate it in a quantitative manner, generally based on the sieve analysis of the aggregate.

One method of determining the relative size and grading of the aggregates is based on the *surface area* of the aggregate. The surface area of an aggregate is the number of square inches or square feet of surface area of the particles per unit weight of aggregate. This surface area may be calculated by formula, assuming that all of the particles are spheres, determining by actual count the number of particles per unit of weight. Such a formula is given by R. B. Young and W. D. Walcott in the *Proceedings* of the American Society for Testing Materials, vol. 20, part II, page 138. It is $A = 236.1\sqrt[3]{\frac{n}{s^2}}$ where A = surface area in square feet per hundred pounds, s = specific gravity of sand, and n = number of grains per gram in any size of separation. It may also be approximated by assuming that an aggregate that would be classified[10] as 0–100 has a surface area of 13,700 sq. in. per lb. and that each coarser size group as determined by the

[10] See Art. 11.

standard sieves named above has a surface area one-half that of the next smaller group.[11]

A more widely used method of determining the relative size and grading is based on the *fineness modulus* of the aggregate. The fineness modulus of an aggregate is the sum of the percentages in the sieve analysis, divided by 100, using sieves Nos. 100, 50, 30, 16, 8, 4, $\frac{3}{8}$ in. and $1\frac{1}{2}$ in., as adopted as the standard set of sieves for "Sieve Analysis of Aggregates for Concrete" (1922),[12] and expressing the sieve analysis as the total percentages coarser than each sieve. The coarser the aggregate the larger its fineness modulus and the less water required to bring the concrete made with it to a normal consistency.

While there is no material relation between the surface area of a graded aggregate and the fineness modulus of the aggregate, the methods proposed for proportioning concrete which involve either the fineness modulus or the surface area give more scientifically proportioned concretes than other methods heretofore proposed. For the majority of aggregates the two methods give results in rather close agreement with one another.

10. Proportioning by the Water Ratio Theory. In the practical application of the water ratio theory, the workability of the mix must be considered. The resultant concrete must be sufficiently plastic so that it may be puddled readily into the angles and corners of the forms and around the reinforcement without bringing excessive quantities of water and fine particles to the surface. The proportion of fine to coarse aggregate must be such that there is no honeycombing in the structure, and, when the forms are removed, the surfaces and corners of the members are smooth and sound throughout. These requirements generally make it necessary to use at least one-half as much of the fine as of the coarse aggregate. While the most economical mixture contains a relatively large proportion of the coarse aggregate and has a comparatively stiff consistency, a workable mix must be obtained if uniform concrete is to result. On the other hand, ease of placing should not be allowed to become the predominant

[11] See *Proceedings*, Am. Soc. for Testing Materials, vol. 19, part II, p. 478, for further discussion of Surface Area and Surface Modulus.

[12] See Appendix D.

factor when this will result in extremely wet consistencies. With consistencies represented by a slump greater than 9 in., the accumulation of water and fine particles on the surface, and the segregation of the aggregates tend to produce concrete lacking in uniformity.

The tests made at the Structural Materials Laboratory, in addition to establishing the principle of the water ratio theory,

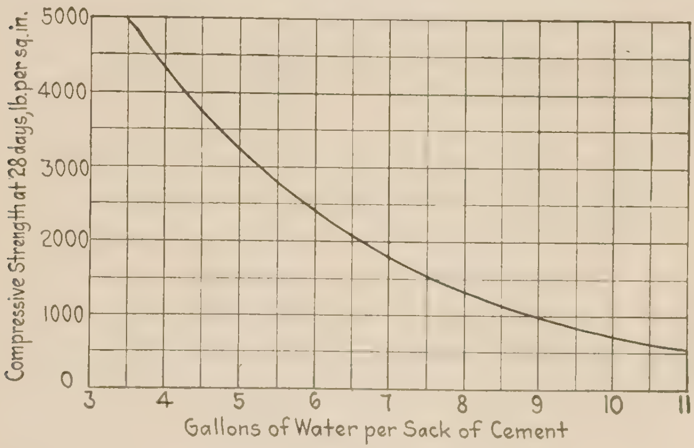


Fig. 1.

furnished sufficient data so that definite concrete strengths may be forecast for various water cement ratios. Figure 1[13] shows graphically the effect of the quantity of mixing water upon the strength of the concrete. From it may be obtained the number of gallons of water per sack of cement required to produce concrete of a definite strength. This amount of water includes the moisture contained in the aggregates.[14]

[13] From *Bull.* 9, Structural Materials Research Laboratory. Mixes made according to this curve may be expected to show a minimum strength equal to, or greater than, the specified strengths. If rigid control is exercised in the making of the concrete, ultimate strengths greater by from 300 to 500 lb. per sq. in. may be expected.

[14] A portion of the moisture in the aggregates is in the interior of the particles. Such moisture, if it remains in the interior until after the cement has set, does not affect the water cement ratio. If a bone dry aggregate is used, a certain amount of water will be absorbed by the aggregates and the

The amount of moisture in the aggregate may be determined as follows: Take a sample of the aggregate in the same degree of compactness and in the same condition of moisture as it will be in when measured for use on the work. Determine its volume and weight. Dry thoroughly and obtain the weight a second time. The difference in weight is the moisture content for the given volume. This may be converted into gallons of water per cubic foot of aggregate and the proper reduction made in the amount of water to be added in mixing.

An approximation of the moisture content of the aggregates may be assumed as follows:

Very wet sand	¾ gal. per cu. ft.
Moderately wet sand	½ gal. per cu. ft.
Moist sand	¼ gal. per cu. ft.
Moist gravel or crushed rock	¼ gal. per cu. ft.

Example of Proportioning by the Water Ratio Theory. Approximate Method. A concrete having a minimum strength of 2000 lb. per sq. in. is desired. The aggregates are sand in a moderately wet condition and pebbles in a moist condition. Both aggregates have been examined for organic impurities, silt, and clay and have been declared suitable for concrete work. From Fig. 1, the curve shows that the total water required is 6.6. gal. per sack of cement. A first batch is made up using one bag of cement, 2 cu. ft. of sand, and enough coarse aggregate to produce the required consistency. In a moderately wet condition 2 cu. ft. of sand contain approximately 1 gal. of water. Therefore, $6.6 - 1.0 = 5.6$ gal. of water are to be added when mixing. If such a mix shows too much or too little sand, the amount may be changed for the next batch, and the correction to the amount of water to be

---

water cement ratio will be changed. The amount of water absorbed by different aggregates is variable. It may be determined in accordance with Recommended Practice of the American Society for Testing Materials (1920 *Proceedings*, Part 1, Appendix 1 of *Report* of Committee C-9) or average quantities may be assumed as follows:

Average sand	1.0 per cent by weight
Pebbles and crushed limestone	1.0 per cent by weight
Trap rock and granite	0.5 per cent by weight
Porous sandstone	7.0 per cent by weight

added again computed. In this approximate method no account is taken of the water absorbed by the aggregates, nor of the water contained in the coarse aggregate.

Exact Method. Assume the same aggregates and the same desired concrete strength as before. The moisture content of the aggregates is determined as follows:

Weight of $\frac{1}{10}$ cu. ft. sand moderately wet and loose	8 lb. 12 oz.
Weight of 8 lb. 12 oz. moderately wet and loose when dried	8 lb. 6 oz.
Water per cu. ft. of moderately wet and loose sand 3 lb. 12 oz. =	.45 gal.
Weight of $\frac{1}{2}$ cu. ft. moist pebbles	51 lb. 8 oz.
Weight of 51 lb. 8 oz. moist pebbles when dried	50 lb. 2 oz.
Water per cu. ft. of moist pebbles 2 lb. 12 oz. =	33 gal.
Water absorbed by sand	$.01 \times 8\frac{3}{4} \times 10 = .875$ lb. = .10 gal. per cu. ft.
Water absorbed by pebbles	$.01 \times 51\frac{1}{2} \times 2 = 1.03$ lb. = .12 gal. per cu. ft.
Net deduction for water in sand	$.45 - .10 = .35$ gal. per cu. ft.
Net deduction for water in pebbles	$.33 - .12 = .21$ gal. per cu. ft.

A first batch may be made up consisting of 1 bag cement, 2 cu. ft. sand, and 4 cu. ft. pebbles. The water to be added in mixing is $6.6 - 2 \times .35 - 4 \times .21 = 5.1$ gal. If the above mix has too stiff a consistency, additional water and cement may be added in the ratio of one sack of cement to 6.6 gal. of water and another batch mixed with less aggregate and consequently more added mixing water. For instance: a $1:1\frac{3}{4}:3\frac{1}{2}$ is tried. The water to be added is $6.6 - 1\frac{3}{4} \times .35 - 3\frac{1}{2} \times .21 = 5.3$ gal. Similarly, a mix too harsh may be corrected by changing the proportion of fine to coarse aggregate and computing the proper reductions in the amount of mixing water to be added.

11. Design of Concrete Mixtures. Proportioning by the water ratio theory alone is a trial method. If the grading of the aggregates is considered, proportions of cement and aggregates may be predetermined which, with a given water ratio, will produce concrete of definite strength and of the desired workability at the least cost. When such a procedure is followed, it may truly be called a design of the mix.

The steps in the design are as follows:

1. Determine the water ratio for the specified strength from Fig. 1.

2. Determine the degree of workability required as represented by the slump.

3. Examine representative samples of the aggregate for cleanness and structural quality.

4. Determine the unit weights[15] of the aggregates in the condition in which they are to be used on the work. Determine the moisture content of the aggregates. Determine the unit weights of the aggregates in a dry and rodded condition.

5. Make a sieve analysis of the aggregates and determine their fineness moduli and maximum sizes.

6. Determine the real mix and the fineness modulus of such a mix from Fig. 1*a*.

7. Calculate the percentages of fine and coarse aggregates to produce the required fineness modulus of the mixed aggregate.

8. Determine the unit weight of the mixed aggregate in a dry and rodded condition.

9. Determine the shrinkage factor, that is, the ratio of the volume of the mixed aggregate to the sum of the separated volumes measured dry and rodded.

10. Calculate the nominal mix.

11. Determine the *bulking factor* and calculate the field mix.

12. Calculate the quantity of mixing water to be added to each batch.

The details of the various steps in the design will be illustrated by means of a typical design problem.

1. Concrete of 2500 lb. per sq. in. minimum strength is specified, which, from Fig. 1, requires 5.85 gal. of water per sack of cement.

2. The degree of workability desired is represented by a slump of from 3 to 4 in. With a moderate amount of tamping, a concrete of such a consistency can be puddled around reinforcement that is not closely spaced.

3. The samples of aggregate selected for testing were taken from various locations in the supply. The total weight of the sample of fine aggregate taken was about 50 lb. and that of the coarse aggregate about 100 lb. The actual samples to be used in testing the aggregate were taken from these larger samples by the

[15] See Appendix D.

quartering method as follows: The sample was thoroughly mixed with a trowel and spread in the form of a circle to a depth of from 3 to 4 in. The circle was quartered and two opposite quarters discarded. The remaining two quarters were remixed and the process repeated until the sample of the desired size was obtained. Samples were tested for organic impurities and for the quantity of silt and clay by the standard methods given in Appendix D.

4. For the determination of the unit weight of the fine aggregate a measure having a capacity equal to, or greater than, $\frac{1}{10}$ cu. ft. should be used. For the coarse aggregate the measure used should have a capacity of at least $\frac{1}{2}$ cu. ft. For the determination of the unit weights of the aggregates as used on the work, the measures should be filled in the same manner as the larger measures are to be filled in the field. For the aggregates in the present problem the following results were obtained.

FINE AGGREGATE

Weight of $\frac{1}{10}$ cu. ft. damp and loose	8 lb. 13 oz.
Weight of 8 lb. 13 oz. damp and loose when dried	8 lb. 7 oz.
Weight of $\frac{1}{10}$ cu. ft. dry[16] and rodded	10 lb. 7 oz.
Weight per cu. ft. damp and loose	88.1 lb.
Weight of 88.1 lb. damp and loose when dried	84.4 lb.
Weight per cu. ft. dry and rodded	104.4 lb.
Amount of moisture	$\frac{3.7}{84.4}$ = 4.4 per cent

COARSE AGGREGATE

Weight of $\frac{1}{2}$ cu. ft. damp and loose	48 lb. 13 oz.
Weight of 48 lb. 13 oz. damp and loose when dried	48 lb. 1 oz.
Weight of $\frac{1}{2}$ cu. ft. dry and rodded	53 lb. 12 oz.
Weight per cu. ft. damp and loose	97.6 lb.
Weight of 97.6 lb. damp and loose when dried	96.1 lb.
Weight per cu. ft. dry and rodded	107.5 lb.
Amount of moisture	$\frac{1.5}{96.1}$ = 1.6 per cent

5. Sieve analyses of the aggregates were made according to the Standard Method of Test for Sieve Analysis of Aggregates for Concrete given in Appendix D.

The following results were obtained:

[16] See Appendix D for Standard Method of Test for Unit Weight of Aggregate for Concrete.

Sieve number or size in inches	Fine Aggregate			Coarse Aggregate		
	Weight of sample 500 g.			Weight of sample 4500 g.		
	Weight retained	Percentage retained	Percentage coarser than each sieve	Weight retained	Percentage retained	Percentage coarser than each sieve
100	111.5	22	96	....	..	100
50	80.5	16	74	....	..	100
30	181.0	36	58	....	..	100
16	37.5	8	22	....	..	100
8	70.5	14	14	105	2	100
4	0	..	0	652	14	98
3/8 in.	0	..	0	1286	29	84
3/4 in.	0	..	0	1611	36	55
1 in.[17]	0	..	....	848	19	
1½ in.	0	..	0	0	0	0
	Fineness modulus 2.64			Fineness modulus 7.37		

Aggregates are classified as to size according to the following rules:

a. At least 15 per cent shall be retained on the sieve next smaller than that considered the maximum size. Thus, a graded sand with 15 per cent or more retained on the No. 8 sieve is classified as the 0—No. 4 size. If 14 per cent or less is retained on the No. 8 sieve, the sand is classified as the 0—No. 8 size. A coarse aggregate having 15 per cent coarser than the 2-in. sieve is considered as a 3-in. aggregate.

b. Not more than 15 per cent of a given aggregate shall be finer than the sieve considered as the minimum size.

Applying the above rules to the given aggregates, the range of size for the fine aggregate is 0—No. 8 and that for the coarse aggregate No. 4 to 1½ in.

6. From Fig. 1*a*,[18] for a concrete having a consistency represented by a slump of from 3 to 4 in., a strength of 2500 lb. per sq. in. and a maximum size of aggregate of 1½ in., the volume of

[17] 1- and 2-in. sieves are used in determining the size of the aggregate but not in calculating the finenes modulus.

[18] From "Design and Control of Concrete Mixtures," Portland Cement Association, Chicago, Ill.

mixed aggregate for each volume of cement is 4.6 and the fineness modulus of such a mix is 5.75.

7. Let x be the percentage of fine aggregate, y the percentage of coarse aggregate, m_f the fineness modulus of the fine aggregate,

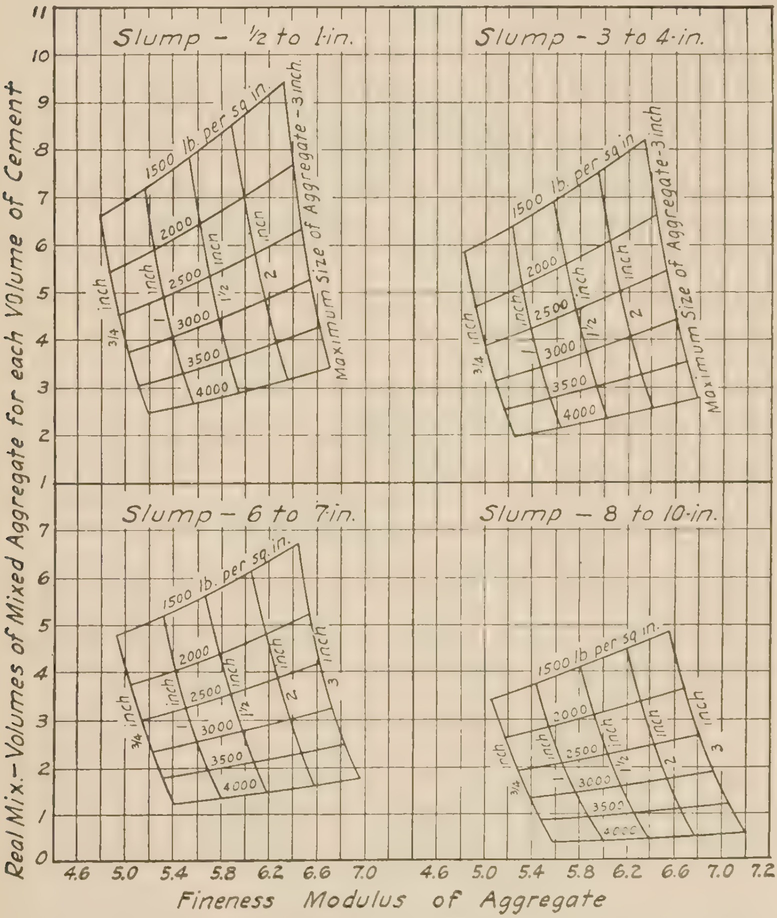


FIG. 1a.—Relation of size and grading of aggregate and quantity of cement to strength of concrete. This figure is based on the relation between strength and quantity of mixing water showed in Fig. 1.

m_c the fineness modulus of the coarse aggregate, and m the fineness modulus of the mixed aggregate. Then,

$$m_f x + m_c y = m \quad \text{and} \quad \frac{x + y}{100} = 1$$

from which

$$x = 100 \frac{m_c - m}{m_c - m_f}$$

In the present problem the percentage of fine aggregate is $100 \times \frac{7.37 - 5.75}{7.37 - 2.64} = 34$ and the percentage of coarse aggregate is 66.

8. The aggregates were thoroughly mixed in the above proportions (34 lb. of fine aggregate and 66 lb. of coarse aggregate). Using the ½-cu. ft. measure, the weight of ½ cu. ft. of the mixed aggregate is 61 lb. 8 oz., or the weight per cubic foot 123.0 lb.

9. The weight of 1 cu. ft. of the aggregates measured separately is $.34 \times 104.4 + .66 \times 107.5 = 106.5$ lb. Since 1 cu. ft. of the mixed aggregate weighs 123.0 lb. per cu. ft., the shrinkage factor is $\frac{106.5}{123.0} = .87$.

10. In order to obtain the required volume of mixed aggregate it is necessary to furnish a greater volume of the aggregates measured separately. To supply 1 cu. ft. of mixed aggregate $\frac{1}{\text{shrinkage factor}}$ cu. ft. of the aggregates measured separately is required. In the present instance the volume of aggregates measured separately is $\frac{4.6}{.87} = 5.3$ cu. ft., and the amount of fine aggregate required is $.34 \times 5.3 = 1.80$ cu. ft., and the amount of coarse aggregate $.66 \times 5.3 = 3.50$ cu. ft. The nominal mix is, therefore, 1:1.80:3.50.

11. The nominal mix is for aggregates measured in a dry and rodded condition. From (4) the relation between the aggregate in a damp and loose condition and the aggregate in a dry and rodded condition may be obtained. When dried, 1 cu. ft. of damp and loose sand weighs 84.4 lb., while 1 cu. ft. of dry and rodded sand weighs 104.4 lb. Therefore, in order to furnish 1 cu. ft. of dry and rodded sand, it is necessary to furnish $\frac{104.4}{84.4} = 1.24$ cu. ft. of damp loose sand. This ratio of the unit weight of the dry and rodded sand to the unit weight of the damp and loose sand when dried is called the *bulking factor*. The bulking factor for the coarse aggregate is $\frac{107.5}{96.1} = 1.12$.

The field mix is $1{:}1.80 \times 1.24{:}3{:}50 \times 1.12 = 1{:}2.23{:}3.92$.

12. For most aggregates the water normally absorbed by them may be taken from the average values given in Art. 10 without appreciable error. The percentage of moisture in the aggregates was determined in (4). The water correction for the sand is, therefore,

$$2.23 \times 84.4(.044 - .01) = 6.40 \text{ lb.} = .77 \text{ gal.}$$

and for the coarse aggregate

$$3.92 \times 96.1(.016 - .01) = 2.26 \text{ lb.} = .27 \text{ gal.}$$

The amount of mixing water to be added for each sack of cement is, therefore,

$$5.85 - .77 - .27 = 4.81 \text{ gal.}$$

If the mix as designed is too *harsh* (lacking in fine aggregate) or *over-sanded* (lacking in coarse aggregate), more fine or coarse aggregate may be added without affecting the strength of the resultant concrete. Such changes, will, however, require the addition of more cement and water in the proper proportions if the same degree of workability is to be maintained. If any such change is deemed desirable, a redesign of the mix is advisable. For instance, in the present case, if it were desired to use less fine aggregate (say 30 per cent), the fineness modulus of the combined aggregate mix, with this proportion of fine aggregate, is $.30 \times 2.64 + .70 \times 7.37 = 5.95$, and the real mix as obtained from Fig. 1*a* is 1:4.8. With this revised real mix, the nominal and field mix and the amount of water to be added in mixing may be calculated as before.

Instead of performing the calculations of steps 6 to 10, the nominal mix may be taken from the tables in Appendix E. When these tables are used, the water to be added in mixing need not necessarily be computed, and the slump test may be used as a measure of the strength. The design by the use of the tables then resolves itself into tests of the aggregates for impurities, a sieve analysis, the determination of the bulking factor, and the calculation of the field mix.

12. Selection of Method of Proportioning. The selection of the most economical method of proportioning from those pre-

viously described depends upon the quantity of concrete to be placed, and the relative importance of producing uniform concrete.

Where the quantity of concrete to be placed is large and uniformity is important, the full design, as given in the previous article, should be made. Sieve analyses of the aggregates should be made at frequent intervals, and when any change in the grading appears, the proper correction should be made in the proportions. Changes in moisture content should also be noted, and corrections to the bulking factor[19] and the amount of water to be added in mixing should be made accordingly.[20] Frequent slump tests should be made in order to give warning of an appreciable change in moisture content as well as to check the amount of water supplied at the mixer.

Where the degree of uniformity of the concrete is not so important, the tables of Appendix E may be used. For the average run of aggregates these tables will give excellent results. With this method, frequent slump tests become increasingly important.

For smaller quantities of concrete, the proportioning by the water ratio theory alone may be more economical. When this method is used, the individual in charge of the work should be experienced in the making of concrete, if economic mixtures are to be produced. The exact method should be used for all but the smaller jobs.

13. Quantities of Materials Required. Probably the best-known method of estimating the quantites of cement and aggre-

[19] When the aggregates are uniform as to grading, a table may be calculated or a curve plotted showing the bulking factor for various percentages of moisture content. The latter may then be readily determined in the field from small samples and the bulking factor read from the table or curve.

[20] The volume occupied by a given weight of sand is about the same when the sand is thoroughly saturated with water as when the sand is in a dry and rodded condition, although for intermediate percentages of moisture the sand may bulk as much as 40 per cent. This fact may be used to advantage in measuring the quantity of sand for a concrete mix. When a sand is thoroughly saturated with water, or *inundated*, both the amount of dry and rodded sand and the amount of water per unit of measure are constant, no matter what the condition of moisture of the sand may be as it is supplied in the work. Commercial applications of this method of measuring the sand have been very successful in aiding the production of uniform concrete.

gates required for a given volume of concrete is by means of the empirical rule devised by Wm. F. Fuller, which is:

$$C = \frac{11}{c + s + g}$$

$$S = C \times s \times \frac{3.8}{27}$$

$$G = C \times g \times \frac{3.8}{27}$$

where C is the number of barrels of cement per yard of concrete.

S is the number of cubic yards of sand per cubic yard of concrete.

G is the number of cubic yards of coarse aggregate per cubic yard of concrete.

c:s:g represents the proportions of cement, sand, and coarse aggregate.

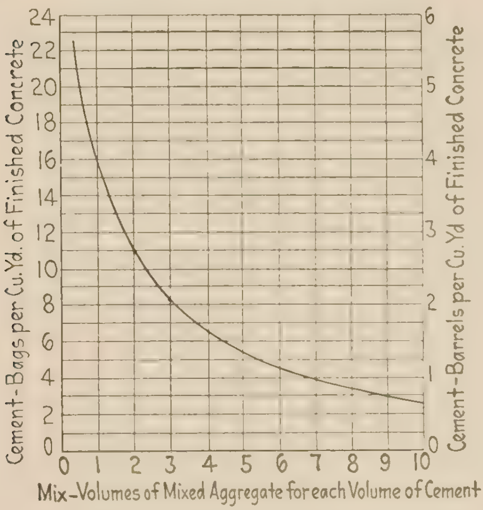


FIG. 1*b*.

This rule is based on aggregates measured loose, and for well-graded aggregates gives results slightly greater than are actually required.

Figure 1*b* is based on the results of measurements made at the Structural Materials Research Laboratory. From it the amount of cement in bags or barrels may be obtained when the real mix

is known. The amounts of aggregate may then be determined by proportion.

Illustration. For the mix as designed in Art. 11 according to Fuller's rule, the quantities required per cubic yard of concrete are as follows:

$$C = \frac{11}{1 + 2.23 + 3.92} = 1.54 \text{ bbl.}$$

$$S = 1.54 \times 2.23 \times \frac{3.8}{27} = .48 \text{ cu. yd.}$$

$$G = 1.54 \times 3.92 \times \frac{3.8}{27} = .85 \text{ cu. yd.}$$

From Fig. 1*b* for the real mix of 1:4.6, 5.6 sacks or 1.40 bbl. of cement are required. The amount of sand is $5.6 \times 2.23 = 12.5$ cu. ft. = .46 cu. yd., and the amount of coarse aggregate is 5.6×3.92 cu. ft. = .81 cu. yd.

14. Mixing. Machine-mixed concrete is usually more uniform in quality than that mixed by hand and is, of course, more economical where there is any great amount of concrete involved. Where, however, the amount of concrete to be mixed is small, hand mixing may be more economical, but whenever this method is used, careful superintendence is necessary to insure that the work is thoroughly done.

The time of mixing is a very important factor in the strength of concrete, so this element should receive detailed attention. The effect of the duration of the mixing operation is shown by Fig. 2, which is based upon tests made by Professor H. H. Scofield at Purdue University.[21] While these concretes are much stronger than the average concrete, this does not affect the significance of the results. They show that there is a decided advantage gained by operating the mixer much longer than the usual time allowed. Similar tests made at the Structural Materials Laboratory, Chicago, by Duff A. Abrams[22] with a wider range of concretes show similar results. The results differed from the tests of Fig. 2 in that the increase of strength was not so rapid over the first portion of the curve, *i.e.*, a concrete mixed from ½ to 1½ min.

[21] See *Eng. Contr.*, Jan. 27, 1915.

[22] See *Can. Eng.*, July 25, 1918.

showed a larger proportional strength to a concrete mixed 10 min. than did the tests of Professor Scofield. Furthermore, the time of mixing was not carried beyond 10 min. in this second set of tests, but the results seemed to show that had this been done, a somewhat further increase in strength would have resulted.

The Joint Committee recommends that "the mixing of each batch shall continue not less than 1 min. after all the materials

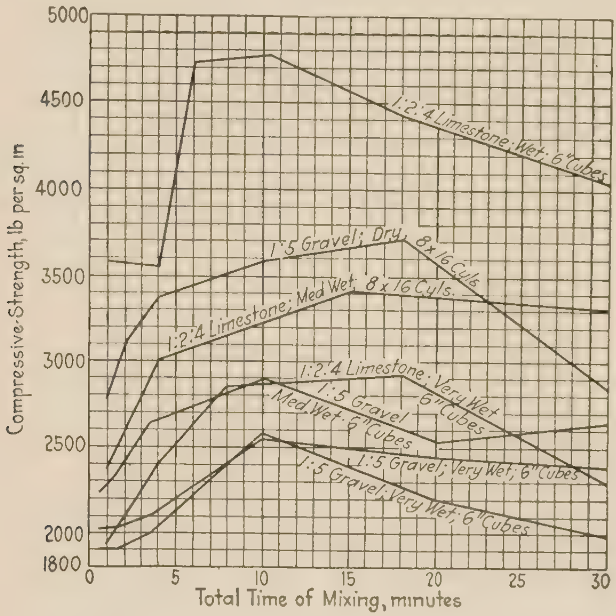


Fig. 2.

are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200 ft. per min. . . . Hand mixing . . . shall be done on a water-tight platform. The cement and fine aggregate shall first be mixed dry until the whole is of uniform color. The water and coarse aggregate shall then be added and the entire mass turned at least three times, or until a homogeneous mixture of the required consistency is obtained."

15. Regaging. The Joint Committee recommends that the remixing of concrete which has partially hardened shall not be permitted, and most specifications set a limit of elapsed time between the mixing and final deposition in the forms. While it is

generally advisable not to permit retempering on account of the difficulty of insuring that all of the concrete is thoroughly and properly remixed, when this operation is performed in an approved manner with enough water added to bring the concrete to its original consistency, it is beneficial to the strength. The additional working given the concrete no doubt accounts for this increase. The retempering must be done before the concrete reaches its final set, and the best results are obtained with concrete regaged from 1 to 3 hours after the first mixing. The set of remixed concretes is permanently retarded. This by itself would make it inadvisable to allow this practice in cold weather.

16. Depositing Concrete in the Forms. The concrete should be placed in the forms as soon as possible after the mixing is completed, in a manner which shall prevent separation of the ingredients. It should not all be dumped in one place and allowed to flow horizontally, but deposited in approximately uniform layers, in order to prevent the separation of the coarse aggregate from the mortar. Forms should be as tight as possible to prevent the loss of cement carried by the escaping water, and should be filled continuously without stoppage in order to prevent the formation of laitance or "days' work" planes. Laitance is a whitish substance consisting of the finest particles of the cement together with some of the silt and clay from the aggregates. It is brought to the surface of freshly mixed concrete where excess water is used (as it usually is in reinforced concrete) and, as it hardens very slowly and never acquires much strength, it constitutes a plane of weakness. Where continuous deposition is impossible, the laitance should be scraped off and the surface of the old concrete roughened before deposition is resumed. Sufficient puddling and tamping should be done to insure that the concrete completely fills the forms and is in thorough contact with the reinforcement. Forms of considerable height should be provided with means of depositing the concrete without dropping it through too great a distance. Besides the separation that is bound to take place unless this is done, both forms and reinforcement become coated with hardened concrete long before they are completely filled, and this may form planes of weakness in the top of the structure.

17. Rodding. Experiments made at the University of Texas 1917–1919[23] show that concrete of the ordinary consistency used in reinforced concrete work can be increased in strength about 100 per cent by rodding. A $\frac{5}{8}$-in. pointed round rod was used for the rodding and was pushed into the concrete from ten to twenty times at each rodding, the intervals varying from 10 to 30 minutes. Concrete rodded at intervals for $2\frac{1}{2}$ hours attained a strength of about 100 per cent more than unrodded concrete of the same mix. Further rodding had no effect upon the strength except that in a few cases a decrease was noted when the rodding was carried on for several hours. The direct effect of rodding is to expel entrapped air and excess water, and to compact the concrete. Concrete of normal consistency cannot be used for reinforced concrete work, but by using this rodding process, concrete of even greater strength can be obtained with an original very mushy consistency.

Curing Conditions

18. The principal variations in curing conditions which affect the process of hardening and the strength of the concrete are variations in moisture and temperature conditions. While it is important that the amount of water used in mixing be controlled so that the consistency is as nearly normal as practical, it is just as important that the concrete be not allowed to dry out immediately, if the maximum strength obtainable is to be attained. All concrete should be protected against premature drying out for at least 1 week, and for a longer time if the temperature is near the freezing point. This may be done by sprinkling with water at intervals, or by covering with damp or wet burlap. In road construction, water may be held over the entire surface by damming the edges with loose earth and forming a series of ponds. The importance of keeping the concrete moist while hardening cannot be too strongly emphasized. Tests show that a concrete allowed to dry out immediately will usually reach a strength of not more than 50 per cent of the strength of similar concrete kept moist over the entire period of curing. Figure

[23] See *Proceedings*, Am. Soc. for Testing Materials, vol. 20, part II, p. 219

3 shows this relation[24] graphically. All test specimens were tested at 4 months, having had various intervals of storage in damp sand. Each value is the average of 24 tests (four each for six consistencies).

The relation between the mean temperature during the curing period and the strength of concrete is illustrated by Fig. 4.[25] The tests from which the curves were plotted covered a wide range of temperature conditions, and the results were fairly consistent. A knowledge of the effect of the mean temperature upon the strength is very necessary in determining the time when forms may be removed and loads applied, and a careful study of Fig. 4 will furnish the necessary information for deter-

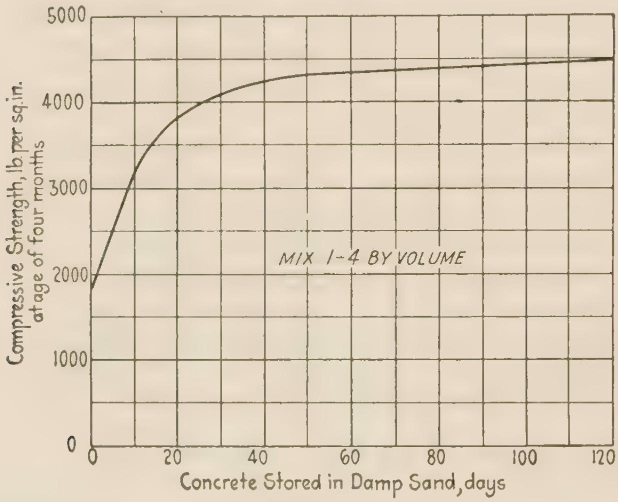


FIG. 3.

mining the relative length of time the forms should be kept in place under different temperature conditions.

By combining high temperatures with a saturated condition of the atmosphere, it would follow that accelerated hardening of the concrete would be obtained. These conditions are brought into being by the application of live steam to concrete while hardening. This method is especially useful in the manufacture

[24] Taken from *Bulletin* 2, Structural Materials Research Laboratory, Lewis Institute, Chicago, Ill.

[25] Taken from *Bulletin* 81, Engineering Experiment Station, University of Illinois.

of concrete blocks, tile, small pipe, etc. where the saving in forms, storing space, and time is important. By placing the concrete products in a confined space, and applying the steam under pressure, a still more rapid increase in strength will be attained. The steam should not be applied until after the concrete has obtained an initial set. Results of tests show that up to 80 lb. per sq. in. gauge-pressure steam has an accelerating action

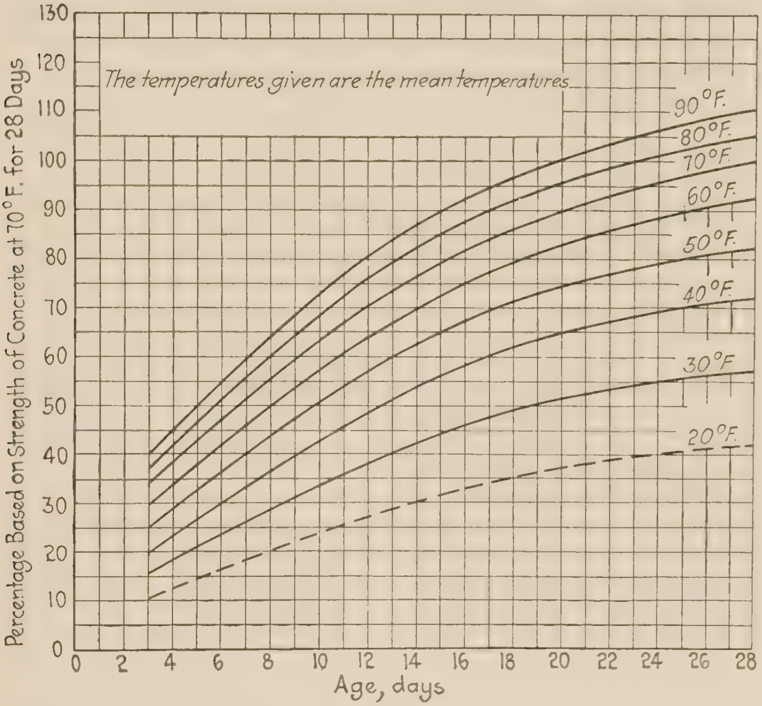


FIG. 4.

on the hardening of concrete, and that the compressive strength increases with the pressure and time of exposure. The application of the steam, too, permanently accelerates the hardening after the exposure to steam ceases. Concrete so treated has reached a compressive strength in 2 days (exposure to steam under pressure 24 hours) greater in some cases by 100 per cent than unsteamed concrete has reached in 28 days.

19. Freezing. The effect of low temperatures in delaying the hardening of concrete is shown in Fig. 4. When water reaches

a temperature of 39 deg. Fahrenheit some subtle change occurs which decreases its chemical ability for combination. This change becomes more marked as the freezing point is approached, and concretes placed with the temperature near the freezing point take several times as long to obtain a final set as concretes cured at normal temperatures. In case of dry atmospheric conditions much of the water may evaporate before the final set takes place, and insufficient water be left to combine chemically with the cement. In case the temperature falls below the freezing point before final set, the expansion of the water while freezing exerts a force sufficient to destroy the cohesion between the particles of the green concrete.

The injurious effect of freezing is lessened by two factors, namely, that concrete is a very poor conductor of heat, and that the chemical action of setting and hardening generates a certain amount of heat to combat the freezing action of the atmospheric conditions. Thus the serious injury is usually confined to the surface of the concrete, and rarely penetrates more than an inch or two in depth. In massive members this may not seriously impair the strength, but be harmful only to the appearance. In the smaller members, however, a large percentage of the strength may be lost.

Various methods are used to prevent the freezing of concrete, namely, heating the materials, covering the green concrete, adding certain salts to the mixture to lower the freezing point of water, etc. The Joint Committee recommends that:

"In freezing weather suitable means shall be provided for maintaining a temperature of at least 50 deg. Fahrenheit for not less than 72 hours after placing, or until the concrete has thoroughly hardened . . . Salt, chemicals, or other foreign materials shall not be mixed with the concrete for the purpose of preventing freezing, unless approved by the Engineer."

Effects of Miscellaneous Agencies and Conditions on the Strength and Durability of Concrete

20. Salts, Hydrated Lime, and Water-proofing Compounds. Common salt is often added to the mix to lower the temperature at which the water will freeze. The addition of salt lowers the

freezing point about 1 deg. Fahrenheit for each 1 per cent of salt added to the mixing water. This has been shown to be beneficial to the strength of concretes cured at freezing temperatures up to 12 per cent. More than this amount of salt has generally proved injurious. With normal temperatures the addition of common salt is injurious, the decrease in strength being roughly proportional to the percentage of salt added. The set is retarded in all cases, and in reinforced concrete the salt is likely to cause corrosion of the steel.

Calcium chloride is used for the same purpose as common salt. Not such a large percentage is beneficial to strength as in the case of common salt, but up to 4 per cent concretes cured at any temperature are benefited, and the setting is accelerated. Tests made at the University of Wisconsin[26] indicated that the best effect was obtained with a mixture of 2 per cent of calcium chloride and 9 per cent of common salt.

The use of hydrated lime in quantities up to 15 per cent of the weight of the cement has been advocated by various authorities on the theory that it improved the workability of the concrete or increased its strength and water-tightness. In lean mixtures it is true that the addition of hydrated lime does have a marked effect in producing a more plastic and better working concrete. In the richer mixes this effect is less pronounced. Some tests have shown a slight increase in strength with the use of a small percentage of hydrated lime. It appears that if the hydrated lime is added without decreasing the amount of cement or increasing the amount of water, such an increase usually occurs, but if some of the cement is replaced by hydrated lime, a reduction in strength can be expected.

Various water-proofing compounds in powdered or liquid form are sometimes used to make a more impervious concrete. They are either added to the mixing water, mixed with the cement on the job, or added to the cement during its manufacture. Their function is to fill the voids or pores of the concrete with a more or less soapy, insoluble filler, and thus prevent the percolation of water through the concrete. The results obtained are varied. Some practically impervious concrete has been produced, while

[26] *Wisconsin Eng.*, October, 1913.

on other work the water-proofing has not been successful. Practically all of the compounds in use detract from the compressive strength of the concrete. Fully as impervious concrete can generally be obtained by using a slightly richer mix, well-graded fine aggregate, as stiff a consistency as possible, and thoroughly puddling the concrete as it is placed.

Certain classes of mineral oils have been used in concretes for damp-proofing them or reducing their permeability, but the results obtained do not warrant their use.

21. Alkali. The action of alkali on concrete is a problem peculiar to the prairie regions of the west. These regions, because they have a low rainfall and poor drainage, present extraordinary conditions in respect to the amounts of soluble salts present in the soil. These salts are mainly sodium, magnesium, and calcium sulphates. Seepage water from shallow excavations commonly shows concentrations of from 1 up to 6 per cent or more. Chemical action between these sulphates and the cement causes the decomposition of the concrete, the physical action resembling exactly that of frost. Cases have been cited where concrete immediately above and free from such contact was in first-class condition. The action is slow and retarded by impermeability, but while there is reason to believe that well-made structures of dense concrete will stand up indefinitely in sea-water where constantly immersed, there is no doubt that equally well-made concrete will not stand up long in contact with ground waters of high alkali content. The life of a structure in contact with such waters may be lengthened by following the same precautions that should be used in making and placing concrete which is subject to the action of sea-water, and by providing drainage such that as small an amount of alkali water as possible shall come in contact with the finished concrete.[27]

[27] The Joint Committee specifies that: "Concrete in alkali waters or below the ground line of alkali soils shall contain a minimum of $1\frac{3}{4}$ bbl. (7 bags) of Portland cement per cubic yard in place . . . Concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams, or work planes. Metal reinforcement or other corrodible metal shall not be placed closer than 2 in. to the surface of members exposed to alkali soils or waters. In foundations and in heavy structures the metal reinforcement shall not be placed closer than 3 in. to the surface."

22. Oils, Acids, and Sewage. Concrete thoroughly hardened is unaffected by mineral oils such as ordinary petroleum or engine oils. Various animal and vegetable oils may slightly weaken and disintegrate a concrete, but such cases are rare.

Acids which seriously injure other materials will also injure concrete. The condition where this is most likely to occur is in the discharge of acids in sewage. Strong sulphuric acid in contact with the concrete converts the carbonate of lime into sulphate of lime, which is soft and easily corroded. Two factors, however, tend to make this effect less marked; first, the likelihood of the acid being so much diluted by the water of the sewage as to be practically harmless, and second, the greasy substance which is usually found to coat the perimeter of a sewer under the water line prevents the full action of the acid upon the cement.

23. Manure. Manure is sometimes used to cover fresh concrete in freezing weather. Since dry manure is a poor conductor of heat, and since during decomposition it generates heat, it is quite effective in preventing freezing of the concrete. Unless the work is first covered by some impermeable material, the uric acid in the manure is likely not only to discolor the green concrete but partially to disintegrate it. If the manure becomes wet during the early stages of the hardening of the concrete, unless the latter is efficiently protected, the disintegrating effect may be quite marked. Thoroughly hardened concrete is sometimes discolored by contact with manure, but its strength is not impaired.

24. Electrolysis. Although in most structures the danger of action by electrolysis is negligible, there are certain conditions where reinforced concrete may be seriously damaged by the flow of an electric current between the concrete and the steel. If electrically positive reinforcement is in contact with concrete, it will become corroded, provided the concrete is sufficiently moist and the voltage sufficiently high. The corrosion of the reinforcement with its consequent expansion causes cracking in the concrete. Strong currents, however, of high enough voltage are not usually found under actual conditions, so danger from this source is very rare. Common salt, even in amounts less than 1 per cent, increases the rate of corrosion of the reinforcement

since it increases the conductivity of the concrete. This rate of increase is so tremendous that special care should be taken in structures exposed to contact with sea-water to prevent the flow of stray electric currents. In constructions where stray electric currents may be expected, no salt should be used in the concrete. Electrically negative reinforcement in contact with concrete produces a softening effect upon the latter, which may extend for 1/4 in. or more all around the reinforcement. This softening effect eventually completely destroys the bond between the concrete and the steel. It manifests itself at all voltages, the rate being approximately proportional to the voltage.

25. Sea-water. Almost invariably, specifications forbid the use of sea-water in mixing concretes. The detrimental effect to the concrete itself is usually not so large except in very lean mixtures, but in reinforced concrete construction where sea-water is used in mixing, the corrosion of the steel is likely to be serious, and may eventually result in the complete destruction of the work.

The reliability of concrete and reinforced concrete when exposed to the action of sea-water is variable, but, under favorable conditions and with proper care, structures comparing favorably in durability with those of timber or steel can be constructed. A richer and denser mix should be used than for ordinary construction, in order to insure against the infiltration of water into the concrete, which causes, in the case of plain concrete, complete disintegration due to chemical reaction, or in the case of reinforced concrete, failure due either to electrolysis or to rusting of the steel. Just enough fresh water should be used in mixing so that the concrete settles around the reinforcing rods with light tamping. The forms should be oiled to insure a smooth surface, and if practical, a richer mortar coat should be applied next the forms at the same time the other concrete is placed. Reinforcing material should be protected by at least 3 in. of concrete. Construction joints should be avoided, and the concrete should not be exposed to the sea-water until it is thoroughly hardened.[28]

[28] The Joint Committee specifies that: "Plain concrete in sea-water from 2 ft. below low water to 2 ft. above high water, or from a plane below to a

Structural Properties of Concrete

26. Compressive Strength. The ultimate strength of a concrete normally increases with age. This increase proceeds very rapidly for the first few days after the concrete is placed, but becomes more gradual as time goes on, though continuing at a more reduced rate for an indefinite period. The compressive strength of concrete at the age of 28 days is generally used as a measure of the quality of the concrete. This assumes proper mixing and placing and suitable curing conditions. The compressive strength of the concrete is based on tests of 6- by 12-in. or 8- by 16-in. cylinders made in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the

---

plane above wave action, shall contain a minimum of $1\frac{3}{4}$ bbl. (7 bags) of Portland cement per cubic yard in place. Other plain concrete in sea-water or exposed directly along the sea coast shall contain a minimum of $1\frac{1}{2}$ bbl. (6 bags) of Portland cement per cubic yard in place. Porous or weak aggregates shall not be used . . .

"Sea-water shall not be allowed to come in contact with the concrete until it has hardened for at least 4 days. Concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams or work planes. The placing of concrete between tides shall be a continuous operation, . . . where it is impossible to avoid seams or joints the surface of the set concrete shall be roughened . . . , thoroughly cleaned of foreign matter and laitance, and saturated with water. The new concrete placed in contact with the hardened or partially hardened concrete shall contain an excess of mortar to insure bond. To insure this excess mortar at the juncture of the hardened and newly deposited concrete, the cleaned and saturated surfaces of the hardened concrete, including vertical and inclined surfaces, shall first be slushed with a coating of neat cement grout against which the new concrete shall be placed before the grout has attained its initial set. Concrete shall be deposited in sea-water only when so directed by the Engineer . . .

"Metal reinforcement shall be placed at least 3 in. from any plane or curved surface, except at corners when it shall be at least 4 in. from adjacent surfaces. Metal chairs, supports, or ties shall not extend to the surface of the concrete. Where unusually severe conditions of abrasion are anticipated, the face of the concrete from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall be protected by creosoted timber, dense vitrified shale brick, or stone of suitable quality, as designated on the plans or as required by the Engineer."

Field, of the American Society for Testing Materials"[29] and tested in a well-equipped laboratory by a competent operator.

The ultimate compressive strength expressed in pounds per square inch is used as a basis for determining the unit stresses to be used in design, for it has been found that practically all of the other structural properties of a concrete are proportional to the compressive strength.

27. Tensile Strength. The tensile strength of concrete is a property of little importance, because it is so low in comparison with the compressive strength that it is usually neglected altogether in the design of reinforced concrete structures. It may be roughly estimated as having a value of about 10 per cent of the compressive strength.

28. Transverse Strength. The transverse or flexural strength of concrete is low as compared with its compressive strength, but much greater than the strength in pure tension. The transverse strength is measured by the stress developed in beam action. In a reinforced concrete member this strength is usually disregarded, and steel reinforcement is placed in the member to develop the flexural stresses on the tension side. Load tests, however, on reinforced concrete structures have shown that the transverse strength of the concrete contributes to a marked degree in increasing the capacity of the structure. Tests made by Duff A. Abrams[30] indicate that at 28 days, the transverse strength varies from 26 per cent of the compressive strength for a 1000-lb. per sq. in. concrete to 15 per cent for a 4000-lb. per sq. in. concrete.

29. Shearing Strength. The shearing strength of concrete is important in that failure by shear on a diagonal plane often occurs in short compression specimens. The direct shear must not be confused with the combination of shear and diagonal tension that occurs in the web of a beam. The resistance of concrete to direct shear is difficult to determine, as it is almost impossible to eliminate the effect of bearing, diagonal tension, and other stresses, so that different series of tests show quite a

[29] See *Standards* 1924, Am. Soc. for Testing Materials, p. 762.

[30] See *Bulletin* 11, Structural Materials Research Laboratory, Lewis Institute, Chicago.

variation from one another. For most concrete the shearing strength is at least 60 per cent of the compressive strength, and will need to be considered in design only in exceptional cases.

30. Elasticity. Concrete is not a perfectly elastic material, there being a slight decrease in the ratio of stress to strain as the stress increases. Concrete also shows a permanent set under the smallest loads, but within working limits this permanent set does not continue to increase under repeated applications of the load. Within these limits, therefore, there is a fairly constant relation

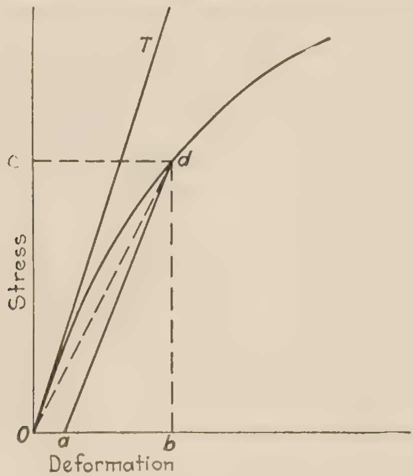


FIG. 5.

between stress and strain, which may be considered as the modulus of elasticity of the concrete.

Since the stress-strain line is curved almost from the beginning, the method of calculating the modulus of elasticity needs to be considered. Figure 5 is a typical stress-strain diagram with the curvature somewhat exaggerated. A load producing a stress *Oc* has been applied and removed a sufficient number of times until the permanent set *Oa* shows no further increase, and all points on the stress-strain line *ad* fall on an approximately straight line. The deformation measured from the original position, for a stress *Oc*, is *Ob*, and the slope of the line *Od* is called the "secant modulus." The slope of the tangent to the curve, represented by *OT*, is known as the "initial modulus" or "initial tangent modulus."

In reinforced concrete design the principal use of the modulus of elasticity is to determine the value of the relative stresses carried by the steel and concrete, assuming that there is perfect bond between the two materials. For such a computation the deformation should be measured from the original position, and the "secant modulus" should be used. In the case of a beam where the stress in the concrete varies according to the "straight-line" theory, the "secant modulus," while not exactly representing the conditions, is nearer to the exact conditions than the "initial modulus." The relation between the "secant modulus" and the "initial modulus" is not a constant, but for all but the smallest loads the former is the smaller.

There is a relation between the modulus of elasticity of a concrete and its compressive strength, but this relation is not linear. Stanton Walker[31] expresses this relation for usual concretes as $E = 33{,}000\ S^{\frac{5}{8}}$, where E = the initial modulus, and S the compressive strength of the concrete. The tests from which this relation was determined covered a wide range of consistencies, mixes, times of mixing, curing conditions, and age at time of test. These tests also showed that the modulus of elasticity increases as the aggregate becomes coarser (within certain limits), that it increases with age, the richness of the mix, and time of mixing, and is less for wet than for dry consistencies.

The value for this initial modulus for concrete at the age of 28 days varies from about 1,500,000 to 5,500,000 lb. per sq. in. with a somewhat narrower range for the usual concretes. The values most often used in design are generally somewhat smaller, the Joint Committee specifying as follows:

Compressive strength of concrete at 28 days in lb. per sq. in.	Ratio modulus of elasticity of steel to that of the concrete
1500–2200	15
2200–2900	12
2900 or more	10

[31] See *Bulletin* 5, Structural Research Materials Laboratory, Lewis Institute, Chicago.

The modulus of elasticity of the steel is taken as 30,000,000 lb. per sq. in.

31. Elastic Limit. For the same reasons as those given at the beginning of the previous section, there can be no *elastic limit* in the true sense of the term. There appears to be a stress, however, below which repetition of the same load does not cause appreciable increase in set, while beyond this stress, repetition of load causes increased set indefinitely, and final failure far below the normal ultimate strength. This stress may be considered as the *elastic limit.* Tests show quite a range of values, varying from 25 to 90 per cent of the ultimate compressive strength, but for the average concrete, it is probably in the neighborhood of from 40 to 60 per cent of the ultimate compressive strength.

32. Contraction and Expansion. Concretes expand as the temperature is raised, and contract as the temperature is lowered. The coefficient of expansion per degree of temperature change increases somewhat with the richness of the mix, but the range of values is small. Tests made in the laboratories of Cornell University gave a range of values of from .00000677 for a 1:1½:3 concrete to .00000537 for a 1:3:6 concrete, with an average for all tests of .00000604. Other tests have shown a close agreement. The value generally used is .000006 per deg. Fahrenheit.

Concretes expand in volume if kept wet or immersed in water, and contract if exposed to air. This property is not confined to freshly placed concrete, but is characteristic of concretes of many years' service. A concrete which dries out in air may be expected to contract from .02 to .05 per cent, and when immersed in water may expand at least one-half of this amount.

This tendency to change in volume with different moisture conditions and changes in temperature does, of course, set up stresses of both tension and compression in a restrained reinforced concrete structure. The tensile stresses often exceed the amount that the concrete can sustain, and cracks result.

33. Bond. The adhesion of new concrete to work previously placed, becomes an important consideration in certain classes of construction. Very few tests of this property, however, have been made. Tests made by Hector St. George Robinson in 1912 indicate that a thorough cleaning of the old surface is beneficial to

bond. When the old surface was roughened, cleaned, either treated with hydrochloric acid or coated with cement grout, a joint of about 80 per cent efficiency was obtained. Merely wetting the surface gave an efficiency of about 40 per cent, and wetting and roughening something more than 50 per cent. If the old concrete is not thoroughly wetted, it will draw the moisture from the new concrete, often leaving not enough in the latter for a normal consistency, and resulting in weak concrete near the joint as well as producing a joint of low efficiency. (For bond between concrete and steel see Art. 46.)

34. Weight. The weight of a concrete varies somewhat with the proportions of the mix, the consistency, and the character of the aggregate. The richer concretes are slightly heavier, and the wetter consistencies are lighter, except when cinders are used as the coarse aggregate. A stone or gravel concrete will usually weigh between 140 and 150 lb. per cu. ft., with an average of about 145 lb. per cu. ft. In reinforced concrete the steel adds from 3 to 5 lb. per cu. ft., and the weight of reinforced concrete (including the steel) is usually taken as 150 lb. per cu. ft. The weight of cinder concrete may be taken as 115 lb. per cu. ft.

Other Properties of Concrete

35. Resistance to Fire. Concrete is not only incombustible, but also a poor conductor of heat. Hence it is a splendid fire-resisting and fire-proofing material. Clay products and building stones are equally non-combustible, but they possess greater conductivity and a higher coefficient of expansion. The low coefficient of expansion lessens the tendency to crack when heated, and the low conductivity prevents the transference of the heat of the fire to the interior of the mass and to the reinforcing steel. Tests of conductivity have shown that when the surface of a mass of concrete is exposed for hours to a high heat, the temperature at a depth of 1 in. beneath the surface is considerably lower, while at a depth of 3 in. or more the rise in temperature is very slight.

The low thermal conductivity of concrete is to a large extent due to voids in the material. Neat cement, with a void content

about twice that of the average concrete, shows a corresponding decrease in its conductivity. It is also partly due to the absorption of the heat of vaporization by the water of combination in the hardened cement. The absorption of heat by the surface material as it becomes dehydrated retards the dehydration of the concrete beneath. The surface concrete which is injured by heat, but which remains in place, affords protection for the material farther in, as it is a poorer conductor than the original concrete.

The experience gained from some of the great fires, for example, those of Baltimore and the Edison Plant, etc., has shown that concrete exposed to high heat for a considerable length of time becomes calcined to a depth of from $\frac{1}{4}$ to $\frac{3}{4}$ in. but shows no tendency to spall off except at exposed corners and edges.

The Joint Committee specifies as follows for concrete covering over steel reinforcement:

"Metal reinforcement in fire-resistive construction shall be protected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders, and columns provided aggregate showing an expansion not materially greater than that of limestone or trap rock is used; when impracticable to obtain aggregate of this grade, the protective covering shall be 1 in. thicker and shall be reinforced with metal mesh having openings not exceeding 3 in., placed 1 in. from the finished surface."

An idea of the severe test which concrete may be expected to pass as a fire-resisting material may be obtained from the following specification of the Building Code of the City of New York for fire-proof partition walls.

"A vertical panel of not less than 14 ft. long and 9 ft. high shall be subjected to a fire continuous for not less than 1 hour at an average temperature of 1700 deg. Fahrenheit during the latter half hour, followed by an application for not less than $2\frac{1}{2}$ minutes of a hose stream from a $1\frac{1}{8}$ in. nozzle at 30 lb. nozzle pressure, without passage of flame during the test."

36. Weathering Qualities. The principal weathering agencies affecting the durability of concrete are variation in temperature, wind, rain, and variation in moisture conditions. Changes in temperature and moisture conditions cause more or less expansion

and contraction in concretes, which in turn are apt to cause cracking that may result in ultimate failure. Cracking due to variations in temperature is likely to be confined principally to the surface of a structure, and may be made less harmful by the use of steel reinforcement so placed that a multitude of small cracks, which often are not visible to the naked eye, replace a few large and deep cracks. Expansion and contraction due to moisture changes are oftentimes more serious, as the moisture may penetrate the concrete further and cause dangerous stresses to be introduced. The expansion and contraction of rich mixes are considerably more than those of the leaner mixtures, when moisture and temperature conditions vary. This circumstance is often responsible for the cracking off of a rich surface coat floated or plastered on a leaner base. The surface material not only tends to expand and contract more on account of its comparative richness, but it protects the underlying material from going through the extensive temperature and moisture changes which it itself is experiencing. To prevent the ultimate spalling off of this surface layer, as lean and as thin a surface coat as possible should be used, and where practicable, it should be applied before the leaner base has set so as to make the bond between the two as strong as possible.

37. Abrasive Resistance. The extensive use of concrete in the construction of roads, pavements, and floors makes its resistance to wear or abrasion an important consideration. In general, a concrete of high compressive strength will have a high resistance to abrasive action. Abrasion either wears away the cement and sand grains or it pulls the sand grains out of the cement matrix. It follows, therefore, that with soft aggregates more cement will be needed in order to keep the wear low, while with hard and durable aggregates just sufficient cement is needed to hold the aggregate against the abrasive action. The quantity of mixing water used, however, the length of time of the mixing, and the curing conditions have more effect on the abrasive resistance than the hardness of the aggregate, and a good wearing surface can be produced with inferior aggregates if other conditions are favorable. Provided the proper precautions are taken, and a good quality concrete produced, the actual wear on the

surface of either a pavement or a floor will not be of any serious amount, no matter how heavy the traffic.

38. Porosity. The porosity of a concrete is important principally as a determining factor of other properties of concrete, such as fire-resistance, absorption, permeability, etc. It is the percentage of void space in terms of the total volume. It is dependent upon the consistency of the mix and the composition and grading of the aggregates. Wet consistencies produce less porous concretes, unless the concrete eventually becomes very dry, when the reverse is true. Well-graded aggregates make a less porous concrete than others, and up to certain limits the greater the proportion of coarse aggregate the less the porosity. Concretes show from 12 to 20 per cent porosity.

39. Absorption. The absorptive property of a concrete is determined by the amount of water it will absorb when exposed to damp conditions or when immersed in water. This property is of more importance in connection with mortar or stucco used as a plaster over metal lath, which must be protected to prevent corrosion. In this case a mortar showing the least absorption will be the most durable in the average. The average concretes will absorb from 8 to 14 per cent of water measured by volume, while some mortars will absorb still more. The amount of absorption increases with the wetter consistencies and dry curing conditions, and decreases as the proportion of cement is increased.

40. Permeability. The permeability of concrete is measured by the rate at which water under a given pressure will pass through a given thickness of the material. It is an important consideration where water-tightness of floors or wall is required and where the percolation of water may cause serious damage. Permeability decreases with age and the richness of the mix, and generally with the continuation of the flow. Wetter consistencies seem to make less permeable concrete, particularly when the face of the concrete in contact with the water is constantly damp. A well-graded sand, with some very fine particles, tends to make the percolation of water more difficult. Ideal curing conditions are also beneficial in decreasing the permeability.

CHAPTER II

GENERAL PROPERTIES OF REINFORCED CONCRETE

Reinforcement

41. Types. The reinforcing steel in reinforced concrete construction must be of such form and size that it easily may be incorporated as a part of the structure and provide sufficient surface to bond thoroughly together the two materials. In order to prevent the great concentration of stress at any point in the concrete, and in order to furnish sufficient area for bond strength, it is necessary to use the steel in comparatively small sections. With the small sections required, economy of manufacture requires the use of steel in the form of round or square bars. These vary in size from 1/4 in. in diameter up to 1 1/2 in. in diameter. Bars of all diameters are not always readily obtainable, and designers should confine their selections wherever possible to the sizes manufactured by all bar companies. These are indicated in Table I by bold-faced type. In order to insure prompt delivery the number of sizes and lengths of bars to be used on a job should be kept as small as possible. The following extras in cents per 100 lb. are standard with all mills for both round and square bars.

Size Extras

3/4 in. and larger	Base
5/8 in.	10 cts.
1/2 in.	20 cts.
3/8 in.	40 cts.
1/4 in.	$1.00

Lengths less than 10 ft. are subject to the following extras:

Lengths over 60 in. and less than 120 in.	5 cts.
Lengths 48 in. to 60 in. inclusive	10 cts.
Lengths 24 in. to 48 in. inclusive	15 cts.
Lengths 12 in. to 24 in. inclusive	30 cts.
Length 12 in. or less . . . not less than	40 cts.

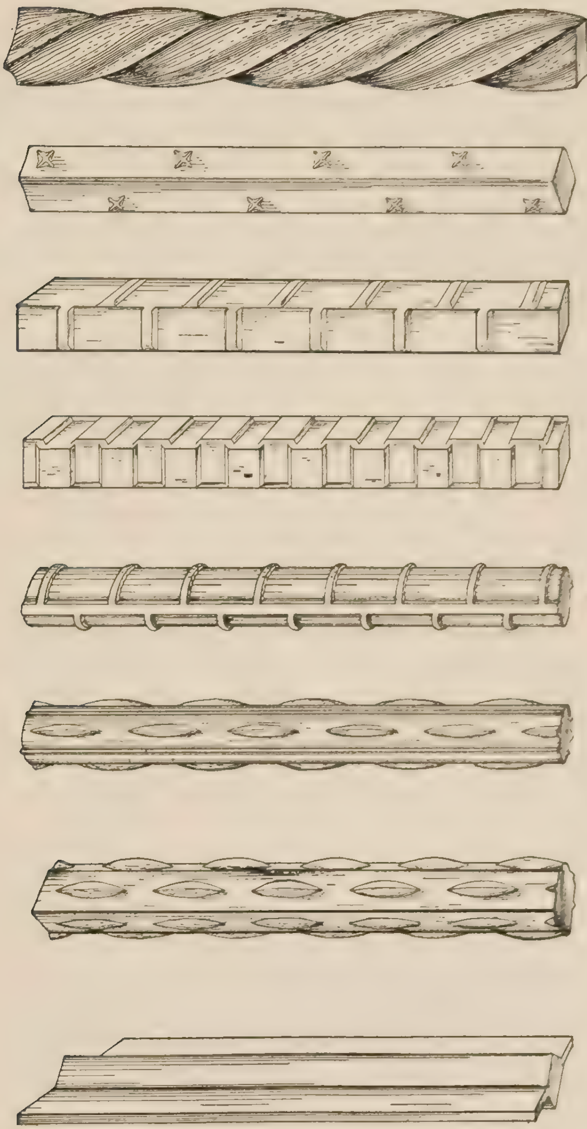


FIG. 6.

Small quantities of the same shape and size are subject to the following extras:

Less than 2000 lb. to 1000 lb.	20 cts.
Less than 1000 lb.	50 cts.

There is also an extra charge for cutting less than 2000 lb. of any size to a specific length, whether or not the above extras apply. They are as follows:

Less than 2000 lb. to 1500 lb.	10 cts.
Less than 1500 lb. to 1000 lb.	20 cts.
Less than 1000 lb. to 500 lb.	40 cts.
Less than 500 lb.	60 cts.

Plain round and square bars are often used, the necessary bond strength being furnished by the adhesion of the steel and concrete. Plain flat bars are not desirable, as the adhesion between them and the concrete is considerably less than for round or square bars. Deformed bars have been devised to furnish a bond between the concrete and steel independent of adhesion. This is accomplished by providing projections or depressions or both on the surface of the bar. Some deformed bars are so shaped that the area of the section is constant throughout the length, while others have considerable difference between sectional areas taken at different points. Some of the common types of deformed bars in use are illustrated in Fig. 6.

Wire fabric and expanded metal in various forms are used to a considerable extent in slabs and other thin concrete sections. These types of reinforcement are easy to place, and since the metal is so well distributed in small sections, it is especially well adapted to resist the cracking likely to occur from changes in temperature and moisture conditions. Some of the forms of this type of reinforcement are shown in Fig. 7.

42. Grade. Three grades of steel[1] are in use for reinforcing bars, namely, structural steel, intermediate, and hard. Some authorities prefer that bars of the structural steel grade only be used. The brittleness of the hard or high-carbon steel is to be feared especially in light members subject to sudden impact stresses. High-carbon steel when used should be thoroughly

[1] See *Standards* 1924, Am. Soc. for Testing Materials, pp. 141 and 145.

inspected and tested in order to prevent brittle or cracked material from being used in the completed structure. Structural steel

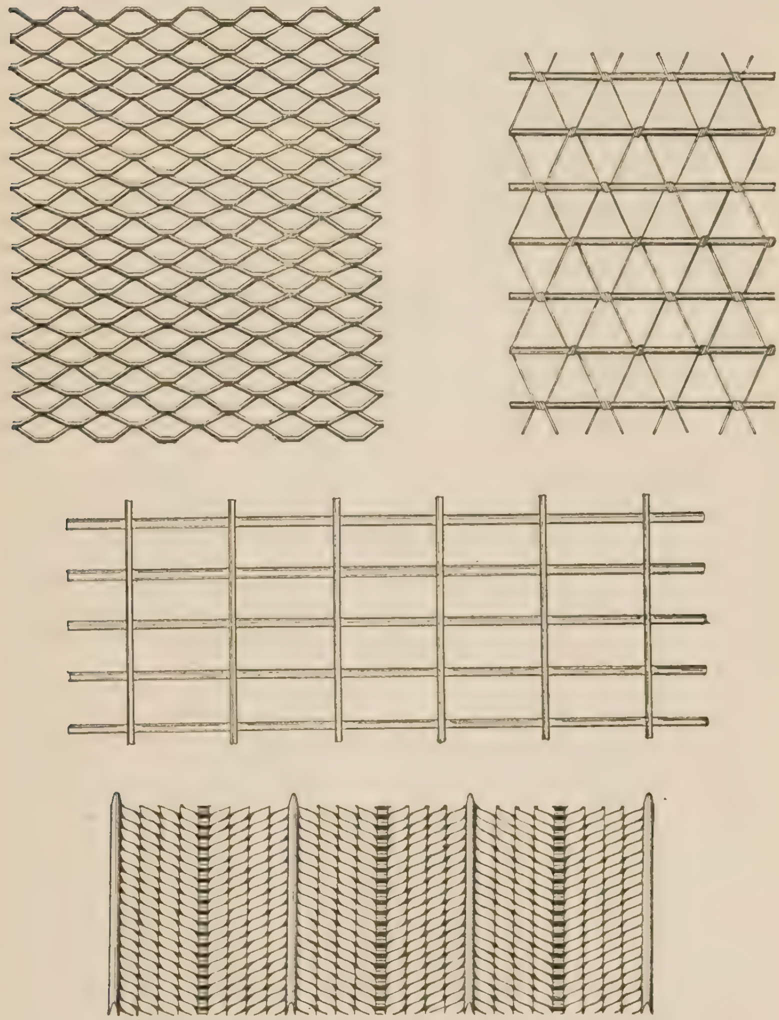


FIG. 7.

should have an ultimate strength of from 55,000 to 70,000 lb. per sq. in. The intermediate grade should range from 70,000

to 85,000 lb. per sq. in., and the hard grade should have an ultimate strength of 80,000 lb. per sq. in. or greater.

43. Coefficient of Expansion. The coefficient of expansion of steel is approximately 0.0000065 per degree Fahrenheit, and this value may be used in all design.

44. Modulus of Elasticity. The modulus of elasticity of all grades and kinds of steel is nearly the same, and may be taken as 30,000,000 lb. per sq. in.

Advantages of Concrete and Steel in Combination

45. Since concrete is only about one-tenth as strong in tension as in compression, it cannot be used economically by itself for the construction of any member sustaining or likely to sustain flexural stresses. Its compressive strength is sufficiently high to be of structural importance, and it is a good fire-proof material; it is durable, and materials for its manufacture can be obtained in almost any locality.

Steel, on the other hand, when not embedded in concrete, cannot withstand successfully great heat, and is subject to corrosion. Its tensile strength is high in almost any shape of section. To resist compression by itself it must be made in forms of less concentrated cross-section than the bar, in order to have lateral rigidity.

When the two materials are so arranged in a structural member, subject to both tension and compression, that the steel will resist the tension and the concrete the compression, the greatest advantage over other types of construction occurs. The member is more fire-proof than one constructed of steel or timber alone, and is often more economical. The steel is used in its cheapest form, the bar, and protected by its covering of concrete. In compression members such as columns, the use of the steel is not so economical, but a reinforced concrete column is again the most permanent and fire-proof construction obtainable. Columns of structural steel incased in concrete may be as durable, but the initial cost is greater. Plain concrete columns are not safe construction on account of the possible bending or shearing forces which may develop.

Bond between the Concrete and the Steel

46. All reinforced concrete construction is based on the assumption that the two materials are thoroughly bonded together. The high value of the adhesion of concrete to steel rods embedded in it was known long before the days of reinforced concrete, and use of this property was made in anchor bolts, rods, etc. Most of the tests of bond have been made by embedding a short reinforcing bar in a block or cylinder of concrete and pulling it out in a testing machine. In such tests the concrete surrounding the bar is in compression, and the conditions do not correspond to those ordinarily existing in beams or slabs. Other tests have been made with the two bars embedded, one in each end, of a concrete cylinder, and tension applied to each rod to determine the bond stress. Still other tests have been made with rods embedded in small reinforced concrete beams, the middle portion of the rods being left exposed. The results of tests of different types seem to show that a correct value of the bond resistance can be obtained by properly made tests of the simple kind first mentioned.

From an extensive series of bond tests made at the University of Illinois[2] conclusions were reached as follows:

Bond between concrete and steel may be divided into two principal elements, adhesive resistance and sliding resistance. The source of adhesive resistance is not known, but its presence is a matter of universal experience with materials of the nature of mortar and concrete. Sliding resistance arises from inequalities of the surface of the bar and irregularities of its section and alignment together with the corresponding conformations in the concrete. The adhesive resistance must be overcome before sliding resistance comes into action. In other words, the two elements of bond resistance are not effective at the same time at a given point. Many evidences of the tests indicate that adhesive resistance is much the more important element of bond resistance.

Pull-out tests with plain bars show that a considerable bond stress is developed before a measurable slip occurs. After the adhesive resistance is overcome, a further slip without an opportunity of rest is accompanied by a rapidly increasing bond

[2] *Bulletin* 71, Engineering Experiment Station, University of Illinois.

stress until a maximum bond resistance is reached at a definite amount of slip.

The true relation of slip of bar to bond stress can best be studied by considering the action of a bar over a very short section of the embedded length. The difficulties arising from secondary stresses made it impracticable to conduct tests on bars embedded very short lengths. The desired results were obtained by varying the forms of the specimens in such a way that the effect of different combinations of dimensions could be studied.

Pull-out tests with plain bars of the same size embedded different lengths furnish data which suggest the values of bond resistance over a very short length of embedment, or indicate values of bond resistance which are independent of the length of embedment. Tests with bars of different size, which were embedded a distance proportional to their diameters, give the true relation when the effect of size of bar is eliminated. Two series of tests of this kind on plain round bars of ordinary mill surface gave almost identical values for bond resistance after eliminating the effect of length of embedment and size of bar, and we may consider that these values represent the stresses which were developed in turn over each unit of area of the embedded bar as it was withdrawn by a load applied by the method used in these tests. These tests showed that for concrete of the kind used (a 1:2:4 mix stored in damp sand and tested at the age of about 60 days) the first measurable slip of bar came at a bond stress of about 260 lb. per sq. in., and that the maximum bond resistance reached an average value of 440 lb. per sq. in. Concluding that the adhesive resistance was overcome at the first measurable slip, it will be seen that the adhesive resistance was about 60 per cent of the maximum bond resistance. This ratio did not vary much for a wide range of mixes, ages, size of bar, condition of storage, etc.

Sliding resistance reached its maximum value for plain bars of ordinary mill surface at a slip of about .01 in. The constancy in the amount of slip corresponding to the maximum bond resistance for a wide range of mixes, ages, sizes of bar, conditions of storages, etc. is a noteworthy feature of the tests. With further slip the sliding resistance decreased slowly at first, then

more rapidly, until with a slip of .1 in. the bond resistance was about one-half its maximum value.

Bond Resistance in Terms of Compressive Strength of Concrete. Pull-out tests with plain round bars show end slip to begin at an average bond stress equal to about one-sixth the compressive strength of 6-in. cubes from the same concrete; the maximum bond resistance is equal to about one-fourth the compressive strength of 6-in. cubes. These values were about the same for a wide range of mixes, ages, and conditions of storage. In terms of the compressive strength of 8- by 16-in. concrete cylinders these values would be about 13 per cent for first end slip and 19 per cent for the maximum bond resistance.

Distribution of Bond Stress along a Bar. The tests indicate that bond stress is not uniformly distributed along a bar embedded any considerable length and having the load applied at one end. Slip of bar begins first at the point where the bar enters the concrete, and the bond stress must be greater here than elsewhere until a sufficient slip has occurred to develop the maximum bond resistance at this point. Slip of bar begins last at the free end of the bar. After slip becomes general, there is an approximate equality of bond stress throughout the embedded length.

Variation of Bond Resistance with Size, Shape, and Condition of Surface of Bar. The maximum bond resistance was not materially different for bars of different diameters.

Rusted bars gave bond resistances about 15 per cent higher than similar bars with ordinary mill surface.

The tests with flat bars showed wide variations of bond resistance and were not conclusive. Square bars gave values of unit stress about 75 per cent of those obtained with plain round bars.

T-bars gave lower unit bond resistance than plain round bars, but gave about double the bond resistance per unit of length than was found for the plain round bars of the same sectional area.

With polished bars the bond resistance is due almost entirely to adhesion between the concrete and the steel. Numerous tests with polished bars embedded in 1:2:4 concrete and tested at 60 days indicated a maximum bond resistance of about 160 lb.

per sq. in. or about 60 per cent of the bond resistance of bars of ordinary surface at small amounts of slip.

Adhesive resistance must be destroyed, sliding resistance largely overcome, and the concrete ahead of the projections must undergo an appreciable compressive deformation before the projections on a deformed bar become effective in taking bond stress. The tests indicate that the projections do not materially assist in resisting a force tending to withdraw the bar until a slip has occurred approximating that corresponding to the maximum sliding resistance of plain bars. As slip continues a larger and larger portion of the bond stress is taken by direct bearing of the projections on the concrete ahead.

In determining the comparative merits of deformed bars, the bars which longest resist beginning of slip should be rated highest, other considerations being equal.

The concrete cylinders of the pull-out specimens with deformed bars were reinforced against bursting or splitting, because it was desired to study the load-slip relation through a wide range of values. In only a few tests was the maximum bond resistance reached at an end slip less than .1 in. It should be recognized that, in general, the bond stresses reported for deformed bars at end slip of .05 and .1 in. could not have been developed with bars embedded in unreinforced blocks. These high values of bond resistance must not be considered as available under the usual conditions of bond action in reinforced-concrete members. In the tests in which the blocks were not reinforced, evidence of splitting of the blocks was found at end slips of .02 to .05 in.

The normal components of the bearing stresses developed by the projections on a deformed bar may produce very destructive bursting stresses in the surrounding concrete. The bearing stress between the projections and the concrete in the tests with certain types of commercial deformed bars was computed to be from 5800 to 14,000 lb. per sq. in. at the highest bond stresses considered in these tests. The large slip and the high bearing stresses developed in the later stages of the tests show the absurdity of seriously considering the extremely high values that are usually reported to be the true bond resistance of many types of deformed bars.

Round bars with standard V-shaped threads gave much higher bond resistance at low slips than the commercial deformed bars. The average bond resistance at an end slip of .001 in. was 612 lb. per sq. in. The maximum bond resistance was 745 lb. per sq. in. These were the only deformed bar tests in which failure came by shearing the surrounding concrete.

The 1-in. twisted square bars gave a bond resistance per unit of surface at an end slip of .001 in. only 88 per cent of that for the plain rounds. Following an end slip of about .01 in., these bars showed a decided decrease in bond resistance, and a slip of five to ten times this amount was required to cause the bond resistance to regain its first maximum value. After this, the bond resistance gradually rose as the bar was withdrawn. Some of the bars were withdrawn 2 or 3 in. before the highest resistance was reached. The apparent bond stresses at these slips were very high; but, of course, such stresses and slips could not be developed in a structure and could not have been developed in the tests had the blocks not been reinforced against bursting. Such values are entirely meaningless under any rational interpretation of the tests.

Anchoring of Reinforcing Bars. The tests with plain round bars, anchored by means of nuts and with washers, only showed that the entire bar must slip an appreciable amount before these forms of anchorage come into action. Anchorages of the dimensions used in these tests did not become effective until the bar had slipped an amount corresponding to the maximum bond resistance of plain bars. With further movement the apparent bond resistance was high, but was accompanied by excessive bearing stresses on the concrete.

Influence of Method of Curing Concrete. Tests on specimens stored under different conditions indicate that concrete stored in damp sand may be expected to give about the same bond resistance and compressive resistance as that stored in water. Water-stored specimens gave values of maximum bond resistance higher in each instance than the air-stored specimens; the increase for water storage ranged from 10 to 45 per cent. The difference seemed to increase with age. The presence of water not only did not injure the bond for ages up to 3 years, but it was an impor-

tant factor in producing conditions which resulted in high bond resistances. It was found, however, that specimens tested with the concrete in a saturated condition gave lower values for bond than those which had been allowed to dry out before testing. The bars in specimens which had been immersed in water as long as 3½ years showed no signs of rust or other deterioration.

Influence of Freezing of Concrete. Specimens made outdoors in freezing weather, where they probably froze and thawed several times during the period of setting and hardening, were almost devoid of bond strength.

Influence of Age and Mix of Concrete. Pull-out tests made at early ages gave surprisingly high values of bond resistance. Plain bars embedded in 1:2:4 concrete and tested at 2 days did not show end slip of bar until a bond stress of 75 lb. per sq. in. was developed. Bond resistance increases most rapidly with age during the first month. The richer mixes show a more rapid increase than the leaner ones. The tests on concrete at ages of over 1 year showed that the bond resistance of specimens stored in a damp place may be expected ultimately to reach a value as much as twice that developed at 60 days.

The load-slip relation of leaner and richer mixes was similar to that for 1:2:4 concrete. For a wide range of mixes the bond resistance was nearly proportional to the amount of cement used. This relation did not obtain in a mix from which the coarse aggregate had been omitted.

Effect of Continued and Repeated Load. When the application of load was continued over a considerable period of time or when the load was released and reapplied, the usual relation of slip of bar to bond resistance was considerably modified. The few tests which were made indicate that the bond stress corresponding to beginning of slip is the highest stress which can be maintained permanently or be reapplied indefinitely without failure of bond.

Effect of Concrete Setting under Pressure. Bond resistance of plain bars is greatly increased if the concrete is caused to set under pressure. With a pressure of 100 lb. per sq. in. on the fresh concrete for 5 days after molding, the maximum bond resistance was increased 92 per cent over that of similar bars in

concrete which had set without pressure. The greater density of the concrete and its more intimate contact with the bar seem to be responsible for the increased bond resistance. Light pressure gave an appreciable increase in bond resistance. With polished bars the effect of pressure was slight.

As might have been expected, the compressive resistance of concrete setting under pressure was increased in much the same ratio as the bond resistance. At the age of 80 days the initial modulus of elasticity in compression for concrete which set under a pressure of 100 lb. per sq. in. was about 37 per cent higher and the compressive strength was increased by about 73 per cent over that of concrete which had set without pressure. The density of the concrete, as determined by the unit weights, was increased about 4 per cent by a pressure of 100 lb. per sq. in. on the fresh concrete. The increase in strength and density was relatively greater for the low than for the high pressure. A pressure continued for 1 day, or until the concrete had taken its final set and hardening had begun, seems to have produced the same effect in increasing the strength and elastic properties of the concrete as when the pressure was continued for a much longer period.

The comparison of the bond stresses developed in beams and in pull-out specimens from the same materials is of interest. Such a comparison should be made for similar amounts of slip. In the pull-out tests the maximum bond resistance came at a slip of about .01 in. for plain bars. The mean bond resistance for the deformed bars tested was not materially different from that of the plain bars until a slip of about .01 in. was developed; with a continuation of slip the projections came into action and with much larger slip high bond stresses were developed. The beam tests showed that about 79 to 94 per cent of the maximum bond resistance was being developed when the bar had slipped to .001 in. at the free end; hence the bond stress developed at an end slip of .001 in. was used as a basis of the principal comparisons in the pull-out tests. It is recognized, however, that, under certain conditions, the stresses developed at larger amounts of slip may have an important bearing on the effective bond resistance of the bar.

The pull-out tests and beam tests gave nearly identical bond stresses for similar amounts of slip in many groups of tests, but it seems that this was the result of a certain accidental combination of dimensions in the two forms of specimens and it did not indicate that the computed stresses in the beams were the correct stresses. It is believed, however, that a properly designed pull-out test does give the correct value of bond resistance, and gives values which closely represent the bond stresses which actually exist in a beam or other member as slipping is produced from point to point along the bar. The relative position of the bar during molding may be expected to influence the values of bond resistance found in the tests.

A working bond stress equal to 4 per cent of the compressive strength of the concrete tested in the form of 8- by 16-in. cylinders at the age of 28 days (equivalent to 80 lb. per sq. in. in concrete having a compressive strength of 2000 lb. per sq. in.) is as high a stress as should be used. This stress is equivalent to about one-third that causing the first slip of bar and one-fifth the maximum bond resistance of plain round bars as determined from pull-out tests. The use of deformed bars of proper design may be expected to guard against local deficiencies in bond resistance due to poor workmanship and their presence may properly be considered as an additional safeguard against ultimate failure by bond. It does not seem wise, however, to place the working bond stress for deformed bars much higher than that used for plain bars.

The Joint Committee recommends a bond stress of .04 of the ultimate compressive strength for plain bars and .05 for deformed bars.

47. Length of Embedment of Reinforcing Bars to Develop Full Strength in Bond.

Let

f_s = allowable unit tensile stress in the steel

A_s = the area of the bar

o = the circumference or perimeter of bar

i = diameter or thickness of bar

u = allowable unit bond stress between the concrete and the steel

x = required length of embedment

For round bars

$$xou = A_s f_s \text{ or } \pi x i u = \frac{\pi i^2 f_s}{4}$$

For square bars $\qquad 4iux = i^2 fs$

For any bar $\qquad x = \dfrac{f_s i}{4u}$

48. Reinforced Concrete in Tension. Early tests of reinforced concrete seemed to indicate that the ultimate strength in tension was far greater than that of plain concrete. This was due to the fact that the bond between the concrete and the steel causes a uniform stretching of the concrete, and the cracks which occur when the concrete is stressed are so numerous and minute as to be difficult to detect when they first begin to open up, and do not become visible until a stretching occurs corresponding to a tensile stress much greater than the ultimate strength of concrete.

A reinforced concrete beam carrying its design load is more heavily stressed on the tension side than the ultimate strength of plain concrete, provided enough steel is embedded on the tension side to develop the full allowable compressive strength of the concrete. The presence of the cracks above referred to, therefore, greatly decrease the tension that can be taken by the concrete, and most moment formulas now in use for the design of reinforced concrete beams neglect entirely the tensile strength of the concrete.

The effect of temperature and moisture changes on plain concrete is discussed in Arts. 18 and 36. If a structure having a large area of exposed surface is restrained by outside forces, these changes cause stresses to be set up in the concrete which will in turn cause cracks to appear on the exposed surface. In order to prevent the appearance of large and unsightly cracks, such surfaces should be reinforced with sufficient steel (generally about ¼ of 1 per cent of the cross-section of the concrete) to cause the stretching due to the tension in the concrete to be distributed uniformly over the whole surface, and thus make the cracks so numerous as to be invisible.

CHAPTER III

BEAMS AND SLABS

49. Stresses in Homogeneous Beams. Before commencing any discussion of reinforced concrete beams, a summary of the principles relating to homogeneous beams should be thoroughly reviewed. Briefly, these may be set forth as follows:

1. At any cross-section there exist external forces which may be resolved into components normal and tangential to the section. These components which are normal to the section are stresses of tension and compression; their function is to resist the bending moment at the section. The tangential components added together constitute a stress known as the resisting shear.

2. The neutral axis passes through the center of gravity of the cross-section.

3. The intensity of stress normal to the section increases directly with the distance from the neutral axis, and is a maximum at the extreme fiber. The intensity of stress at any given point in the cross-section is represented by the formula $f = My/I$ in which

f = the unit fiber stress at a distance y in. from the neutral axis.
M = the external bending moment at the section in inch-pounds.
I = the moment of inertia of the cross-section about the neutral axis.

4. The longitudinal shear in pounds per square inch (v) at any point in the cross-section is given by the equation $v = VQ/Ib$ in which

V = the total shear at the section in pounds.
Q = the statical moment about the neutral axis of that portion of the cross-section lying between an axis through the point in question parallel to the neutral axis, and the nearest face (upper or lower) of the beam.
I = the moment of inertia of the cross-section about the neutral axis.

b = the width of the beam at the given point.

The statical moment mentioned above is the product of the area of the portion considered and the distance of its center of gravity from the neutral axis.

5. In a beam with constant cross-section, the maximum values of f and v will occur where M and V, respectively, are a maximum.

6. At any point in the beam there exists a vertical shear, the intensity of which is equal to that of the longitudinal or horizontal shear.

7. The intensity of shear (horizontal and vertical) along a vertical cross-section in a rectangular beam varies as the ordinates of a parabola, the intensity being zero at the top and bottom of the beam and a maximum at the neutral axis. The maximum is $\frac{3}{2}$ the average intensity or $\frac{3}{2} \times \frac{V}{bd}$, since at the neutral axis $Q = \frac{bd^2}{8}$ and $I = \frac{bd^3}{12}$, in the equation $v = \frac{QV}{Ib}$.

8. Due to the action of shearing forces (horizontal and vertical) and flexure stresses, there exist, at any point in a beam, inclined stresses of tension and compression, the maximum values of which form an angle of 90 degrees with each other. The intensity of the inclined stress at any point is given by the equation $t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2}$ in which

f = the intensity of horizontal fiber stress.

v = the intensity of vertical or horizontal shearing stress at the point.

The inclined stress makes an angle α with the horizontal of such an amount that $\tan 2\alpha = \frac{2v}{f}$.

9. Since the horizontal and vertical shearing forces are equal and the flexural stresses zero at the neutral plane, the inclined tensile and compressive forces at any point in that plane form an angle of 45 degrees with the horizontal, the intensity of each being equal to the unit shear at the point. At the end of a simply supported beam where the bending moment is zero, these stresses act at practically 45 degrees with the horizontal for the entire

depth of the beam. Since the shear is zero at the point of maximum moment, the stresses here are horizontal.

50. Assumptions in the Theory of Flexure. The common theory of flexure assumes:

1. A plane cross-section before loading remains a plane cross-section after loading.
2. The stress is proportional to the deformation.

The first of these two assumptions implies that the unit deformations of the fibers at any section are proportional to their distance from the neutral axis, and the second that the unit stresses in the fibers vary as the distances of the fibers from the neutral axis.

The common theory of flexure does not apply for wide ranges of stress. In the design of structures, however, the stresses used are only a comparatively small percentage of the ultimate, and the errors in the above assumptions are small and on the side of safety. For stresses in excess of those commonly used in design, the relation between stress and deformation is not constant; the stress-deformation diagram for such cases assumes the form of a parabola.

In the following discussion, unless exception is noted, a straight-line variation between stress and deformation is assumed. Furthermore, the tensile strength of the concrete is neglected.

51. Plain Concrete Beams. Plain concrete beams are inefficient as flexual members since failure on the tension side of the beam occurs when but a small portion of the ultimate compressive strength of the concrete has been developed on the compressive side of the beam. The strength of a plain concrete beam may be expressed by the equation $M = \frac{f_t I}{e}$ in which

f_t = the working unit stress of concrete in tension.

e = the distance from the neutral axis to the extreme tensile fiber.

Illustrative Problem. How great a moment can be developed by a plain rectangular concrete beam whose cross-section is 8 × 14 in. if the safe working stress of concrete in tension is 125 lb. per sq. in.?

$$M = \frac{f_t I}{e} = \frac{f_t b a^2}{6} = \frac{125 \times 8 \times \overline{14}^2}{6} = 32{,}670 \text{ in.-lb.}$$

This assumes that the neutral axis is in the middle of the beam. On account of the inequality of strength of concrete in tension and compression this assumption is not theoretically correct, but for purposes of design usually gives satisfactory results.

Rectangular Reinforced Beams and Slabs

52. Flexure Formulas. The tensile and transverse strength of plain concrete is very low and unreliable (see Art. 48). Its practical uses are limited to structures or parts of structures in which no tensile stresses are induced, *i.e.*, to arches, piers, and certain massive constructions. In order to make concrete available for use in structural members involving tension, such as beams, for example, steel rods are embedded in the tension side of the beam. It is, of course, assumed that the rods are embedded so that the union between the steel and concrete

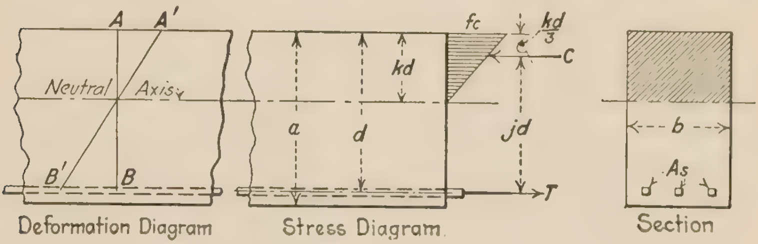


FIG. 8.

is sufficient to make the two materials act as one. The purpose of the steel is to carry the tensile stresses. The concrete sustains the compressive and shearing stresses, because its resistance to these is comparatively large.

Figure 8 represents a portion of a rectangular reinforced concrete beam. Let AB represent any cross-section before the load is applied to the beam, and $A'B'$ the same cross-section after the load is applied. AA' represents the shortening of the extreme upper fiber per unit of length, and BB' the unit elongation of the steel. In the stress diagram, f_c represents the unit compressive stress in the extreme fiber at the section AB. The total compression $C = (\frac{1}{2}f_ckd)b = \frac{1}{2}f_ckbd$, and the total tension, neglecting that in the concrete (see Art. 50), is $T = A_sf_s$, in which f_s is the

unit stress in the steel and A_s the cross-sectional area of the steel. For the general notation used in the following discussion, see Appendix A.

For equilibrium the total compressive resistance of a beam must equal the total tensile resistance. From Fig. 8

$$\tfrac{1}{2} f_c k d b = A_s f_s \qquad (a)$$

From the assumption that deformations vary as the distance from the neutral axis

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} \qquad (b)$$

Since $E = \dfrac{\text{unit stress}}{\text{unit deformation}}$

$$AA' = \frac{f_c}{E_c} \text{ and } BB' = \frac{f_s}{E_s}$$

and

$$\frac{AA'}{BB'} = \frac{nf_c}{f_s} \qquad (c)$$

Equating (*b*) and (*c*)

$$\frac{nf_c}{f_s} = \frac{kd}{d - kd} \qquad (d)$$

from which

$$f_s = \frac{nf_c(1 - k)}{k} \text{ or } f_c = \frac{f_s k}{n(1 - k)} \qquad (1)$$

This equation gives the relation between the actual stresses in the steel and the concrete in any beam at any stage of loading, provided the value of k is known.

The actual percentage of steel, p, in terms of the effective cross-section of the concrete is

$$p = \frac{A_s}{bd}$$

Hence from equation (*a*),

$$k = \frac{pf_s}{\tfrac{1}{2} f_c} \qquad (e)$$

Substituting the value of f_s from equation (1)

$$k = \frac{2pn(1 - k)}{k}$$

Solving for k,

$$k = \sqrt{2pn + \overline{pn}^2} - pn \qquad (2)$$

This value of k is independent of the unit stresses in the steel and concrete but is dependent upon the proportion of steel in

the beam and the ratio of the moduli of elasticity of the two materials. It is to be used in reviewing, *i.e.*, in calculating unit stresses or resisting moments of a beam whose dimensions and amount of reinforcement are known.

From Fig. 8,

$$jd = d - \frac{kd}{3},$$

or

$$j = 1 - \frac{k}{3} \tag{3}$$

The resisting moment of a beam is dependent upon the strength of both the steel and the concrete. The resisting moment of each is equal to the total stress in each, *i.e.*, compression in concrete and tension in steel, multiplied by the lever arm of the couple, jd, or

$$M_c = (\tfrac{1}{2}f_c kbd)jd = \tfrac{1}{2}f_c kjbd^2 \tag{4}$$

$$M_s = A_s f_s jd \tag{5}$$

and since $A_s = pbd$, M_s may also be written as

$$M_s = pf_s jbd^2.$$

The internal moments in steel and concrete are each equal to the external bending moment at all stages of loading (for equilibrium), but if the maximum allowable value of the resisting moment of the concrete, M_c, is reached before that of the steel, M_s, it means that the beam will fail in compression before the maximum fiber stress in the steel is reached; *i.e.*, the beam has more steel than is theoretically required—it is overreinforced.

In designing a reinforced concrete beam, it is desirable to place in the beam an amount of steel such that the limiting unit stresses or limiting resisting moments as expressed by equations (4) and (5) shall be reached simultaneously. If this ideal steel ratio is obtained

$$M_c = M_s = \tfrac{1}{2}f_c kjbd^2 = A_s f_s jd = pf_s jbd^2$$

or,

$$M = Kbd^2 \tag{6}$$

in which $K = \tfrac{1}{2}f_c kj$ or $pf_s j$.

If the ratio $\frac{f_s}{f_c} = r$, equation (d) reduces to the form $\frac{n}{r} = \frac{k}{1 - k}$ and by solving for k

$$k = \frac{n}{n + r} \tag{7}$$

This value of k depends only upon the unit stresses in the steel and in the concrete and upon the value of the ratio n. *Therefore it cannot be used in review,* since the *simultaneous* values of f_s and f_c are not known.

An expression for the ideal steel ratio may be obtained as follows: Since with this ideal percentage of steel $\frac{1}{2}f_ckj = pf_sj$, then $p = \frac{k}{2r}$, and since for any given values of f_s and f_c, $k = \frac{n}{n + r}$ the value of p becomes

$$p = \frac{n}{2r(n + r)} \tag{8}$$

The values of M_c and M_s will be equal only when the amount of steel placed in the beam is such that the actual steel ratio, $p = \frac{A_s}{bd}$, equals the quantity given by equation (8). On account of the commercial sizes of reinforcing steel in use, the actual ratio will be greater or less than the ideal. In the former case M_s will be greater than M_c and the strength of the beam will be limited by that of the concrete. For underreinforced beams (p less than the ideal ratio) the reverse will be true.

The value of the external bending moment in each case varies according to the method of supporting the beams and the type of loading. For example, a simply supported beam, *i.e.*, one resting on two supports, one at each end, and not restrained in any way, with a uniformly distributed load, may be assumed as having a moment equal to $\frac{1}{8}wl^2$; a partially continuous beam (continuous over one support only), with the same type of loading $\frac{1}{10}wl^2$; and a fully continuous beam (continuous over two or more supports) $\frac{1}{12}wl^2$ in which w = the load per unit of length and l = the span. For other loadings and methods of support see Chapter VI.

The span length l of freely supported beams and slabs is generally taken as the distance between the centers of supports, but need not exceed the clear span plus the depth of the beam or slab. The span length for continuous or restrained members built monolithically with the supports is often considered as the clear distance between faces of supports. Many designers use the distance between the centers of supports, as the effective span length, both for continuous and simply supported beams.

53. The equations previously developed are summarized below.

$$f_s = \frac{nf_c(1-k)}{k} \text{ or } f_c = \frac{f_s k}{n(1-k)} \tag{1}$$

$$k = \sqrt{2pn + pn^2} - pn \qquad \text{(review)} \tag{2}$$

$$j = 1 - \frac{k}{3} \tag{3}$$

$$M_c = \tfrac{1}{2} f_c kjbd^2 \tag{4}$$

$$M_s = A_s f_s jd = pf_s jbd^2 \tag{5}$$

$$M = Kbd^2 \text{ in which } K = \tfrac{1}{2} f_c kj \text{ or } pf_s j \tag{6}$$

$$k = \frac{n}{n+r} \qquad \text{(design only)} \tag{7}$$

$$p = \frac{n}{2r(n+r)} \tag{8}$$

It is to be noted that in designing, either equation (4) or (6) may be used to determine the cross-section required to insure against crushing of the concrete under any given bending moment, and equation (5) to determine the area of steel necessary to develop the full strength of the concrete in compression.

The above equations refer to flexural stresses only (tension and compression) and do not provide for the shearing stresses that exist in the beam. These are considered separately in Arts. 65 to 82.

Economic and constructional considerations are usually best served when the cross-section of the beam is so proportioned that b is from one-half to three-quarters of d. Also, in order to keep the amount of mill or carpenter work as small as possible, the width of beam b should be chosen so that a plank of standard width may be used for the bottom form. In fulfilling this requirement, most designers consider the nominal width of the plank and hence proportion all beams for widths of even integral inches. Some designers, however, prefer to consider the actual width of the lumber available after dressing and proportion their beams accordingly.

In the problems throughout this work the nominal width, only, is considered.

54. Placing the Reinforcement. In placing the reinforcement, three general requirements must be fulfilled. First, there must

be sufficient space between the rods to permit proper placing of the concrete around them; second, there must be sufficient section along the plane of the rods properly to transmit the stresses of tension and shear; third, there must be sufficient concrete below the steel to afford ample protection for the steel.

It is advisable in this connection to follow the recommendations of the Joint Committee. They require that "the minimum clear distance between parallel bars shall be 1½ times the diameter of round bars or 1½ times the diagonal of square bars, . . . but in no case shall the spacing between bars be less than 1 in. . . . Metal reinforcement in wall footings and column footings shall have a minimum covering of 3 in. of concrete. At surfaces of concrete exposed to the weather, metal reinforcement shall be protected by not less than 2 in. of concrete . . . Metal reinforcement in fire-resistive construction shall be protected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders, and columns . . . In structures where the fire hazard is limited, the metal reinforcement shall not be placed nearer the exposed surface than ¾ in. in slabs and walls or 1½ in. in beams, girders, and columns.

The following values of the depth of concrete below the steel in beams and girders are used in general practice: 1 in. in the clear for beams whose depth is lessthan 10 in., 1½ in. for depths between 10 and 20 in., and 2 in. for depths greater than 20 in. Where two or more layers of bars are used, there should be a minimum clear distance between the rows of 1 in.

55. Allowable Unit Stresses. The allowable unit working stresses in the concrete as specified by the Joint Committee (see Art. 3) are given in Appendix B. These stresses are given as a percentage of the ultimate compressive strength, f'_c, of the concrete at the age of 28 days. Thus, the safe working unit stress in flexure is $0.40\ f'_c$, which gives, for a 2000-lb. concrete, a value of 0.40×2000, or 800 lb. per sq. in. This is somewhat in excess of the working stress that has been in general use for a concrete of this strength. While the Joint Committee's recommendations may be followed with safety if the work is done with careful attention to the details of proportioning and placing, a somewhat lower unit stress in flexure is specified in many munici-

pal building codes. To emphasize this fact, some of the following problems are solved with assumed working stresses which do not agree with the recommendations as given in Appendix B. Where such exception is taken, the assumed allowable unit stresses are given in the data of the problem.

56. *Illustrative Problems.*

I. A rectangular reinforced concrete beam has a total cross-section of 8 × 14 in. and a length of 20 ft.-0 in. It is reinforced with four ½-in. square bars in one row, the centers of the bars being 1½ in. above the lower surface of the beam. Assuming a 2000-lb. concrete, and following the recommendations of the Joint Committee, Appendix B, what is the resisting moment of the beam?

$$p = \frac{A_s}{bd} = \frac{4 \times .25}{8 \times 12.5} = .0100$$

$$k = \sqrt{2 \times .01 \times 15 + (.01 \times 15)^2} - .01 \times 15 = .418$$

$$j = 1 - \frac{.418}{3} = .861$$

$$M_c = \tfrac{1}{2} \times 800 \times .418 \times .861 \times 8 \times \overline{12.5}^2 = 180{,}300 \text{ in.-lb.}$$

$$M_s = 1.00 \times 16{,}000 \times .861 \times 12.5 = 172{,}500 \text{ in.-lb.}$$

Therefore the beam is underreinforced, the strength of the steel governs, and the resisting moment of the beam is 172,500 in.-lb.

II. Use the beam of the preceding problem and find the value of the unit stresses in the steel, f_s, and in the concrete, f_c, if a uniform live load of 175 lb. per lin. ft. is applied to it.

The weight of the beam $= \dfrac{8 \times 14}{144} \times 150 = 115$ lb. per ft. The total load carried by the beam = 175 + 115 = 290 lb. per ft. The actual external bending moment is

$$M = \frac{290 \times \overline{20}^2 \times 12}{8} = 174{,}000 \text{ in.-lb.}$$

$k = .418$ and $j = .861$ as in the preceding problem.

$$174{,}000 = 1.00 \times f_s \times .861 \times 12.5$$

$$f_s = 16{,}150 \text{ lb. per sq. in.}$$

$$f_c = \frac{.418 \times 16{,}150}{15(1 - .418)} = 775 \text{ lb. per sq. in.}$$

This value for f_c could also have been found from the equation for the resisting moment of the concrete.

III. Determine the cross-section of concrete and area of steel required for a simply supported rectangular beam with a span of 18 ft.-0 in. which is to carry a live load of 375 lb. per lin. ft. Assume the allowable unit stress in the concrete as 650 lb. per sq. in., that in the steel as 16,000 lb. per sq. in., and the ratio n as 15.

Assume weight of beam as 225 lb. per lin. ft. Total load to be carried = 600 lb. per lin. ft. Actual external bending moment is

$$M = \frac{600 \times \overline{18}^2 \times 12}{8} = 292{,}000 \text{ in.-lb.}$$

$$r = \frac{16{,}000}{650} = 24.6$$

$$k = \frac{15}{15 + 24.6} = .379$$

$$j = 1 - \frac{.379}{3} = .874$$

$$292{,}000 = \tfrac{1}{2} \times 650 \times .379 \times .874 \times bd^2$$

from which $bd^2 = 2710$ in.^{3}

Since the best proportioned rectangular beam is one in which b equals $\frac{1}{2}$ to $\frac{3}{4}$ of d, and since b should be kept in even inches in order to simplify the form work, b is taken as 10 in. and d as 16.5 in. By adding $2\frac{1}{2}$ in. below the center of the steel, the total cross-section is 10 $\times$ 19 in., giving a weight of

$$\frac{10 \times 19}{144} \times 150 = 200 \text{ lb. per lin. ft.}$$

Since this does not agree with the assumed value, it becomes necessary to check back to see whether any revision should be made in the design. The revised bending moment equals

$$M = \tfrac{1}{8} \times 575 \times \overline{18}^2 \times 12 = 280{,}000 \text{ in.-lb.}$$

$$280{,}000 = \tfrac{1}{2} \times 650 \times .379 \times .874 \times bd^2$$

$$bd^2 = 2600 \text{ in.}^3$$

To satisfy this, $b = 10$ in., $d = 16.1 = 16.5$ in., using the next $\frac{1}{2}$ in. above the theoretical. Since these results agree with those assumed in the revision, the design is satisfactory.

With these values of b and d

$$280{,}000 = A_s f_s jd \text{ from which}$$
$$A_s = 1.21 \text{ sq. in.}$$

Four $\frac{5}{8}$-in. round rods, area 1.23, in one row, are selected.

If the ideal steel ratio had been used in determining the quantity of steel to be placed in the beam, the procedure would have been as follows:

From equation (8) $p = \dfrac{15}{2 \times 24.6(15 + 24.6)} = .0077$

and $A_s = .0077 \times 10 \times 16.5 = 1.27$ sq. in.

This, it will be noticed, is slightly in excess of the area required as determined by the first method. This difference may be accounted for as follows: The value of A_s, as computed by this latter method, represents an area of steel that will develop, in tension, the full compressive strength of a beam whose effective dimensions are 10×16.5 in. But in order fully to develop a moment of 280,000 in.-lb. as required by the problem, an effective cross-section of only 10×16.1 in. was needed. The value of $d = 16.5$ in. was selected to simplify the dimensioning of the plans. The development of the full strength of a 10×16.5-in. beam therefore furnishes an excess of steel over that required to provide for the maximum external bending moment in the beam, and hence is a waste of material. In this case the four $\frac{5}{8}$-in. rods would not be sufficient. A greater difference between the theoretical value of d required and that furnished would emphasize this to a greater extent. The latter method, therefore, is not recommended for general use.

57. Tables for Rectangular Beam Problems. Many of the computations in the design and review of rectangular beams may be eliminated by the use of previously prepared tables. For example, in design, the value of $k = \dfrac{n}{n + r}$ depends only upon the ratio of the moduli of elasticity and the allowable unit stresses of the two materials. Table IV gives, for the most common values of n and for all practical combinations of f_s and f_c, the corresponding values of k as determined by the above equation.

Similar values of $j = 1 - \frac{k}{3}$, $K = \frac{1}{2}f_c kj = pf_s j$ (for use in the formula $M = Kbd^2$), and $p = \frac{n}{2r(n + r)}$ are also given.

In problems involving the design of rectangular beams, if for any reason it should be desirable or necessary to use a value of d greater than that theoretically required to provide for the bending moment, the value of j may still be found from Table IV. The arguments, however, are then f_s, the allowable unit stress in the steel, and K, the quotient of $\frac{M}{bd^2}$. The corresponding value of the unit concrete stress will also be given in this table. This will be less than the allowable, because an effective depth in excess of that required for moment was used.

Similarly, Table V, for use in the review of beams, gives the values of $k = \sqrt{2pn + \overline{pn}^2} - pn$, and $j = 1 - \frac{k}{3}$ for sufficient values of the variables p and n to make the solution of ordinary problems possible with but a slight amount of interpolation.

58. *Illustrative Problems Involving the Use of Tables.*

I. The use of Table IV in designing a beam may be shown by its application to Problem III of Art. 56.

From this table, for values of $n = 15$, $f_c = 650$, and $f_s = 16{,}000$

$$K = 107.7 \text{ and } j = .874$$

Therefore,

$$280{,}000 = 107.7bd^2 \text{ and}$$
$$bd^2 = 2600 \text{ in.}^3 \text{ as before.}$$

Selecting $b = 10$ in.

and $d = 16.5$ in.

$$280{,}000 = A_s \times 16{,}000 \times .874 \times 16.5 \text{ from which}$$
$$A_s = 1.21 \text{ sq. in.}$$

The use of the equation $M = Kbd^2$ is identical with the use of $M_c = \frac{1}{2}f_c kjbd^2$, since $K = \frac{1}{2}f_c kj$.

II. Problem I of Art. 56 may be solved by combining Tables IV and V. The problem might be reworded as follows: A rectangular reinforced concrete beam has a total cross-section of 8 × 14 in. and a length of 20 ft.-0 in. It is reinforced with four ½-in. square bars in one row, the centers of the bars being 1½ in. above

the lower surface of the beam. As the load is increased, which will fail first, theoretically, the concrete or the steel? *i.e.*, is the beam over- or underreinforced? What is the resisting moment of the beam? Use a 2000-lb. concrete, and allowable unit stresses as given by the Joint Committee.

The ideal percentage of steel required to give equal strength in tension and compression with the allowable unit stresses for a 2000-lb. concrete is given in Table IV as .0107.

The actual value of p in the beam $= \frac{1.0}{8 \times 12.5} = .0100$

The beam is, therefore, underreinforced, and its strength is limited by that of the steel.

Table V shows $k = .418$ and $j = .861$ and the resisting moment $M = 1.0 \times 16{,}000 \times .861 \times 12.5 = 172{,}500$ in.-lb.

Note: Table IV has been used *only* to determine the relative strength of the steel and the concrete in the beam, and *not* to determine the values of k and j.

59. Slabs. A slab is a rectangular beam of comparatively large ratio of width to depth. There are, however, certain modifications entering into the design and review of a slab which it was not necessary to consider in the solution of rectangular beam problems.

60. Slabs Supported on Two Sides Only. The simplest form of slab is one of indefinite length, supported only by two beams running its full length. From Fig. 9 it is seen that if a 12-in. strip of slab were cut out at right angles to the supporting beams, a rectangular beam 12 in. wide, with depth equal to the thickness of the slab, and length equal to the distance center to center of supports, would result. This strip may then be analyzed by the same formulas which were used in problems on rectangular beams, the bending moment being computed for a width of 1 ft. The load per square foot now becomes the load per linear foot on the imaginary beam. Since all of the load on the slab must be transmitted to the two supporting beams, it follows that all of the reinforcing steel should be placed at right angles to them, with the exception of any bars that may be placed in the other direction to take care of shrinkage and temperature stresses. A slab thus consists of a series of rectangular beams side by side.

The ratio of steel in a slab is most readily determined by dividing the sectional area of one bar by the area of concrete between two successive bars, the latter being the product of the depth to the center of the bars and the distance between them, center to center. The spacing required to furnish a certain area per foot of width is found by dividing the area of one bar by the area required per inch of width.

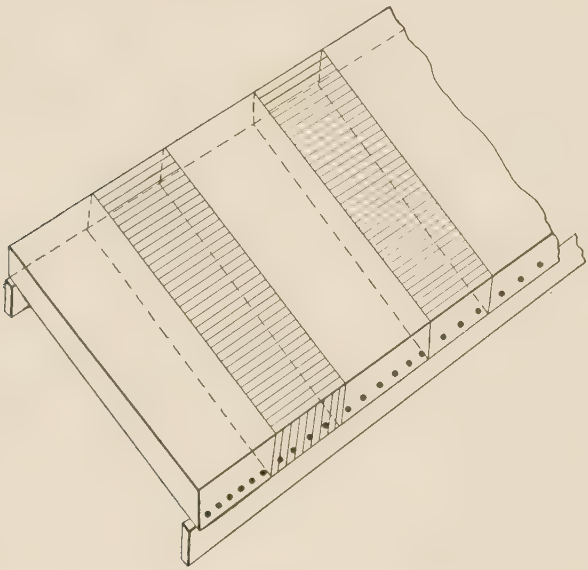


Fig. 9.

61. Slabs Supported on Four Sides. When a slab is square, or nearly so, and the nature of the construction makes it desirable to have a beam or girder along each side of the slab, it is advantageous to reinforce it in both directions. In the case of a square slab one-half of the total load is transmitted to each pair of beams or girders provided there is a uniform distribution of the load on the slab. If, however, one dimension of the slab is much greater than the other, so much of the load is transferred over the short direction to the supports that reinforcement in the long direction is of little value in carrying loads. Numerous tests bear out these conclusions. For slabs nearly square somewhat more than one-half of the load is transmitted in the short direc-

tion to the longer beams, and the remainder at right angles to the shorter beams. The usual assumption of load distribution is that the part, w_1, of the total load, w, which is transferred in the short direction may be represented by the equation

$$w_1 = \left(\frac{l}{l'} - \frac{1}{2}\right)w$$

in which l = the longer and l' the shorter dimension of the slab. Use of this equation indicates that for slabs whose ratio $\frac{l}{l'}$ is greater than $1\frac{1}{2}$, all of the load must be considered as distributed to the longer beams, and the shorter rods designed accordingly. The only reinforcement required in the long direction is then that necessary to prevent shrinkage and temperature cracks and to assist in binding the entire structure together.

After the proportion of the total load to be transmitted in each direction has been determined, a strip of slab 12 in. in width is considered in each direction and each strip designed or reviewed as previously explained.

When a slab is reinforced in both directions, the value of d determined for one set of rods fixes that to be used in the computations of the other set. The rods running at right angles to one another are placed one above the other, the upper resting directly on the lower. In a square slab it is customary to use the d for the upper row in all computations and to place the same reinforcing, similarly spaced, in the lower row. On account of the larger value of d, this provides a slight excess of steel in the lower row, but is desirable, as it simplifies the details of construction. In rectangular slabs, it will generally be found economical to place the shorter bars, which carry the larger part of the load, as the lower row, thus making the d for these bars as great as possible. In this case also, it is general practice to space the reinforcement equally in both directions in order to insure its correct placing and to facilitate construction. The short bars then govern the design except in slabs nearly square.

62. Theoretical Discussion of Load Distribution. The above discussion of load distribution on a concrete slab treats the loading on each system as uniformly distributed, thus resulting in an equal spacing of rods throughout the slab. This is theoretically

incorrect. An element parallel to a supporting beam, close to the beam, sustains practically none of the load; the element at right angles to this supports the entire load at that point.

In a square slab the loading curve on any element actually approximates a parabola with ordinates varying from a minimum at the center to w lb. per sq. ft. at the ends, w being the total load per sq. ft. on the slab. The center minimum depends upon the location of the element in the slab, decreasing from $\frac{w}{2}$ for an element at the center of the slab to zero for one at the supporting beams.

The center bending moment for an element at the center of the slab as found by this analysis is a trifle larger than that found by assuming a uniform distribution of the load to the various elements over the entire slab, thus requiring a slightly closer spacing of the rods over this area. For a strip closer to the support, the spacing of rods could be increased, since the center moment decreases as the support is approached.

Some designers take into account the fact that in slabs supported along four sides the bending moment is greater near the center of the slab than near the edges, and assume that two-thirds of the previously calculated moments are carried by the center half of the slab and one-third by the outside quarters.

To assume the load uniformly distributed along the various elements and to space the rods equally throughout the slab simplifies the construction and is usually preferable to solving by the more exact analysis which gives a variable spacing of the rods.

In a rectangular slab of length l and width l', the amount of load carried by each system of reinforcement is found analytically by equating the deflections of two strips, one parallel to each of the edges of the slab. The deflection of a beam uniformly loaded may be shown to be proportional to the load per ft. multiplied by the fourth power of the span. Hence, since the deflections of the two strips must be equal at the center

$$w_1 l'^4 = w_2 l^4$$

therefore

$$\frac{w_1}{w_2} = \frac{l^4}{l'^4}$$

The amount of load per sq. ft. carried by each of the two sets of bars is thus inversely proportional to the fourth power of the dimensions. The recommendation given in the preceding article agrees very closely with the results of the above analysis within practical limits, and is satisfactory for the majority of designs.

63. Placing of Reinforcement. The insulation at the bottom should follow the recommendation of the Joint Committee unless conditions warrant some change (see Art. 54). In general, a depth below the center of the steel varying from ¾ in. for a slab whose d is less than 3¼ in. to 1¼ in. for a slab whose d is greater than 4¾ in. may be used.

The lateral spacing of rods, except where used only to prevent shrinkage and temperature cracks, should not exceed 2½ times the thickness of the slab. The bars should not be placed closer together than 2½ diameters, center to center, nor should there be less than 1 in. in the clear laterally, between the bars.

64. *Illustrative Problems.*

I. Design a reinforced concrete slab, fully continuous, supported on two sides only, to sustain a live load of 120 lb. per sq. ft. The span of slab is 11 ft.-0 in. A 2000-lb. concrete is to be used. $f_s = 16{,}000$.

Assume a 4½-in. slab and consider a 12-in. strip at right angles to the supporting beams. The maximum external bending moment on this strip, which may be considered a rectangular beam 12 in. wide, is

$$M = \tfrac{1}{12} \times 176 \times \overline{11}^2 \times 12 = 21{,}300 \text{ in.-lb.}$$

Since $M = Kbd^2$, in which K from Table IV = 146.7, the required effective cross-section of the imaginary beam is

$$bd^2 = \frac{21{,}300}{146.7} = 145 \text{ in.}^3$$

Since $b = 12$ in., $d = 3.48$, and taking the nearest ½ in. above the theoretical, the depth to steel is made 3½ in., and the total thickness of 4½ in. as assumed is satisfactory. No revision is necessary.

The area of steel per foot of slab width may now be obtained.

$$M = A_s f_s jd \text{ in which } j \text{ from Table IV} = .857.$$

$$21{,}300 = A_s \times 16{,}000 \times .857 \times 3.5$$

from which $A_s = .444$ sq. in. per ft. of width

or $$\frac{.444}{12} = .037 \text{ sq. in. per in. of width.}$$

Selecting ½-in. round rods, the maximum allowable spacing is

$$\frac{.1963}{.037} = 5.3 \text{ in.}$$

To simplify construction, a spacing of 5¼ in. is used throughout the slab. Since this gives a suitable arrangement, the ½-in. round rods are satisfactory.

The required effective cross-section determined above could have been found from equations (7), (3), and (4) of Art. 53 in the order named, if no tables had been available.

II. A floor panel is to be 9 ft.-0 in. by 10 ft.-0 in.; the slab is to be fully continuous and reinforced in both directions. Design the slab to carry a live load of 300 lb. per sq. ft. Assume $f_c = 650$, $f_s = 16{,}000$ and $n = 15$.

Let w_1 be the part of the total load that is transmitted in the short direction.

$$w_1 = \left(\frac{l}{l'} - \frac{1}{2}\right)w = \left(\frac{10}{9} - \frac{1}{2}\right)w = .61w$$

Assuming the weight of the slab to be 62.5 lb. per sq. ft. (or a total thickness of 5 in.), the total load on the slab is

$$w = 300 + 62.5 = 362.5 \text{ lb. per sq. ft.}$$

Design of Transverse or Short Direction.

$$w_1 = .61 \times 362.5 = 222 \text{ lb. per sq. ft.}$$

The actual external bending moment per ft. of slab width is

$$M_1 = \tfrac{1}{12} \times 222 \times 9^2 \times 12 = 17{,}900 \text{ in.-lb}$$

$$k = \frac{n}{n + r} = \frac{15}{15 + 24.6} = .379$$

$$j = 1 - \frac{k}{3} = 1 - \frac{.379}{3} = .874$$

$$17{,}900 = \tfrac{1}{2} \times 650 \times .379 \times .874 \times bd^2$$

from which $bd^2 = 166 \text{ in.}^3$

Since $b = 12$ in., $d = 3.72$ in.

By using a value of $d = 4$ in. and allowing 1 in. of concrete below the center of steel for fire-proofing, the total thickness of slab = 5 in. as assumed. No revision is necessary.

$$17{,}900 = A_s \times 16{,}000 \times .874 \times 4$$

from which $A_s = .320$ sq. in. per ft. of slab width.

By using ½-in. round rods, the maximum spacing equals

$$\frac{.1963}{\frac{.320}{12}} = 7.2 \text{ in. which is reduced to 7 in.}$$

The above computations could be much simplified by the use of Table IV.

From this $K = 107.7$ and since $M = Kbd^2$

$$bd^2 = \frac{17{,}900}{107.7} = 166 \text{ as before.}$$

By selecting $d = 4$ in. and taking the value of $j = .874$ from the same table, $17{,}900 = A_s \times 16{,}000 \times .874 \times 4$ or

$$A_s = .320 \text{ sq. in. per ft. of slab width.}$$

Design of Longitudinal or Long Direction. Let the part of the total load that is transmitted in the long direction be w_2.

Then $w_2 = 362.5 - 222 = 140.5.$

The actual external bending moment on a 12-in. strip in this direction is $M_2 = \frac{1}{12} \times 140.5 \times \overline{10}^2 \times 12 = 14{,}050$ in.-lb. $k = .379$ and $j = .874$ as in the preceding computations for the short direction, and $K = 107.7$. Therefore

$$bd^2 \text{ required} = \frac{14{,}050}{107.7} = 131 \text{ in.}^3$$

As before stated, these rods are placed directly on top of the transverse rods, the latter being placed underneath so as to give them the benefit of the maximum lever arm about the center of the compressive forces, since they carry the bulk of the load. The value of d, then, for the longitudinal rods is fixed, and equals 3½ in. Therefore

$$bd^2 \text{ as furnished} = 12 \times \overline{3\tfrac{1}{2}}^2 = 147$$

Since this is greater than that required, the strength of the concrete is sufficient to carry the load in this direction.

The area of the steel required is found from the steel resisting moment equation, $14{,}050 = A_s \times 16{,}000 \times .874 \times 3\frac{1}{2}$
from which $A_s = .288$ sq. in. per ft. of slab width.

The spacing required for $\frac{1}{2}$-in. round rods is

$$\frac{.1963}{\frac{.288}{12}} = 8.2 \text{ in.}$$

Since this is so nearly equal to the spacing adopted in the other direction, it is reduced to that value, or 7 in.

III. A typical floor panel, 10 ft.-0 in. by 12 ft.-0 in., is reinforced in both directions with $\frac{1}{2}$-in. square bars 8 in. center to center, the center of the lower row of bars being placed 1 in. above the lower surface of the slab. The total thickness of slab is 5 in., $f_s = 16{,}000$, $f_c = 650$, and $n = 15$. What live load per square foot will the panel sustain?

Investigation of Short Direction. The bars in the short direction are placed beneath the others. Their d, therefore, is 4 in. The percentage of steel in the short direction is

$$p = \frac{.25}{8 \times 4} = .0078$$

For a 12-in. strip parallel to the short sides of the slab:

The resisting moment of the concrete is

$$M_c = \tfrac{1}{2} \times 650 \times .380 \times .873 \times 12 \times 4^2 = 20{,}800 \text{ in.-lb.}$$

The resisting moment of the steel is

$$M_s = .0078 \times 16{,}000 \times .873 \times 12 \times 4^2 = 20{,}900 \text{ in.-lb.}$$

The values of k and j are taken from Table V.

The smaller of these two resisting moments must not be exceeded by the actual external bending moment. The values shown above indicate that the slab is slightly overreinforced in the short direction, *i.e.*, there is more steel than is required to develop the full compressive strength of the concrete. This fact could have been determined by comparing the actual percentage of steel with the ideal ratio for the given allowable unit stresses. The percentage furnished, .0078, is greater than the ideal, .0077, as given in Table IV.

The external bending moment equals $\frac{1}{12}w_1l'^2$, and its maximum allowable value equals 20,800 in.-lb. Therefore,

$$20{,}800 = \frac{1}{12} \times w_1 \times \overline{10}^2 \times 12$$

from which $w_1 = 208$ lb. per ft.

This is the total load that can safely be carried in the short direction. Since this is $\frac{l}{l'} - \frac{1}{2} = .70$ of the total load on the slab w, the total load that can be carried on the slab before the short rods will reach their allowable stress (as governed by the concrete in this case) equals

$$\frac{208}{.70} = 297 \text{ lb. per sq. ft.}$$

Investigation of Long Direction. The effective depth of the long bars is $3\frac{1}{2}$ in.

$$p = \frac{.25}{8 \times 3\frac{1}{2}} = .0089;\ k = .400;\ j = .866$$

For a 12-in. strip of the slab parallel to the long sides: Table IV shows that the slab is overreinforced in this direction and the resisting moment of the 12-in. strip is limited by the strength of the concrete.

$M_c = \frac{1}{2} \times 650 \times .400 \times .866 \times 12 \times \overline{3.5}^2 = 16{,}600$ in.-lb.

The actual bending moment is $\frac{1}{12}w_2l^2$, hence

$$16{,}600 = \frac{1}{12} \times w_2 \times \overline{12}^2 \times 12$$

from which $\qquad w_2 = 115$ lb. per ft.

This represents the load that can be carried safely in the long direction; it is equal to but .30 of the total load on the slab.

$$115 = .30w, \text{ hence } w = 382 \text{ lb. per sq. ft.}$$

From the two investigations above, it is seen that a total load of 382 lb. per sq. ft. could be placed on the slab without overstressing it in the long direction. This load, however, would considerably overstress the slab in the short direction. The maximum total load per square foot that can be placed on this slab is thus determined by the strength in the transverse direction, and equals 297 lb. as computed above.

The slab itself weighs 62 lb. per sq. ft., hence the safe live load equals $297 - 62 = 235$ lb. per sq. ft.

IV. A fully continuous floor panel 9 ft.-0 in. by 9 ft.-0 in. is to support a live load of 300 lb. per sq. ft. Determine the required thickness of slab and the arrangement of the reinforcement. Assume maximum allowable unit stresses of 750 and 16,000 for the concrete and steel, respectively and $n = 15$.

Assume $t = 4\frac{1}{2}$ in.; then the total load per square foot is 356 lb. Since the slab is square, one-half of this load will be transmitted in each direction. Hence $w_1 = 178$ lb. For a 12-in. strip of slab in either direction,

$$M = \frac{1}{12} \times 178 \times \overline{9}^2 \times 12 = 14{,}400 \text{ in.-lb.}$$

Since $M = Kbd^2$, in which K (Table IV) is 133.8,

$$bd^2 = \frac{14{,}400}{133.8} = 107.5 \text{ in.}^3$$

$b = 12$ in., therefore d must be at least 2.98 in.

In a square slab, since the moments for strips in either direction are equal, the strength of the strip parallel to the upper row of bars will govern the design. Selecting an effective depth of 3 in. for this row, allowing 1 in. of insulation below the center of the lower row, and assuming $\frac{1}{2}$-in. bars, the total thickness of slab required is $3 + \frac{1}{2} + 1$, or $4\frac{1}{2}$ in. as assumed.

For the upper row,

$$A_s = \frac{14{,}400}{16{,}000 \times .862 \times 3} = .348 \text{ sq. in. per ft. of width}$$

or $\frac{.348}{12} = .029$ sq. in. per in. of width. The maximum spacing of $\frac{1}{2}$-in. round bars is then $\frac{.1963}{.029}$ or $6\frac{3}{4}$ in.

The spacing of bars in the lower row is made the same for uniformity. Since the effective depth for this row is $\frac{1}{2}$ in. greater than for the upper row, this arrangement is safe but not excessively uneconomical.

Diagonal Tension, Shear, and Bond

65. Stresses in a Concrete Beam. The preceding paragraphs contain an outline of the methods of calculating the maximum fiber stresses in the concrete and steel of a reinforced concrete beam, and of so proportioning the amounts of steel and concrete

that the working strength of any part of the beam in flexure is not exceeded.

As indicated in Art. 49, there are other internal stresses existing in a concrete beam which, if not properly cared for, may in themselves cause failure of the beam. These stresses are: (1) shearing stresses, or those tending to make one plane of concrete, either vertical or horizontal, slide along an adjacent plane; (2) diagonal tension stresses, or those which cause cracks in the concrete along inclined planes near points of maximum shear; and (3) in reinforced beams, bond stresses, or those tending to cause the steel to pull away from the concrete when under stress and thus destroy the unity of the beam.

66. Shearing Stresses. If a pile of boards is used to support a load, the boards being free to slip on each other, it is noticeable that the ends overlap even when the boards are of equal length (see Fig. 10). Slipping has occurred along the surfaces of contact. If, however, they are glued together, piled as before, the slipping is prevented, but the *tendency to slip* still exists and is known as the shearing stress in surfaces parallel to the neutral axis. These shearing stresses exist in beams of any material as long as the two sides of the surface considered form a continuous substance.

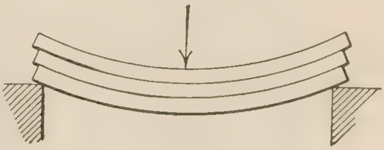


Fig. 10.

In addition to the horizontal shearing stresses described above, there exist vertical stresses of the same nature, *i.e.*, a tendency for one side of the beam to slide upward past the other side. These two kinds of shearing stresses are of the same intensity per unit of area at any point in the beam.

67. Intensity of Shearing Stress in a Plain Concrete Beam. As before stated (Art. 49), the value of the unit shear in a plain concrete beam (or any homogeneous beam) equals

$$v = \frac{QV}{Ib} \tag{9}$$

The value of the unit shear as represented by this equation becomes zero at the top and bottom surfaces of the beam, and a

maximum at the neutral axis. The maximum stress equals $\frac{3}{2}$ the average, or $\frac{3}{2}\left(\frac{V}{bd}\right)$ (Art. 49). Between the neutral axis and the extreme surfaces the value of the unit shear varies as the ordinates of a parabola. The absolute maximum value of v occurs where the total external vertical shear is greatest.

68. Intensity of Shearing Stress in a Reinforced Concrete Beam. In a non-homogeneous beam the unit shearing stresses vary in a very different manner from that described above. To derive an equation which expresses the variation of the unit shear at any section of a reinforced concrete beam, consider a short section of the beam as a free body. The forces acting on this element are those of compression, C and C', of tension, T

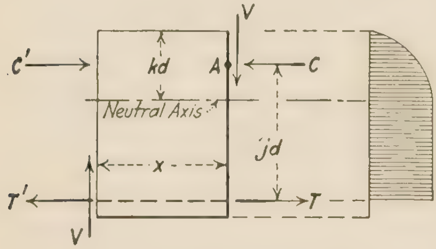


FIG. 11.

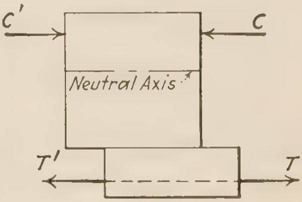


FIG. 11a.

and T', and of total vertical shear, V. Hence in Fig. 11, which represents a section so short that no part of the external vertical load need be considered (the total vertical shear on the left equals that on the right of the element), $T - T'$ = the total shearing stress, or tendency for the upper portion to slide along the lower, on any plane between the steel and the neutral axis, assuming that the concrete will take none of the tension.

This is illustrated in Fig. 11a. If the element is assumed on the left portion of the span, and the beam subjected to a uniform load, the fiber stresses on the right are greater than those on the left. The lower portion tends to slide toward the right and the upper portion toward the left as shown in Fig. 11a. The force producing this horizontal sliding is equal to the difference in the forces acting on each part of the section, *i.e.*, $T - T'$ for the

lower and $C - C'$ for the upper. $C - C'$ must equal $T - T'$, and the shearing strength of any two consecutive horizontal planes between the neutral axis and the tension steel must be sufficient to transmit the effect of one set of these forces to the other, so as to prevent the movement indicated in Fig. 11*a*.

The unit shear for any plane in this region equals

$$v = \frac{T - T'}{bx}$$

bx being the area of the surface under consideration. The external forces acting on this portion of the beam must be in equilibrium, hence the summation of moments about any point such as A on the line of action of the compressive forces must equal zero, or

$$(T - T')jd = Vx$$

Therefore $$T - T' = \frac{Vx}{jd}$$

Substituting this value of $T - T'$ in the above equation for v

$$v = \frac{V}{bjd} \tag{10}$$

This represents the value of the unit shearing stress along any plane between the steel and the neutral axis, it being also the maximum unit shear in the section. The amount of this shear per linear inch of beam equals

$$v_1 = \frac{V}{jd} \tag{11}$$

The value of j for working loads varies between narrow limits, and this variation causes but insignificant differences in values of v. For this reason it is satisfactory to use the value of $j = 7/8$ in all computations involving shear and bond. This is an average for beams in ordinary construction. Where the more exact value of j has previously been determined in the computations for flexure, it may be used, if desired, in the calculations for shear and bond.

Above the neutral axis the shear follows the parabolic law as in the plain concrete beam (see Fig. 11).

69. Inclined Tensile Stresses (Diagonal Tension). Assume an infinitely small cube to be removed from a beam at any section

along the neutral axis. Two pairs of shearing forces, horizontal and vertical, must be considered. These forces form two couples acting as shown in Fig. 12. Since the prism has been assumed at the neutral axis, the flexural stresses of tension and compression are zero. Therefore, the shearing stresses vbx develop inclined stresses of tension in the direction MN, and compression along the line PQ, each equal to $\frac{2vbx}{\sqrt{2}}$. Since the length of each diagonal of the prism is $x\sqrt{2}$, the intensity of these inclined stresses, that is, the amount per unit area, equals $\frac{2vbx}{\sqrt{2}}$ divided by $bx\sqrt{2}$, or v. It follows that at any point along the neutral plane there exist tensile and compressive forces inclined at 45 degrees with the horizontal, and that the value of these forces per unit of area equals the unit shear at that point.

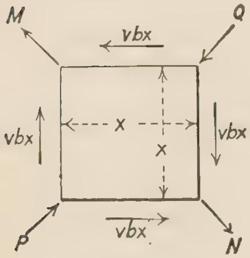


FIG. 12.

Since at all points above and below the neutral axis there exist, in addition to the horizontal and vertical shearing forces, horizontal fiber stresses of tension and compression, the values of the inclined tensile and compressive forces at such points are found by combining the fiber stresses with the shearing stresses.

Treatises on Mechanics prove that the intensity of the inclined stress at any point in a beam may be represented by the equation

$$t = \frac{1}{2}f \pm \sqrt{\frac{1}{4}f^2 + v^2} \qquad (12)$$

and the direction of this stress by the equation

$$\tan 2\alpha = \frac{2v}{f}$$

where f = the fiber stress per unit of area.

v = the intensity of vertical or horizontal shearing stress at the point.

α = the angle made by the stress t with the horizontal.

In the equation for the angle of inclination of the inclined stress, two values of α, differing by 90 degrees, will satisfy, show-

ing that the maximum compressive stress and the maximum tensile stress at any point make an angle of 90 degrees with each other.

On account of the comparatively large compressive strength of concrete, the inclined compressive stresses as determined above may be neglected; failure, if any, occurs because of the opening of the concrete due to tensile stresses in excess of its strength.

For ordinary beams of homogeneous materials, such as beams of steel or timber, a determination of the normal, or flexural stresses, and the shearing stresses described above, gives sufficient information for purposes of design. In concrete beams, both plain and reinforced, the inclined stresses of maximum tension induced by the shearing and bending stresses are usually fully as important as the maximum fiber stresses, and it is necessary to make some provision for them. This is because of the very low strength of concrete in tension. Hence the necessity for the further investigation of such stresses.

70. Diagonal Tension in Plain Concrete Beams. Examination of the above equations shows that at the center of a homogeneous beam, where the moment is a maximum, the direction of the lines of maximum tension is horizontal for the entire depth of the beam. As the end of the beam is approached, the shear becomes large and the bending moment small (assuming a simply supported beam). The effect of the shear on the diagonal tension is great, while the horizontal fiber stress has little weight in determining the inclination and amount of the inclined tensile stress near the support. At the end of the beam, where the horizontal tension is zero, the diagonal tension stresses lie at practically 45 degrees throughout the entire depth of the beam.

Fig. 13.

Figure 13 illustrates the variation in direction of the maximum tensile stresses in a homogeneous rectangular beam. As seen above, the exact direction at any point depends upon the relation between shear and bending moment at the point. Lines of maximum compression run at right angles to those shown.

71. Diagonal Tension in Reinforced Concrete Beams. In reinforced beams, due to the concentration of the tension in the steel, the direction of maximum tension at various depths is somewhat different from that in plain or homogeneous beams. Large shearing stresses exist immediately above the steel, and the maximum tensile stresses become considerably inclined at that plane, the exact direction depending upon the relation between the shear and horizontal fiber stress.

Fig. 14.

Figure 14 represents the general direction of the inclined tensile stresses in a uniformly loaded beam, the wavy lines representing the probable planes of rupture. The diagonal cracks near the bottom are approximately vertical at the center and become more and more inclined as the end of the beam is approached.

In beams with tensile reinforcement the value of t as expressed by equation (12) is indeterminate, since the value of f, the horizontal fiber stress in the concrete, is variable. This is due to the fact that as the loading increases, the concrete cracks more and more, and the amount of tensile stress carried by it therefore decreases. The excess is picked up by the steel, and immediate failure prevented. In a plain beam, increasing the loading after the concrete commences to crack results in eventual rupture of the beam. Therefore the exact amounts of the inclined tensile stresses are unknown. It is seen, however, from a study of equation (12), that the vertical shearing stresses furnish a means of comparing or measuring the diagonal tensile stresses existing in the beam. It must be remembered that the vertical shearing stress is not the numerical equivalent of the diagonal tensile stress, nor is there any definite ratio between them.

By limiting the allowable unit shearing stress to a value which has been found by actual tests to be low enough to insure against failure by diagonal tension, it may be considered that the danger of such failure has been eliminated. This limit of the allowable shearing stress is considerably below the safe working stress of concrete in direct shear because of the fact that, when the shear in a beam is still low, the diagonal tension may be excessive.

Failure occurs not by direct shear, but by the cracking of the concrete along inclined planes.

72. Methods of Strengthening Beams against Diagonal Tension. An examination of equation (12) shows that diagonal tension at any point varies with both the shear and horizontal tension in the concrete. In order to reduce the danger of failure by diagonal tension, heavy shearing stresses should be avoided and the horizontal tension in the concrete kept as small as possible. This latter may be accomplished by furnishing a large amount of steel at points of heavy shear, thus reducing the horizontal deformation and consequently the tension in the concrete.

When relatively heavy shearing stresses exist (the shear being a measure of the indeterminate diagonal tension) it becomes necessary to provide some form of web reinforcement. The Joint Committee recommends 2 per cent of the compressive strength of the concrete as the safe limit of shearing stress for beams without web reinforcement, and 6 per cent for beams in which adequate provisions have been made to care for the inclined stresses. If the longitudinal bars are adequately anchored by means of hooks at both ends, or by some other satisfactory method, somewhat higher stresses are allowed, *i.e.*, $0.03\,f'_c$ for the concrete when no web reinforcement is provided, and not more than $0.12f'_c$ for beams with web reinforcement. This latter value should be used only in cases of extreme refinement in the design and placing of the reinforcement.

73. Types of Web Reinforcement. A study of Fig. 13 shows that the most efficient web reinforcement consists of an arrangement of steel as shown in Fig. 15, the inclined portions being either a continuation of the horizontal rods or additional rods rigidly connected to the horizontal rods at their lower ends. Such an arrangement is, however, not practical. Slight variations between the inclination of the reinforcing rods and the lines of maximum tension have but little effect on the efficiency of the system; hence in practice the most commonly used methods of

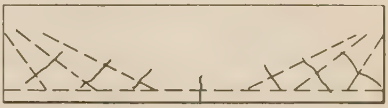


Fig. 15.

arranging diagonal tension reinforcement are divided into three groups: (1) vertical rods or stirrups, attached to or looped about horizontal rods; (2) inclined rods secured to the horizontal rods in such a way that there shall be no slipping; (3) longitudinal rods, some of which are bent up at regions of large shear. A combination of vertical stirrups and bent bars properly arranged gives the most practical effective protection against diagonal tension failure. See Fig. 28.

When separate members, either vertical or inclined, are used as diagonal tension reinforcement, care must be taken to see that they are properly connected to the longitudinal steel so that slipping is prevented. When web reinforcement comes into action as the principal tension web resistance, the bond stresses between the longitudinal bars and the concrete are not distributed as uniformly along the bars as they otherwise would be, but tend to be concentrated at and near stirrups, and at and near the points where bars are bent up. When stirrups are not rigidly attached to the longitudinal bars, and the proportioning of bars and stirrups is such that the local slip of bars occurs at stirrups, the effectiveness of the stirrups is impaired, though their presence still gives an element of toughness against diagonal tension failure. It is on the tension side of a beam that diagonal tension develops in a critical way, so proper connection should always be made between stirrups or other web reinforcement and the longitudinal tension reinforcement, whether the latter is on the lower side of the beam or on its upper side. Where negative moment exists, as is the case near the supports in a continuous beam, web reinforcement to be effective must be looped over, or wrapped around, or be connected with the longitudinal tension reinforcing bars at the top of the beam in the same way as is necessary at the bottom of the beam at sections where the bending moment is positive.

The Joint Committee requires that the web reinforcement shall be anchored at both ends by:

(*a*) providing continuity with the longitudinal reinforcement; or

(*b*) bending around the longitudinal bar; or

(*c*) a semicircular hook which has a radius not less than four times the diameter of the web bar.

The committee further specifies that stirrup anchorage shall be so provided in the compression and tension regions of a beam as to permit the development of the safe working tensile stress in the stirrup at a point $.3d$ from either face; *i.e.*, the length of the anchorage beyond this point shall be such that the bond strength of this portion (see Art. 80) equals the tensile strength of the stirrup. Generally a properly anchored stirrup whose diameter does not exceed one-fiftieth of the depth of the beam will meet this requirement.

In case the end anchorage of the web member is not in bearing on other reinforcement, the anchorage should be such as to engage an adequate amount of concrete to prevent the bar from pulling off a portion of the concrete. In all cases the stirrups should be carried as close to the upper and lower surfaces as fire-proofing requirements will permit.

74. Distribution of Diagonal Tension. Tests show that in beams with web reinforcement, both steel and concrete resist the diagonal tension. There are two methods of proportioning the amounts of diagonal tension to be taken by the steel and by the concrete. (1) The web reinforcement is designed to provide for a proportion (usually two-thirds) of the vertical shear, the remainder to be taken by the concrete. (2) The concrete is assigned a definite unit stress in shear and the remainder of the shear is taken by the web reinforcement. The Joint Committee specifies the latter method and recommends for the concrete a maximum unit shear of $0.02f'_c$ lb. per sq. in. for beams whose longitudinal rods are not thoroughly anchored, and $0.03f'_c$ lb. per sq. in. for beams in which the longitudinal rods are adequately anchored.

75. Spacing and Size of Vertical Stirrups. In Art. 68 it was shown that the horizontal shear per linear inch of beam on any plane below the neutral axis equals $\frac{V}{jd}$. Hence for a length of beam, s, the amount of horizontal shear may be represented by the equation $\frac{Vs}{jd}$, in which V equals the average external vertical shear over the length s (at B in Fig. 16). Since the horizontal shear equals the vertical shear, it follows that this

equation also represents the vertical component of the diagonal tensile stress in a length s. The horizontal component is resisted by the horizontal steel, while the vertical component is resisted by the concrete and web reinforcement.

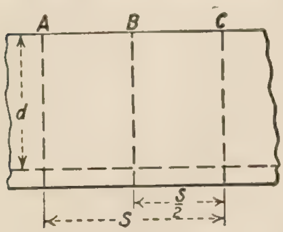


Fig. 16.

If vertical stirrups of an effective area A_v are employed, and if the allowable unit stress in the stirrups is f_v, then the safe strength of each stirrup equals $A_v f_v$. For the first method of distribution given in Art. 74, the vertical component of the diagonal tension to be provided for by the stirrups over a distance s equals $\frac{2}{3} \cdot \frac{Vs}{jd}$. Hence

$$A_v f_v = \frac{2}{3} \cdot \frac{Vs}{jd}$$

and the required spacing of stirrups equals

$$s = \frac{3}{2} \cdot \frac{A_v f_v jd}{V} \tag{13}$$

For the second method of distribution, the vertical component of the diagonal tension to be resisted by the stirrups over a distance s equals $\frac{(V - V_c)s}{jd}$, in which V_c is the amount of total shear that is assigned to the concrete as determined from equation (10), $v_c = \frac{V_c}{bjd}$. The value of v_c to be used is given in the latter part of Art. 74. Let $V - V_c = V'$, then

$$A_v f_v = \frac{V's}{jd}$$

and the required spacing of stirrups is

$$s = \frac{A_v f_v jd}{V'} \tag{13a}$$

The Joint Committee recommends that the longitudinal spacing of vertical stirrups shall not exceed 0.45 of the effective depth of beam, except that in beams where the unit shearing stress exceeds $0.06f'_c$ the limit is $0.3d$. A spacing of less than 4 in. is generally undesirable. Tests indicate that the most effective results are obtained when $s = \frac{1}{3}d$. It is usually satisfactory to select a certain size of stirrup and calculate the spacing

at different points along the beam; if the computed spacings are unsatisfactory, another size of stirrup may be assumed and the corresponding spacings determined.

Common sizes of stirrup rods are ¼-, ⅜- and ½-in. rounds. The diameter varies with the depth of the beam; ¼-in. rods for beams less than 10 in. deep, ½-in. rods for beams 36 in. deep, and intermediate sizes for intermediate depths.

Where web reinforcement is provided by means of vertical stirrups and is required over a comparatively short distance, it is good practice to space the stirrups uniformly over the entire distance, the spacing being calculated for the point of greatest shear (minimum spacing).

76. Bent-up Rods or Inclined Stirrups for Web Reinforcement. When web reinforcement consists of inclined rods, more of the probable planes of rupture are crossed by a given length of rod than is the case when vertical rods are used. Inclined steel is, therefore, more effective than vertical steel of the same amount. In Fig. 17, the diagonal force DE representing the diagonal tension over a distance s is the sum of the components of the horizontal force AB and the vertical force BC in the direction of DE. The components normal to the direction of DE are compressive forces and are absorbed by the concrete. It is assumed that the horizontal force AB is prevented from causing failure in the concrete by the longitudinal steel in the beam. The force BC or its component BE must be otherwise provided for. If this force is taken care of by vertical stirrups or bars, the amount to be resisted is represented by BC. When resolved into components, BE and CE, the amount to be resisted by inclined bars or stirrups is BE, which is $BC(\sin\alpha)$, where α is the angle of inclination of the bar. BC represents the total external shear over the distance s. The amount of stress to be resisted by the inclined bars now is $Vs(\sin\alpha)$. The maximum spacing of inclined rods is calculated in the same manner as for

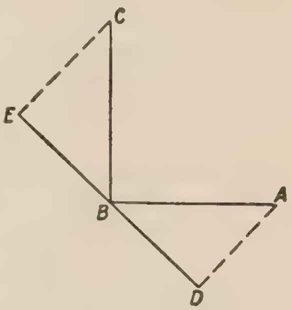


FIG. 17.

vertical stirrups, $V(\sin\alpha)$ being substituted for V in equation (13), giving as a result

$$s = \frac{3}{2}\frac{A_v f_v jd}{V(\sin\alpha)}$$

Since α is usually 45 degrees, this equation may be reduced to

$$s = \frac{3}{2}\frac{A_v f_v jd}{.7V} \qquad (14)$$

For the second method of diagonal tension distribution given in Art. 74, equation (14) becomes

$$s = \frac{A_v f_v jd}{V' \sin\alpha} \text{ or } \frac{A_v f_v jd}{.7V'} \qquad (14a)$$

77. Spacing of Inclined Stirrups. The required spacing of inclined stirrups may be obtained by means of equation (14) or equation (14*a*). When the shearing stress is not greater than $0.06f'_c$, the distance s, measured in the direction of the axis of the beam between two successive inclined stirrups, is limited by the Joint Committee to a maximum value as given by the following equation:

$$s = \frac{45d}{\alpha + 10}$$

in which α is the angle of inclination which the stirrup makes with the longitudinal bar in degrees. When the shearing stress is greater than $0.06f'_c$, the distance s is limited to two-thirds of the value given in the above equation.

While inclined stirrups are more efficient theoretically than vertical stirrups, this advantage is counteracted somewhat because of the difficulty in fastening them to the tension steel and of assuring their correct position after the concrete is poured.

78. Arrangement of Bent-up Rods. The points where horizontal rods may be bent up are governed by the amount of steel required to care for the horizontal fiber stresses caused by the bending moment at different sections along the beam. Since this decreases toward the ends, the amount of fiber stress decreases in the same ratio. Enough steel must always remain at the bottom to care for these stresses; the remainder may be bent up to aid in overcoming the diagonal tension.

The location of the points of bending may be determined graphically as follows: plot the bending moment diagram for the

given loads. Since the amount of tensile steel required at any section of the beam is proportional to the bending moment, the maximum ordinate of the bending moment diagram may also be made to represent the total area of steel reinforcement. Assuming that all of the reinforcing bars are of the same area and will be equally stressed at the point of maximum moment, divide the maximum ordinate into the same number of equal parts as there are bars crossing the section of maximum moment. Draw a horizontal line through each point of division.

The intersection of any one of these horizontal lines with the bending moment curve locates a point beyond which all of the rods in excess of the number represented by the line may be bent up. It is well to exceed the theoretical points by at least 2 in. to allow for the irregularities in the loading. The moment diagram for a uniformly loaded beam, when the condition of the supports is such as to give values of maximum moments equal to either $\frac{wl^2}{8}$ or $\frac{wl^2}{12}$, is shown in Diagram 1. Percentages of steel are given in place of numbers, as explained above.

The bars which are bent should be selected so that the symmetry of the remaining bars about the axis of the beam is not destroyed. As a rule, at least two rods are bent up together from corresponding points on either side of the beam. Sometimes, however, it becomes necessary to depart from this procedure, as when three bars are to be bent. Then either first one, and then two, are bent, or the three at the same place. In either case the odd bar is bent from the middle of the section of the beam if possible.

The Joint Committee recommends that the bending of longitudinal reinforcing bars at an angle across the web of a beam may be considered as adding to diagonal tension resistance for a maximum horizontal distance from the point of bending as given for inclined stirrups in Art. 77. Where the bending is made at two or more points, the distance between the points of bending, or between the point of bending of the bar nearest the support and the edge of the support, should not exceed the value given in Art. 77.

79. Region Where No Web Reinforcement Is Required. As mentioned in Art. 72, web reinforcement is not required in regions where the unit shear is less than a given percentage of the ultimate compressive strength of the concrete; that is to say, the concrete is assumed capable of withstanding all of the diagonal tension as measured by a unit shear of that amount. In a *uniformly loaded* beam, the distance from the support beyond which stirrups are not required is determined as follows:

Let x_1 = the distance to be found, v_1 the unit shear x_1 ft. from the support, and V_1 the total shear at that point.

$$V_1 = \frac{wl}{2} - wx_1 \text{ and } v_1 = \frac{V_1}{bjd}$$

By substituting the value of V_1 from the former in the latter equation, and solving for x_1, this becomes

$$x_1 = \frac{l}{2} - \frac{v_1 bjd}{w} \tag{15}$$

For beams with unsymmetrical or concentrated loads, the points where web reinforcement may be discontinued may be located by constructing the shear diagram for the beam and noting the point or points at which the unit shear is less than the given percentage of the compressive strength of the concrete.

80. Bond Stresses. When steel rods are placed in a beam, there must be sufficient bond between the steel and the concrete to prevent the rods from pulling out when stressed. This bond stress, or tendency for the steel to slide out of the concrete, per square inch of bar surface may be determined as follows.

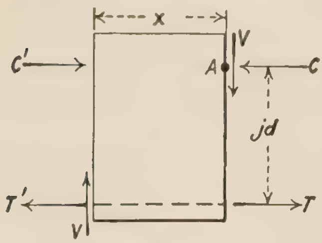


FIG. 18.

If, in the short section of beam discussed in Art. 68, moments about the point A (Fig. 18) are taken

$$(T - T')jd = Vx$$

and

$$\frac{T - T'}{x} = \frac{V}{jd}$$

But $T - T'$ represents the force tending to pull the bars out of the concrete in a length x, hence $\frac{T - T'}{x}$ represents this force per

unit length of beam. Therefore $\frac{V}{jd}$ equals the bond stress per unit of length between the two materials. If u equals the bond stress per unit area of exposed steel surface, and Σ_o the total perimeter of steel,

$$u = \frac{V}{\Sigma_o jd} \tag{16}$$

This equation applies to the steel in tension only.

In order to prevent the pulling out of the rods when stressed, the value of the unit bond stress as computed by equation (16) should not exceed a safe working limit which has been fixed by the results of tests on beams in which such failure has occurred. (See Chap. II.) Where high bond resistance is required, the deformed bar is a suitable means of supplying the necessary strength. As an additional safeguard, end anchorage, consisting of hooks bent through an angle of 180 degrees, may properly be used in special cases. It must be remembered, however, that adequate bond strength throughout the length of the bar is preferable to such anchorage.

The Joint Committee recommends that the unit bond between concrete and plain reinforcing bars shall not exceed 4 per cent of the ultimate compressive strength of the concrete. In the best types of deformed bars this may be increased, but not to exceed 5 per cent of the ultimate compressive strength of the concrete.

Tests show that under favorable conditions the actual bond stress at the ends of simply supported beams with inclined rods for web reinforcement is considerably below the theoretical. If more than two bars are bent up at two or more places, it can be assumed that the inclined rods will assist in reducing the actual bond stress, and the above limits may safely be increased by 50 per cent for the remaining rods. The same increase is also safe for bars which are adequately anchored at both ends by means of hooks or other means, in either simple, continuous, or cantilever beams.[1] The requirements for footings are discussed in Chap. VII.

[1] The Joint Committee allows a greater increase than is recommended above provided adequate anchorage is added, the amount of increase depending entirely upon the amount of anchorage. The Committee on

In a beam with bent rods for diagonal tension reinforcement, each bent rod should have a length for anchorage sufficient to develop enough bond between the steel and concrete to resist the direct tension in the rod. This length is measured from the point where the rod crosses a horizontal plane distant $0.6d$ from the compressive surface to the end of the rod, and may be either straight or bent. It may be computed from the equation

$$l_1 = \frac{f_s l}{4u}$$

as given in Art. 47. Bars in cantilevers, restrained beams, and columns, where the full stress in the steel exists at the point of support, should be embedded in the concrete beyond this point of maximum stress far enough to develop in bond the full stress in the steel. The length of embedment is determined by the

---

Concrete Building Design Specifications of the American Concrete Institute allows a maximum increase of 150 per cent where proper anchorage is provided. The required anchorage is given by this committee in the following specification.

"In members in bending, bond stresses exceeding $0.04f'_c$ or $0.05f'_c$ for plain and deformed bars, respectively, but in no case more than $2\frac{1}{2}$ times the latter, may be used, provided that sufficient additional length of bar is added beyond the theoretical point of zero moment (end of span or point of inflection) to provide for the development of the excess in bond stress over that specified above. The length x to be added for this purpose may be expressed algebraically as follows:

$$xu\Sigma_o = F - F'$$

where x = the length of bar added for anchorage, including the hook, if any;

u = the permissible bond stress given above;

Σ_o = the perimeter of the bars under consideration;

F = the total tension in the bars under consideration;

F' = the total tension in the bars which would be developed in the length y by the computed bond stresses except that no values greater than those specified above be used in the computation;

y = the distance from the point at which the tension is computed to the point of beginning of anchorage.

"The point of beginning of anchorage shall be taken at the edge of the support for freely supported beams, and at the point of inflection (for the loading under consideration) for fixed or continuous beams; anchorage of negative reinforcing bars to be toward the center of the beams from this point. The length of bar added for anchorage may be either straight or bent. The radius of bend shall not be less than 4 bar diameters."

above equation. The length of lap for a splice in a reinforcing rod is likewise determined by the bond required to provide for the stress in the rod at the point of splicing. In determining the length of embedment in all of these cases, a value of u equal to $0.04f'_c$ or $0.05f'_c$ for plain and deformed bars, respectively, should be used even though larger values than these were used in determining the perimeter of bars required at the point of maximum shear.

81. *Illustrative Problem.*

A reinforced concrete beam has a span of 18 ft.-0 in. and is to sustain a live load of 1000 lb. per lin. ft. The reinforcement consists of three ¾-in. round bars placed 18 in. below the upper and 2 in. above the lower surface of the beam. The width of beam is 8 in., and a 2000-lb. concrete is to be used in its construction.

Determine the unit shear at the support, the maximum unit bond stress on the horizontal rods, the distance from the support beyond which stirrups are not required, and the spacing of ⅜-in. round U-stirrups at the support and 2 ft. from the support.

$$\text{The weight of the beam per foot} = \frac{8 \times 20}{144} \times 150 = 167 \text{ lb.}$$

$$\text{The total load} = 1167 \text{ lb. per ft.}$$

$$V \text{ at end} = 9 \times 1167 = 10{,}500 \text{ lb.}$$

The unit shear at the support, from equation (10), is

$$v = \frac{10{,}500}{8 \times .875 \times 18} = 83 \text{ lb. per sq. in.}$$

(This value shows that the beam is safe from diagonal tension failure provided stirrups are placed throughout the beam sufficiently close to satisfy the following computations.)

The maximum unit bond stress is, from equation (16)

$$u = \frac{10{,}500}{3 \times 2.356 \times .875 \times 18} = 94 \text{ lb. per sq. in.}$$

(This shows that deformed bars are necessary to insure against slipping of the steel along the concrete.)

The distance from the support beyond which stirrups are no longer required, *i.e.*, where the unit shear becomes 40 lb. per sq. in. (2 per cent of 2000) is, from equation (15),

$$x_1 = \frac{18}{2} - \frac{40 \times 8 \times .875 \times 18}{1167} = 4.68 \text{ ft.}$$

Assuming the first method of distribution of the total diagonal tension as given in Art. 74, the maximum allowable spacing of $\frac{3}{8}$-in. round U-stirrups at the support, from equation (13) is,

$$s = \frac{\frac{3}{2} \times 2 \times .1104 \times 16{,}000 \times .875 \times 18}{10{,}500} = 8.0 \text{ in.}$$

and the spacing 2 ft. from support is,

$$s = \frac{\frac{3}{2} \times 2 \times .1104 \times 16{,}000 \times .875 \times 18}{10{,}500 - (2 \times 1167)} = 10.2 \text{ in.}$$

This should be reduced to $.45 \times 18 = 8.1$ in. in order to satisfy the recommendation of the Joint Committee, and for uniformity, good practice would space the stirrups 8 in. throughout the entire distance where they are required.

If the second method of distribution given in Art. 74 is used, as recommended by the Joint Committee, the amount of total shear that can be resisted by the concrete is found from the relation $v_c = \frac{V_c}{bjd}$, from which $V_c = 40 \times 8 \times .875 \times 18 = 5050$ lb. The amount to be resisted by the web reinforcement at the support is then $10{,}500 - 5050 = 5450$ lb. From equation (13*a*), with $V' = 5450$ lb. the maximum allowable spacing of the stirrups at the support is,

$$s = \frac{2 \times .1104 \times 16{,}000 \times .875 \times 18}{5450} = 10.2 \text{ in.}$$

and 2 ft. from the support,

$$s = \frac{2 \times .1104 \times 16{,}000 \times .875 \times 18}{5450 - (2 \times 1167)} = 17.8 \text{ in.}$$

As in the preceding case, the spacing throughout the entire distance over which the stirrups are required must be reduced to 8 in. in order to satisfy the recommendations of the Committee.

82. Typical Web Reinforcement Problem. The method of providing for diagonal tension in a beam by means of bending up some of the horizontal steel at points of heavy shear may best be illustrated by making a complete design of a simply supported beam. The connection between the preceding discussion of the Joint Committee's recommendations and this problem should be closely followed.

Design a simply supported rectangular reinforced beam having a span of 16 ft.-0 in., to support a uniform live load of 425 lb. per lin. ft., and three concentrated loads of 12,000 lb. each, these being placed at the quarter points of the span. A 2000-lb. concrete is assumed.

Design for Flexural Stresses.

Assume weight of beam = 440 lb. per lin. ft.

The total uniform load is 865 lb. per lin. ft.

The maximum moment due to uniform load = M_1

$M_1 = \frac{1}{8} \times 865 \times \overline{16}^2 \times 12 = 333{,}000$ in.-lb.

The moment due to concentrated loads = M_2

$M_2 = (18{,}000 \times 8 - 12{,}000 \times 4) \times 12 = 1{,}152{,}000$ in.-lb.

The total maximum moment = 1,485,000 in.-lb.

$M = Kbd^2$, in which K, from Table IV, = 146.7.

$bd^2 = \dfrac{1{,}485{,}000}{146.7} = 10{,}100$ in.^{3}

Selecting a beam whose b is between $\frac{1}{2}$ and $\frac{3}{4}d$, and taking b in even inches to facilitate form work, requires $b = 14$ in., $d =$ 26.9 or 27 in.

Since two rows of steel will undoubtedly be necessary, the total height of beam is 30 in., and the weight per foot is 440 lb. as assumed.

From Table IV, $j = .857$, and the required area of steel is

$$A_s = \frac{1{,}485{,}000}{16{,}000 \times .857 \times 27} = 4.00 \text{ sq. in.}$$

Four $\frac{3}{4}$-in. round bars and four $\frac{7}{8}$-in. round bars will be used, the four larger bars placed in the lower row with their centers 2 in. above the lower surface of the beam, and the four smaller bars in the upper row, the distance center to center of rows vertically being 2 in. With this steel area $p = .0110$, $k = .432$, $j = .856$.

Design for Diagonal Tension Stresses. The rods that can be bent up and used to provide for the diagonal tension stresses are those which are not needed to furnish sufficient surface for the development of the bond stresses at the support.

The maximum end shear = $18{,}000 + 8 \times 865 = 24{,}920$ lb. Since the allowable maximum unit bond stress according to the recommendations in Art. 80, assuming deformed bars, equals

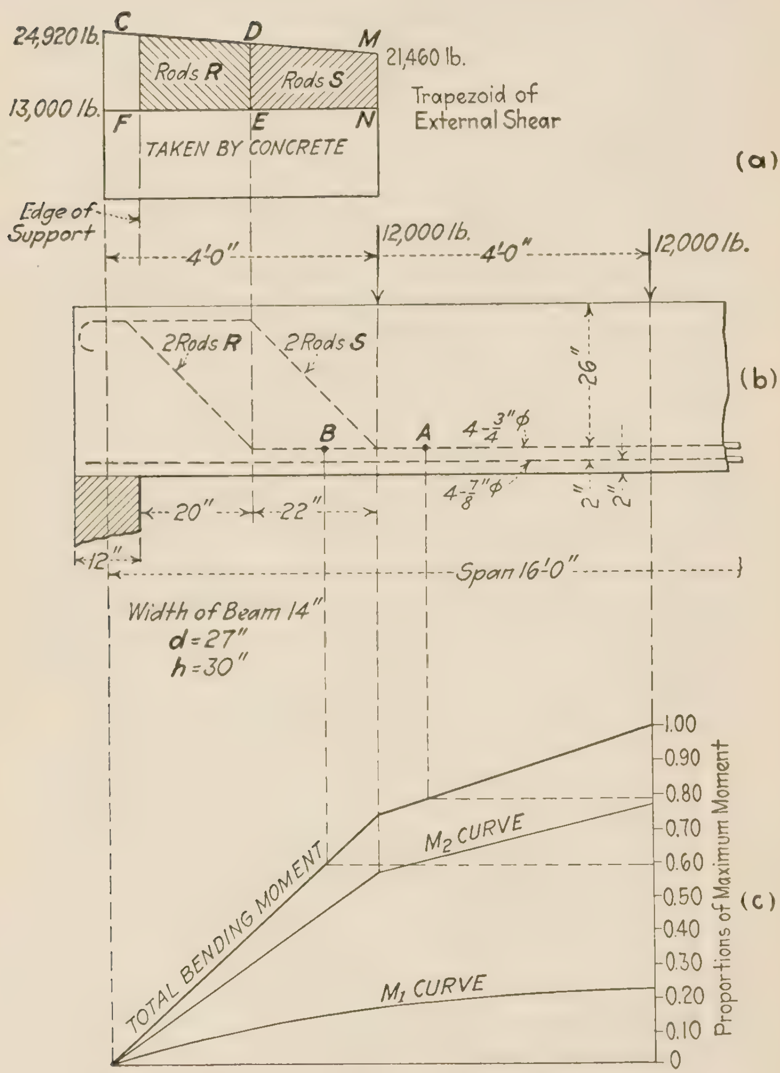

24,920 lb.
C
D
M
21,460 lb.
Rods R
Rods S
13,000 lb.
F
E
N
Trapezoid of External Shear
TAKEN BY CONCRETE
(a)
Edge of Support
4'-0"
12,000 lb.
4'-0"
12,000 lb.
2 Rods R
2 Rods S
26"
(b)
B
A
$4\text{-}\frac{3}{4}"\phi$
$4\text{-}\frac{7}{8}"\phi$
2"
2"
20"
22"
12"
Span 16'-0"
Width of Beam 14"
d=27"
h=30"
TOTAL BENDING MOMENT
M_2 CURVE
M_1 CURVE
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
Proportions of Maximum Moment
(c)

FIG. 19.

$.05 \times 2000$, or 100 lb. per sq. in., the total perimeter of rods required at the support, from equation (16) is,

$$\Sigma_o = \frac{24{,}920}{100 \times .856 \times 27} = 10.7 \text{ in.}$$

The four ⅞-in. bars are required, thus leaving the four ¾-in. bars to be bent up to reinforce the beam against diagonal tension.

As previously stated, a value of $j = \frac{7}{8}$ might be used in all computations involving shear and bond without material error.

At the support, $v = \dfrac{24{,}920}{14 \times .856 \times 27} = 77$ lb. per sq. in.

At the left of the first concentrated load, $V = 24{,}920 - (4 \times 865) = 21{,}460$ lb., and

$$v = \frac{21{,}460}{14 \times .856 \times 27} = 66 \text{ lb. per sq. in.}$$

At the right of the first concentrated load the unit shear is less than the allowable value of 40 lb. per sq. in. and no web reinforcement is required beyond this load.

The amount of external shear that can be resisted by the concrete equals $40 \times 14 \times .856 \times 27 = 13{,}000$ lb. The shaded portions of the shear trapezoid, Fig. 19, represent the amount of external shear at any section to be resisted by the web reinforcement.

In order to preserve the symmetry of the reinforcement at all points, the ¾-in. bars will be bent up in pairs and at 45 degrees with the horizontal. The maximum distance, measured from the point of bending, over which each pair of bars can provide for diagonal tension without overstressing the bars is, from equation (14*a*)

$$s = \frac{2 \times .4418 \times 16{,}000 \times .856 \times 27}{.7(24{,}920 - 13{,}000)} = 39 \text{ in.}$$

Since this is computed for the point of maximum shear, and since it is greater than the arbitrary maximum permitted by the specification as given in Art. 77, *i.e.*, $s = \dfrac{45 \times 27}{45 + 10} = 22$ in., the latter value will govern the final selection of the points of bending and no further investigation of the diagonal tension stresses in the rods will be necessary.

Investigation must now be made to determine whether these rods may be bent up at the proper points to care for all of the diagonal tension, and still leave enough steel at the bottom at all sections to care for the flexural stresses. Part (*c*) of Fig. 19 shows the bending moment diagram plotted to scale. This is constructed by plotting the curves for uniform load bending moment and for concentrated load bending moment on the same coordinate axis, and then adding the two graphically.

In determining the points at which the bars may be bent and still leave sufficient tensile resistance at the bottom of the beam, the difference in sizes of the bars must be considered. One pair of $\frac{3}{4}$-in. bars is equivalent to .21 of the steel. When the first pair is bent, there is left .79 of the total center steel. When the second pair is bent, .58 of the total remains at the bottom of the beam.

Point A, vertically above the point of intersection of a horizontal line through the .79 point of the maximum ordinate of the moment diagram and the total bending moment curve, represents a point to the left of which two $\frac{3}{4}$-in. rods may be bent up. The remaining six rods are sufficient properly to provide for the fiber stress due to bending at any section between point A and the support. Similarly, at point B a total of .42 of the steel, or four rods, may be bent. The rods should be continued at least 2 in. beyond these theoretical points before bending.

The first pair of bars will be bent at the concentrated load, and the next pair 22 in. to the left of this point. The remaining distance to the edge of the 12-in. support is 20 in. Since the amount of shear to be resisted by the web reinforcement is practically constant throughout the entire distance, being only slightly greater as the support is approached, the above arrangement assures practically equal stresses in each pair of bent bars; the slightly smaller distance from the point of bending of bars R to the edge of the support, 20 in., as compared with the distance between the points of bending of bars S and R, 22 in., is sufficient to equalize the total amount of diagonal tension, as measured by the shear, that is to be resisted by each pair of bars.

The bent bars must have a length beyond a point $.6d$ below the top face of the beam equal to $\frac{16{,}000}{4 \times 100} \times \frac{3}{4} = 30$ in. As an

additional safeguard, a hook will be made on the end of each bent bar as shown in Fig. 19.

T-BEAMS

83. Types of T-beams. When a reinforced concrete floor slab is constructed as a monolith with the supporting beam, and the slab and the beam thoroughly tied together by means of stirrups and bent-up bars, part of the slab may be assumed to assist the upper part of the beam in resisting compressive stresses. These two acting together constitute what is known as a T-beam (Fig. 20). The slab is spoken of as the flange, and the beam proper beneath is called the web or stem.

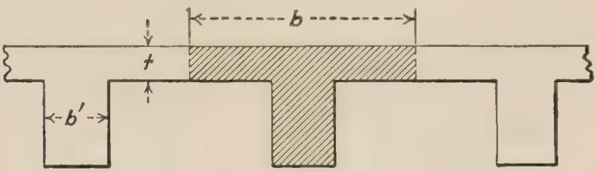


Fig. 20.

The exact width of slab that can be assumed as resisting the compressive forces is a variable. Tests have shown that it is dependent principally upon the relative thickness of the slab and upon the span of the beam. The Joint Committee recommends that the effective width of slab be determined as follows:

(*a*) It shall not exceed one-fourth of the span length of the beam.

(*b*) Its overhanging width on either side of the web shall not exceed eight times the thickness of the slab.[2]

In any case the flange width must not be greater than the distance between adjacent beams.

Another form of T-beam, which is of infrequent occurrence, is one which does not form a part of a floor system, the flange being provided merely to furnish sufficient area in compression. Since

[2] For beams having a flange on one side only, the effective width of flange is limited by the Joint Committee to a maximum of one-tenth of the span length of the beam; its overhanging width from the face of the web shall not exceed six times the thickness of the slab nor one-half the clear distance to the next beam.

the concrete in the lower part of the beam is assumed as taking no tension, its only purpose is to bind the tensile steel and the compressive concrete together. This involves mainly shearing stresses; all of the rectangular section is not required in large beams, so a saving in concrete results when the T-form is used. It is, however, usually more satisfactory to use a rectangular beam with compressive reinforcement to care for cases requiring an excessive amount of concrete rather than to resort to the T-section. A saving in cost of forms, and certain evident structural advantages of the rectangular beam will, in general, outweigh the good points of the T-beam.

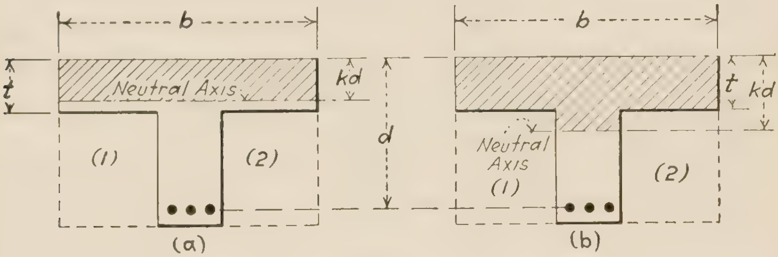


Fig. 21.

The neutral axis of a T-beam may lie either in the flange or in the web, depending upon the relation between the thickness of the flange, the depth of the beam, and the amount of steel. When the neutral axis is in the flange, *i.e.*, when kd is less than t, the equations derived in Art. 52 for rectangular beams must be used, the width of beam being equal to the effective width b of the flange. The reason for this is shown with the aid of Fig. 21*a*, which represents a beam, T-shaped in cross-section. The neutral axis is assumed above the bottom of the flange. The compressive area is represented by the shaded portion of the figure. If the additional concrete, indicated by the areas (1) and (2), had been added when the beam was poured, the physical cross-section would be rectangular in shape, with a width equal to b. No bending strength would be added by the addition of this extra concrete, because the areas (1) and (2) are in the tension portion of the cross-section, and, as stated before, the tensile strength of the concrete is disregarded in all flexure formulas. The original T-shaped beam and the revised rectangular-shaped

beam are equal in flexural strength, and the rectangular beam equations for flexure apply.

When the neutral axis is in the web, *i.e.*, when kd is greater than t, the rectangular beam equations no longer apply. For, in Fig. 21*b*, if the extra concrete represented by the areas (1) and (2) were added to the original T-shaped beam, the resulting rectangular-shaped beam would actually be stronger in flexure than the original beam, because some of the added concrete (those portions of areas (1) and (2) which are above the neutral axis) would be in compression. The application of the rectangular beam equations to this condition would therefore be incorrect in theory. The proper equations for use in this case are derived in the following article in a manner similar to that used in the derivation of the rectangular beam equations, the difference in compression areas being taken into consideration.

When the neutral axis is at the bottom of the flange, *i.e.*, when $kd = t$, then by comparison of Figs. 21*a* and 21*b* it is obvious that both the rectangular beam equations and the T-beam equations will give the same results, *i.e.*, the values of k, j, M_c, M_s, etc., obtained by the one set of equations, will be the same as the corresponding values obtained by the other set of equations.

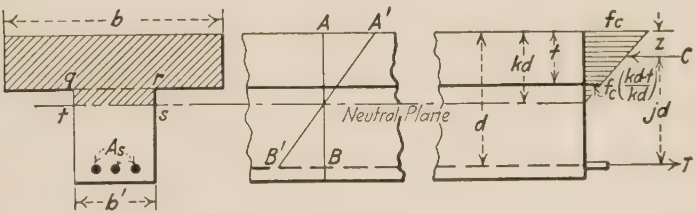


Fig. 22.

84. Flexure Formulas (Neutral axis in the web). Figure 22 represents an element of a T-beam. The amount of compression in the web, represented by the area *qrst* in the cross-section, is usually small in comparison with that in the flange, and hence is neglected in the derivation of equations for ordinary design. From the assumption that deformations vary as the distance from the neutral axis,

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} = \frac{k}{1 - k} \tag{a}$$

Since $E = \dfrac{\text{unit stress}}{\text{unit deformation}}$

$$AA' = \frac{f_c}{E_c} \text{ and } BB' = \frac{f_s}{E_s}$$

$$\frac{AA'}{BB'} = \frac{nf_c}{f_s} = \frac{n}{r} \tag{b}$$

Equating (a) and (b) and solving for k,

$$k = \frac{n}{n + r} \tag{17}$$

This gives an expression for the value of k when n and r are known. This equation can be used only in the design of a true T-beam, that is, one not a part of a floor system, since in such a problem just enough flange width will be provided to bring the unit stress in the concrete to its maximum allowable value simultaneously with that in the steel—the ratio r is known. In a T-beam which is part of a floor system already designed, the compressive area is so large (b is taken as one-fourth the span or $16t$ plus b') that when f_s is a maximum, f_c is only a relatively small value—the ratio r is not known.

The total tension $= A_s f_s$.

The total compression is represented by a trapezoid whose parallel sides are f_c and $f_c \times \dfrac{kd - t}{kd}$, the amount of compression being, therefore,

$$\frac{f_c + f_c \times \dfrac{kd - t}{kd}}{2} \times t \times b = f_c \times \frac{2kd - t}{2kd} \times bt$$

For equilibrium, the total tension must equal the total compression, hence

$$A_s f_s = pbdf_s = f_c \cdot \frac{2kd - t}{2kd} \cdot bt \tag{c}$$

As in the derivation of rectangular beam equations, the relation between the unit stresses in steel and concrete is given by the equation

$$f_c = f_s \cdot \frac{k}{n(1 - k)} \tag{18}$$

Substituting from equation (18) in (*c*) to eliminate unit stresses,

$$k = \frac{np + \frac{1}{2}\left(\frac{t}{d}\right)^2}{np + \left(\frac{t}{d}\right)} \tag{19}$$

The distance of the center of compression (center of gravity of the trapezoid) from the upper face of the beam is

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \tag{20}$$

and the lever arm of the couple formed by the tensile and compressive forces is

$$jd = d - z \tag{21}$$

From equations (19), (20), and (21),

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \cdot \left(\frac{1}{2pn}\right)}{6 - 3\left(\frac{t}{d}\right)} \tag{22}$$

The resisting moments of the steel and concrete equal, respectively,

$$M_s = A_s f_s j d \tag{23}$$

and

$$M_c = f_c\left(1 - \frac{t}{2kd}\right) bi \cdot jd \tag{24}$$

Since the center of gravity of the trapezoid is always above the middle of the slab, the arm of the resisting couple is never less than $d - \frac{1}{2}t$. The average unit compressive stress

$$\frac{f_c + f_c \cdot \frac{kd - t}{kd}}{2} = f_c\left(1 - \frac{t}{2kd}\right)$$

is never so small as $\frac{1}{2}f_c$ except when the neutral axis is at the bottom of the slab, in which case rectangular beam equations apply. The above equations for resisting moments may then be approximated by substituting these limiting values for jd and $f_c\left(1 - \frac{t}{2kd}\right)$ respectively. Then

$$M_s = A_s f_s (d - \tfrac{1}{2}t) \tag{25}$$

and

$$M_c = \tfrac{1}{2} f_c bt (d - \tfrac{1}{2}t) \tag{26}$$

In the design of a continuous T-beam at the support, a slightly larger amount of steel will be required there than at the center. This is due to the fact that the value of j at the support will in all cases be less than at the center. Since the tension steel at the support of such beams is usually provided by bending up from each side one-half of the steel furnished at the center, a slight excess at this latter point is often of advantage. The use of equations (25) and (26) *in design* is therefore justified for all practical purposes (see Art. 96).

85. Shearing Strength of T-beams. Due to the relatively large width of flange, it is safe to say that the compressive strength of the beam will seldom govern the design. Since the only stress that will be imposed upon the concrete below the neutral axis is that of shear (the concrete is assumed incapable of resisting tensile stresses), it follows that the effective cross-section of the stem of the beam, $b'd$, need merely be large enough to keep the horizontal shear below its allowable value.

Since
$$v = \frac{V}{b'jd}$$
the amount of web area required equals
$$b'd = \frac{V}{vj} \tag{27}$$

Figure 11a explains the use of b' instead of b in equation (27).

The value of j may be taken as seven-eighths in the preliminary investigation, and revision made later if necessary. It should be noted that in a continuous T-beam, the negative moment at the support results in a rectangular beam section. The value of j should be selected accordingly if it is deemed necessary to make this revision (see Art. 96).

In long beams with light loads, it is possible that the compressive strength of the beam may govern. In such cases equation (26) may be used to get an approximate value of d; the review of the assumed section will then determine if any revision should be made.

86. Ratio of Depth of Beam to Breadth of Stem. Numerous formulas have been devised to determine the economical depth of a T-beam, but very often the available head room is limited and the results of these formulas would exceed the limitation. The

use of economical depth formulas for shallow beams such as are encountered in ordinary building construction involves, for this reason, an unnecessary computation, practical considerations in most cases governing the design.

A study of numerous successful designs shows that for ordinary beams a ratio between b' and d of one-half to one-third gives satisfactory results. For very large and deep beams a ratio of one-fourth is permissible. In modern building construction, the shallower and wider beam is to be preferred in order to obtain the maximum head room and minimum light obstruction.

Taylor and Thompson, after a comparison of a number of representative designs, suggest the approximate rule to make the depth in inches equal to the span of the beam in feet. The breadth of stem is then fixed by the shearing area required.[2]

87. Diagrams for Design and Review of T-beams. Since equations (19) and (22) depend only upon the relation between t and d, percentage of steel and value of n, all of which would be known in reviewing a T-beam, the values of k and j as expressed by these equations may be obtained from Diagrams 5 and 6, which are constructed with the known quantities as variables. Diagram 5 is based on $n = 12$ and Diagram 6 on $n = 15$. Points which do not fall within the limit of the curved lines indicate that the neutral axis is in the flange (kd is less than t) and so the tables for rectangular beams should be used. Since the value of p must be known, Diagrams 5 and 6 are of no use in design.

Diagrams convenient for use in designing a T-beam may be constructed from equation (24), rewritten in the form

$$\frac{M}{bd^2} = f_c \times \left(1 - \frac{t}{2kd}\right) \times \frac{t}{d} \times j$$

Since k and j are functions of f_s and f_c, the variables are f_s, f_c, $\frac{M}{bd^2}$ and $\frac{t}{d}$. With $f_s = 16{,}000$ (an economical design stresses the steel to the limit), Diagrams 2 and 3 have been plotted so that from the left half the value of f_c corresponding to any given rela-

[2] The Joint Committee recommends that beams in which the T-form is used only for the purpose of providing additional compressive area of concrete shall have a flange thickness not less than one-half the width of web, and a total flange width not more than four times the web thickness.

tion between $\frac{M}{bd^2}$ and $\frac{t}{d}$ may readily be found, and from the right half, the value of j which will give this relation between f_s and f_c. These diagrams cannot be used in the review of a beam since the relation between f_s and f_c is unknown. Diagram 4 is similar to Diagrams 2 and 3, but is based on $f_s = 18{,}000$ and $n = 15$.

88. Types of T-beam Problems. There are three main types of problems that may be encountered in practice.

1. To find the moment of resistance or fiber stresses.

The values of k and j may be found from equations (19) and (22) or from Diagrams 5 and 6, the values of the fiber stresses from equations (23) and (18), or the resisting moments from equations (23) and (24), the smaller of the latter being the resisting moment of the beam. Since the resisting moment of the steel will usually govern in T-beams whose flange is a part of a floor system, it is generally quicker to find the value of M_s from equation (23), and then substitute in equation (18) $f_s = 16{,}000$ (or its limiting value) and determine f_c. If this is less than the allowable, the assumption that the steel governs is correct for the case in question. If it is greater than the allowable, determine from equation (18) the value of f_s that corresponds to the maximum permissible value of f_c, and use this value of f_s in equation (23). The result will be the moment in the steel when the concrete is stressed just to its limit, and hence is the true resisting moment of the beam. (If kd is less than t, then the neutral axis is in the flange; formulas for rectangular beams should then be used, the width being equal to the flange width b of the T-beam.)

2. To design a T-beam in which the flange is a portion of a floor slab already determined.

Find from equation (27) of Art. 85, the cross-section $b'd$ required, and select the width of stem and depth of beam with reference to the most satisfactory shape of beam, spacing of rods, etc. Usually d should be taken as from two to three times b'. The amount of steel may then be determined from equation (25).

In order to determine the steel area more accurately, compute the value of j from equation (22), substituting the value of p corresponding to the approximate area of steel. Equation (23) will

then give the true steel area that is required. A slight variation between the values of p as determined by the approximate method and the true method will cause but insignificant difference in equation (22), so further substituting is unnecessary. The value of k should be found from equation (19) to ascertain whether the neutral axis is in the stem or flange. Equation (2) for rectangular beams would give the same information.

3. To design a T-beam whose flange is not a part of a floor system.

In designing a beam of this type, determine from equation (27) of Art. 85 the shearing area required and select suitable proportions for the breadth and depth of web. From equations (17), (20), and (21), the values of k and j may be determined. Equations (23) and (24) will then give the area of steel and breadth of flange required.

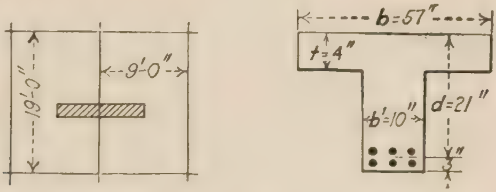


Fig. 23.

In all work on T-beams where flexural stresses are concerned, that is, in the determination of k, j, fiber stresses, resisting moments, and area of steel, the value of b is the width of flange. In the determination of the shearing area required, the value of b' is the width of stem. The reasons for each should be obvious from a study of the foregoing articles.

89. *Illustrative Problems.*

I. A slab floor 4 in. thick is supported by reinforced concrete beams 9 ft.-0 in. center to center (Fig. 23) which together with the slab act as T-beams. The beams are continuous and their span is 19 ft.-0 in. The slab supports a live load of 175 lb. per sq. ft. The section of the beam below the slab is 10×20 in. The reinforcement consists of six $\frac{3}{4}$-in. round bars in two rows, 2 in. center to center vertically, the center of the lower row being 2 in. above the lower surface of the beam. $n = 15$. Determine f_s and f_c at the center of the span.

Weight of slab $= \frac{4}{12} \times 150 = 50$ lb. per sq. ft.

Total load on slab $= 225$ lb. per sq. ft.

Load from slab on each beam $= 9 \times 225 = 2025$ lb. per ft.

Weight of beam below slab $= \dfrac{10 \times 20}{144} \times 150 = 208$ lb. per ft.

Total load on beam $= 2233$ lb. per ft.

$M = \frac{1}{12} \times 2233 \times \overline{19}^2 \times 12 = 805{,}000$ in.-lb.

The breadth of flange cannot exceed either of the following:

$$\tfrac{1}{4} \times 19 \times 12 = 57 \text{ in.}$$

or
$$(16 \times 4) + 10 = 74 \text{ in.}$$

Hence
$$b = 57 \text{ in.}$$

$$p = \frac{2.65}{57 \times 21} = .0022$$

$$\frac{t}{d} = \frac{4}{21} = .19$$

From Diagram 6, $k = .230$ and $j = .928$; $kd = 4.83$ in.

Hence
$$f_s = \frac{805{,}000}{2.65 \times .928 \times 21} = 15{,}500 \text{ lb. per sq. in.}$$

$$f_c = \frac{15{,}500 \times .230}{(1 - .230) \times 15} = 310 \text{ lb. per sq. in.}$$

II. Determine the true resisting moment of the beam whose dimensions and reinforcement are given in the preceding example, assuming a 2000-lb. concrete.

$$M_s = 2.65 \times 16{,}000 \times .928 \times 21 = 827{,}000 \text{ in.-lb.}$$

The corresponding stress in the concrete

$$f_c = \frac{16{,}000 \times .230}{15(1 - .230)} = 319 \text{ lb. per sq. in.}$$

The steel, therefore, governs as assumed, since when it is stressed to its limit, 16,000 lb. per sq. in., the concrete unit stress is well within the allowable.

III. Using the specifications of the Joint Committee for a 2000-lb. concrete, determine the cross-section of the web below the slab and the sectional area of steel required for a continuous T-beam supporting a 5-in. floor slab which sustains a live load of 100 lb. per sq. ft. Distance center to center of adjacent beams, 11 ft.-0 in. Span of beams, 23 ft.-0 in. (Fig. 24).

Weight of slab $= \frac{5}{12} \times 150 = 62.5$ lb. per sq. ft.

Total load on slab $= 162.5$ lb. per sq. ft.

Load on beam from slab $= 11 \times 162.5 = 1790$ lb. per ft.

Assume weight of stem $= \frac{1}{10} \times 1790 = 180$ lb. per ft.

Total load on beam $= 1970$ lb. per ft.

Maximum shear $= V = 1970 \times \frac{23}{2} = 22{,}600$ lb.

Assuming that web reinforcement is to be provided, the allowable unit shearing stress equals $.06 \times 2000 = 120$ lb. per sq. in.

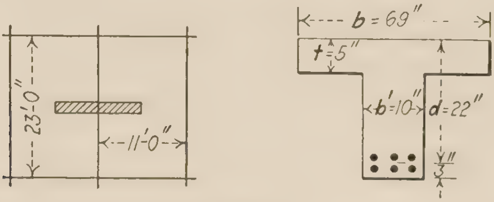


FIG. 24.

Cross-section required for shear $= b'd = \dfrac{22{,}600}{.875 \times 120} =$ 215 sq. in.

Selecting values so that d is from two to three times b', let $b' = 10$ in. and $d = 22$ in.

Since two rows of rods will undoubtedly be necessary, the total height of beam equals $22 + 3 = 25$ in., and the cross-section below the slab equals 10×20 in.

The actual weight of stem below slab $= \dfrac{10 \times 20}{144} \times 150 =$ 210 lb. per ft.

The revised maximum shear $= (1790 + 210) \times \frac{23}{2} = 23{,}000$ lb.

The revised $b'd$ required $= \dfrac{23{,}000}{.875 \times 120} = 219$ sq. in.

This amount is furnished by the selected cross-section.

The maximum moment $= \frac{1}{12} \times 2000 \times 23^2 \times 12 =$ 1,060,000 in.-lb.

$$A_s = \frac{1{,}060{,}000}{16{,}000(22 - 2.5)} = 3.40 \text{ sq. in.}$$

This is furnished by eight $\frac{3}{4}$-in. round rods, the area of which is 3.53 sq. in.

For all practical purposes of design this approximate area of steel would be satisfactory. If the true area were to be determined, the procedure would be as follows:

$$\frac{t}{d} = \frac{5}{22} = .227$$

b is fixed in this case as ¼ span or ¼ (23 × 12) = 69 in. Assuming that the true area of steel will be equal to the approximate value just found,

$$p = \frac{3.53}{69 \times 22} = .0023$$

Equation (22) or Diagram 6 gives $j = .923$ and the true area of steel required becomes

$$A_s = \frac{1{,}060{,}000}{16{,}000 \times .923 \times 22} = 3.26 \text{ sq. in.}$$

The assumed bars are satisfactory.

Equation (19) or Diagram 6 gives $k = .235$. Therefore, $kd =$ 5.2 in. so the equations for T-beams apply as assumed. The maximum unit concrete stress is found by equation (24) to be below the allowable value of 800 lb. per sq. in.

IV. Design a simply supported, isolated T-beam with a span of 30 ft.-0 in. which must support a live load of 3000 lb. per lin. ft. Use working stresses as follows: $f_c = 650$, $f_s = 16{,}000$, $v =$ 120, $n = 15$.

Assume the weight of beam = 950 lb. per lin. ft.

The total load to be carried = 3950 lb. per lin. ft.

The maximum moment = $\frac{1}{8} \times 3950 \times \overline{30}^2 \times 12 = 5{,}330{,}000$ in.-lb.

The maximum shear = $3950 \times \frac{30}{2} = 59{,}300$ lb.

$$b'd \text{ required} = \frac{59{,}300}{120 \times .875} = 565 \text{ sq. in.}$$

Since b' is to be from ½ to ⅓d, the values selected will be $b' = 16$ in. and $d = 36$ in. These are selected in preference to any other possible combination that falls within the limits stated above, in order to keep b' in even inches and to secure as wide a beam as possible to allow for convenient placing of the reinforcement.

The thickness of flange is usually made $\frac{1}{3}d$, hence t will be taken as 12 in.

$$\frac{t}{d} = \frac{12}{36} = .333$$

$$k = \frac{15}{15 + \frac{16,000}{650}} = .379 \text{ and } kd = .379 \times 36 = 13.6 \text{ in.}$$

Therefore, the neutral axis is in the stem and involves a T-section for the problem.

From equations (20) and (21) $z = 4.42$ in. and $j = .877$

$$A_s = \frac{5,330,000}{16,000 \times .877 \times 36} = 10.5 \text{ sq. in.}$$

The width of flange required is determined from equation (24), all quantities of which are known, to be 38.6 in. A width of 40 in. will be used.

Seven $1\frac{1}{4}$-in. square rods will be selected, placed in two rows. The total height of beam is then 39 in., and the weight per foot, 950 lb. as assumed. The design is, therefore, satisfactory (see footnote, p. 111).

Rectangular Beams Reinforced for Compression

90. Use of Beams Reinforced for Compression. A beam is reinforced for compression when its size is limited by structural conditions or when the cross-section necessary to carry a given load without danger of crushing the concrete in the compressive area would be beyond practical limits. The moment in excess of the carrying capacity of the concrete is provided for by placing steel in the compressive portion of the beam. While the effectiveness of steel in compression has been questioned, numerous tests indicate that the steel assumes its proportion of the stress.

A common example of a rectangular beam reinforced for compression occurs at the supports of a continuous T-beam, that is, a floor beam or girder in monolithic beam and girder floor construction. On account of its importance, this is considered separately in Art. 96.

91. Formulas for Design. The notation used in the following derivations is as follows:

M_1 = the moment which can be developed by the limited cross-section of concrete.

M_2 = the moment that must be developed in excess of the compressive strength of the concrete.

M = the total moment to be developed by the beam = $M_1 + M_2$.

A_{s_1} = area of tensile steel for beam without compressive reinforcement necessary to develop the moment M_1.

A_{s_2} = area of additional tensile steel necessary to develop the moment M_2.

A_s = total tensile steel = $A_{s_1} + A_{s_2}$.

A'_s = total area of compressive steel.

f_s = unit stress in tensile steel.

f'_s = unit stress in compressive steel.

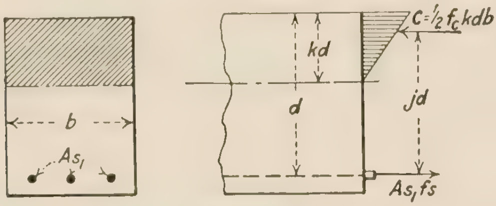


FIG. 25.

Assuming that a rectangular beam which will be called upon to resist a bending moment M is limited in size to an effective cross-section bd, the resisting moment of the concrete M_1 being less than the moment M, compressive steel of an amount A'_s will be required in order to keep the unit stress in the concrete within the allowable limit.

The moment M_1 depends upon the concrete and equals (see Fig. 25 and Art. 52)

$$M_1 = \tfrac{1}{2} f_c kjbd^2 = Kbd^2 \qquad (28)$$

The area of steel required to provide sufficient tensile resistance fully to develop the strength of the concrete is

$$A_{s_1} = \frac{M_1}{f_s jd} \qquad (29)$$

since the resisting moment of the tensile steel = $A_{s_1} f_s jd$.

The values of k, j, and K are found by the same equations as for rectangular beams with only tensile reinforcement.

A moment of an amount $= M_2 = M - M_1$ still remains to be provided for by the necessary amount of compressive steel and additional tensile steel A_{s_2}. The stresses in these two quantities of steel form a couple, the lever arm of which is $d - d'$ (Fig. 26). The resisting moment of the couple is, therefore, $A_{s_2}f_s(d - d')$, and this must be sufficient to develop the moment M_2; hence

$$M_2 = A_{s_2}f_s(d - d') \text{ from which}$$

$$A_{s_2} = \frac{M_2}{f_s(d - d')} \tag{30}$$

The total tensile steel then equals

$$A_s = A_{s_1} + A_{s_2} \tag{31}$$

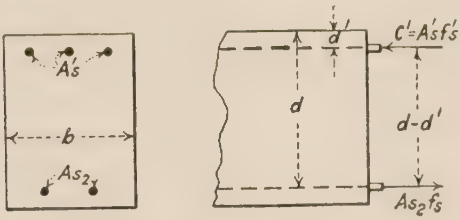


Fig. 26.

Since the beam must be in equilibrium

$$A'_s f'_s = A_{s_2} f_s \tag{a}$$

From the assumption that unit stresses vary as the distance from the neutral axis

$$\frac{f_s}{f'_s} = \frac{d - kd}{kd - d'}$$

$$f'_s = f_s \cdot \frac{k - \left(\frac{d'}{d}\right)}{1 - k} \tag{b}$$

Substituting in equation (a) the value of f'_s from equation (b) in order to eliminate the unit stresses,

$$A'_s f_s \cdot \frac{k - \left(\frac{d'}{d}\right)}{1 - k} = A_{s_2} f_s \text{ and solving for } A'_s$$

$$A'_s = A_{s_2} \cdot \frac{1 - k}{k - \left(\frac{d'}{d}\right)} \tag{32}$$

The position of the neutral axis has not been changed by the addition of the compressive steel, since just enough tensile steel was added to counterbalance it. Therefore, the value of k remains constant throughout the design. On account of the commercial sizes of reinforcing bars, as soon as the rods are selected there will, in all probability, be a slight difference in both tensile steel and compressive steel from the theoretical. Hence the balance in stresses indicated above no longer exists, and in reviewing the beam, the values of k and j will have to be found from equations other than the above. Such equations are derived in the following article.

92. Formulas for Review. The equations for k, j, and the resisting moment to be used in reviewing a rectangular beam

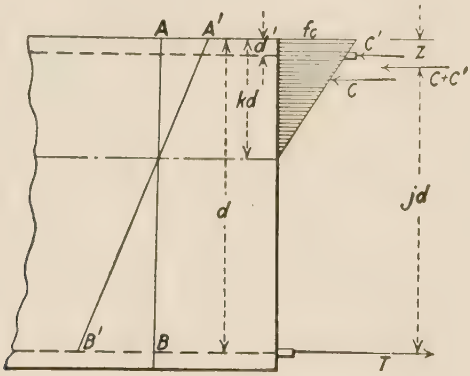


FIG. 27.

with compressive reinforcement are derived in a manner similar to those for rectangular beams with tensile reinforcement only.

The total tension in the steel $= T = A_sf_s = pbdf_s$

The total compression in steel and concrete $= C + C'$

This equals

$$A'_sf'_s + \tfrac{1}{2}f_ckdb = p'bdf'_s + \tfrac{1}{2}f_ckdb = bd(\tfrac{1}{2}f_ck + p'f'_s)$$

in which p = the percentage of tensile steel and p' the percentage of compressive steel in terms of the effective cross-section bd (see Fig. 27).

Since for equilibrium the total tension must equal the total compression

$$pf_s bd = bd(\tfrac{1}{2}f_c k + p'f'_s)$$

or

$$pf_s = \tfrac{1}{2}f_c k + p'f'_s \tag{c}$$

From Fig. 27 it is seen that, since deformations vary as the distance from the neutral axis,

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} \tag{d}$$

but $AA' = \dfrac{f_c}{E_c}$ and $BB' = \dfrac{f_s}{E_s}$.

Hence

$$\frac{AA'}{BB'} = \frac{nf_c}{f_s} \tag{e}$$

Equating (d) and (e) and solving for f_c

$$f_c = \frac{f_s k}{n(1 - k)} \tag{33}$$

From equation (b) of Art. 91

$$f'_s = \frac{f_s\left(k - \dfrac{d'}{d}\right)}{1 - k} \tag{34}$$

Substituting in equation (c) the value of f_c from equation (33) and that of f'_s from equation (34),

$$pf_s = \frac{f_s k^2}{2n(1 - k)} + \frac{p'f_s\left(k - \dfrac{d'}{d}\right)}{1 - k}$$

from which

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \tag{35}$$

By taking the summation of moments of the compressive forces about the top of the beam, the position of their center of gravity may be determined. The resulting equation is

$$z = \frac{\tfrac{1}{3}kdC + d'C'}{C + C'} = \frac{\tfrac{1}{3}kd + d'\dfrac{C'}{C}}{1 + \dfrac{C'}{C}}$$

Since C', the total compressive stress in the steel $= p'bdf'_s$, and C, the total compressive stress in the concrete $= \frac{bdf_ck}{2}$

$$\frac{C'}{C} = \frac{2p'f'_s}{f_ck}$$

Substituting from equations (33) and (34) the values of f_c and f'_s

$$\frac{C'}{C} = \frac{2p'f_s\left(k - \frac{d'}{d}\right)}{(1-k)\cdot\frac{f_sk^2}{n(1-k)}} = \frac{2p'n\left(k - \frac{d'}{d}\right)}{k^2}$$

Substituting this value of $\frac{C'}{C}$ in the equation given above for z,

$$z = \frac{\frac{1}{3}k^3d + 2p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)}$$

From Fig. 27, $jd = d - z$, or $j = 1 - \frac{z}{d}$, and therefore

$$j = \frac{k^2 - \frac{1}{3}k^3 + 2p'n\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)} \tag{36}$$

The resisting moment of the steel is found by taking moments about the center of gravity of the compressive forces

$$M_s = A_sf_sjd \tag{37}$$

The resisting moment of the concrete could be found by taking moments about the center of the tensile steel, but the resulting equation would be extremely cumbersome. It is simpler to use equation (37) in conjunction with equation (33) as explained in Art. 88, in determining the safe resisting moment of the concrete.

93. Diagrams for Review of Beams Reinforced for Compression. In order to simplify the computations for the review of beams of this type, Diagrams 7 and 8 have been constructed, from which the values of k and j as represented by equations (35) and (36) may readily be found. Since these two quantities depend upon the relation between p, p', n, and $\frac{d'}{d}$, the curves have been drawn with these quantities as variables. For inter-

mediate values of $\frac{d'}{d}$, interpolation is necessary to find the true values of k and j.

The equation for the resisting moment of the compressive forces in a concrete beam reinforced for compression is

$$M_c = \tfrac{1}{2}f_ck\left(1 - \frac{k}{3}\right)bd^2 + f'_sp'bd(d - d')$$

By substituting the values of f'_s and j in the above equation

$$M_c = bd^2f_c \cdot R$$

in which

$$R = \frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)$$

Similarly, $M_s = bd^2f_s \cdot N$

in which

$$N = p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1 - k)} \cdot \left(\frac{k}{3} - \frac{d'}{d}\right)$$

Solving these equations for f_c and f_s

$$f_c = \frac{M}{bd^2} \cdot \frac{1}{R}$$

$$f_s = \frac{M}{bd^2} \cdot \frac{1}{N}$$

Diagrams 9 and 10 give values of R and N for different values of p and p', and also f_c and f_s for various values of $\frac{M}{bd^2}$, k having been determined from equation (35). The procedure to be followed in using these diagrams is first to determine p and p'; then enter the proper diagram $\left(\text{this depends on } \frac{d'}{d}\right)$ with the computed values of p and the ratio $\frac{p'}{p}$, the intersection of which determines R (or N); follow along the R (or N) curve thus located, to the intersection with a horizontal line through the existing value of $\frac{M}{bd^2}$; the value of f_c (or f_s) may then be read directly from the curve, interpolating if necessary. The numerical values of R and N are unnecessary to the solution of the problem and are therefore omitted from the curves.

94. Types of Problems. In designing a beam of limited cross-section which is called upon to support a greater load than the

compressive strength of the concrete permits, the rational process is to solve equations (28) to (32) in order. The amounts of tensile and compressive steel are then known, so that proper selection of rods can be made.

In reviewing a beam with compressive reinforcement to find the existing unit stresses under a given load, the values of k and j may be found from equations (35) and (36) or Diagrams 7 and 8, the value of f_s from equation (37), f_c from equation (33), and f'_s from equation (34).

If the safe resisting moment is required, the maximum allowable values of f_s and f_c having been given, k and j should first be found from equations (35) and (36). Then to determine whether the strength of the concrete or of the tensile steel governs, the value of f_s corresponding to the maximum allowable value of f_c should be found from equation (33). If this is greater than the allowable, the safe working limit of the strength of the steel will be exceeded, provided the full strength of the concrete is developed, that is, the steel governs the strength of the beam. Hence the true resisting moment of the beam will be found from equation (37), f_s being taken as the specified limit. If f_s determined as above is less than the allowable, the full strength of the steel cannot be developed without overstressing the concrete. Hence the concrete governs, and the safe resisting moment of the beam may be found from equation (37), the value of f_s being that just computed from equation (33)—the value that results when the concrete is stressed to its maximum.

95. *Illustrative Problems.*

I. A continuous reinforced concrete beam having a span of 20 ft.-0 in. is limited in cross-section to 8 × 18 in. The beam sustains a live load of 500 lb. per lin. ft. $f_s = 16{,}000$, $f_c = 650$, and $n = 15$. Using 2 in. of insulation measured from the center of the bars, determine the area of steel required for tension (A_s) and for compression (A'_s).

$$\text{Weight of beam} = \frac{8 \times 18}{144} \times 150 = 150 \text{ lb. per lin. ft.}$$

Total load carried by beam = 650 lb. per lin. ft.

$$M = \tfrac{1}{12} \times 650 \times \overline{20}^2 \times 12 = 260{,}000 \text{ in.-lb.}$$

From Table IV, $K = 107.7$, $k = .379$, $j = .874$

Assuming that only one row of tensile steel will be required,

$$M_1 = 107.7 \times 8 \times \overline{16}^2 = 220{,}000 \text{ in.-lb.}$$

$$M_2 = 260{,}000 - 220{,}000 = 40{,}000 \text{ in.-lb.}$$

$$A_{s_1} = \frac{220{,}000}{16{,}000 \times .874 \times 16} = 0.98 \text{ sq. in.}$$

$$A_{s_2} = \frac{40{,}000}{16{,}000(16 - 2)} = 0.18 \text{ sq. in.}$$

$$A_s = .98 + .18 = 1.16 \text{ sq. in. of tensile steel.}$$

$$A'_s = .18 \times \frac{1 - .379}{.379 - 2/16} = 0.45 \text{ sq. in. of compressive steel.}$$

To satisfy the above, four 5/8-in. round bars in tension, and two 5/8-in. round bars in compression are selected, each set in one row as assumed.

II. A simply supported concrete beam whose span is 21 ft.-0 in. has a cross-section of 8 × 19 in., and is reinforced as follows: for tension, four 7/8-in. round bars, and for compression, four 7/8-in. round bars, each set in two rows, the center of the row nearest the surface being 2 in. from the surface, and the vertical distance center to center of rows being 2 in.; $n = 15$. Determine the values of the unit stresses in the tensile steel, compressive steel, and concrete, if the beam sustains a live load of 600 lb. per lin. ft.

$$\text{Weight of beam} = \frac{8 \times 19}{144} \times 150 = 160 \text{ lb. per lin. ft.}$$

Total load carried by beam = 760 lb. per lin. ft.

$$M = \tfrac{1}{8} \times 760 \times \overline{21}^2 \times 12 = 505{,}000 \text{ in.-lb.}$$

$$p = p' = \frac{2.41}{8 \times 16} = .0188 \text{ and } \frac{d'}{d} = \frac{3}{16} = .19$$

From Diagram 8, interpolating for $\frac{d'}{d} = .19$, $k = .429$, and $j = .837$

$$f_s = \frac{505{,}000}{2.41 \times .837 \times 16} = 15{,}700 \text{ lb. per sq. in.}$$

$$f_c = \frac{15{,}700 \times .429}{15(1 - .429)} = 785 \text{ lb. per sq. in.}$$

$$f'_s = \frac{15{,}700(.429 - 3/16)}{1 - .429} = 6{,}650 \text{ lb. per sq. in.}$$

III. A reinforced concrete beam has a cross-section of 12 × 30 in. and is reinforced as follows: for tension, eight ¾-in. square bars in two rows, 2 in. center to center, the center of the lower row being 2 in. above the lower surface of the beam, and for compression four ¾-in. square bars in one row, the center of which is 2 in. below the upper surface of the beam. $f_s = 16{,}000$, $f_c = 700$, and $n = 15$. What is the safe resisting moment of the beam?

$$\frac{d'}{d} = \frac{2}{27} = .07$$

$$p = \frac{4.5}{12 \times 27} = .0139$$

$$p' = \tfrac{1}{2}p = .0069$$

From Diagram 8, $j = .886$ and $k = .414$

When f_c is a maximum, the corresponding value of f_s is found from equation (33) to be

$$\frac{700 \times 15(1 - .414)}{.414} = 14{,}900 \text{ lb. per sq. in.}$$

Therefore, the strength of the beam depends upon the concrete; and the safe maximum resisting moment occurs when the steel is stressed to 14,900 lb. per sq. in. and equals:

$$M = 4.5 \times 14{,}900 \times .886 \times 27 = 1{,}600{,}000 \text{ in.-lb.}$$

The full tensile strength of the steel cannot be utilized without overstressing the concrete.

IV. A typical beam in a reinforced concrete floor of the beam and girder type has an effective depth of 25.5 in., a width of stem of 12 in., and reinforcement consisting of four 1-in. square bars in one row, the center of which is 2.5 in. above the lower surface of the beam. One-half of the bars are bent up over the support to provide for negative moment, and the other half continued straight through the support to assist in resisting compressive stresses at that point. The maximum moment in the beam equals 1,500,000 in.-lb. What are the values of f_s and f_c at the support? (See Art. 96.)

$$p = p' = \frac{4.0}{12 \times 25.5} = .0131$$

$$\frac{d'}{d} = \frac{2.5}{25.5} = .10$$

$$\frac{M}{bd^2} = \frac{1{,}500{,}000}{12 \times (25.5)^2} = 192$$

Entering Diagram 9 with $p = .0131$ and $p' = p$, and following the R curve thus determined to the intersection with a horizontal line through the value of $\frac{M}{bd^2} = 192$, it is found that

$$f_c = 650 \text{ lb. per sq. in.}$$

Similarly from Diagram 10

$$f_s = 16{,}600 \text{ lb. per. sq. in.}$$

96. The Design of a Continuous T-beam at the Supports. At the supports of a continuous T-beam the bending moment is negative so that the upper surface becomes the tensile surface, while the lower portion of the section of the beam is in compression. Since in reinforced concrete design the steel is assumed to resist all of the tensile forces, sufficient steel must be placed near the upper surface of the beam over the support to develop the negative moment at that point. Where the moment at the support is assumed numerically equal to the moment at midspan, the tensile steel required near the upper surface over the support is approximately equal to that required in the lower section of the beam at the center of the span to develop the positive bending moment. The length of the lever arm of the resisting couple (jd) is often somewhat smaller at the support than at midspan, in which case the amount of tensile steel required over the support is slightly the greater.

The usual method of providing for this negative bending moment is to bend up about one-half of the reinforcing bars from each adjacent beam and extend them far enough across the support to insure proper development of the negative moment. In the case of a uniformly loaded beam they should be continued to the quarter or third point of the span, the assumed location of the point of zero negative moment. More rods must be bent up or additional rods must be placed in the upper portion over the support if the allowable unit stresses are exceeded.

Since the tension side is uppermost, the flange of the T-beam can no longer be considered effective in resisting stress, hence the form of beam becomes rectangular at the support, the width being equal to the width of the stem.

On account of the small compressive area of concrete (now below the neutral axis) a failure by compression would often result if steel were not added in the compressive area to assist the concrete. Since only a portion of the horizontal rods are bent up over the support, the remaining rods may be brought straight through and extended far enough into the adjacent panel to develop their full strength in bond, and thus furnish the added compressive resistance required.

In determining the length necessary to accomplish this, it will be sufficient to consider only the maximum allowable stress in the rods, and furnish a length from the center of the support to the end of the rod properly to transmit this stress to the concrete. The full working strength of the steel in compression cannot be reached without exceeding the compressive strength of the surrounding concrete. In ordinary design, the stress to be used in determining the required length of embedment need only be equal to the maximum as determined by the concrete, that is nf_c.

Where one half of the longitudinal reinforcement, required at the center of the spans, is bent up from each of two adjacent beams, and the remainder is carried straight through beyond the support far enough to develop its compressive strength in bond, the amounts of tensile and compressive reinforcement are equal. If less compressive reinforcement is required, the bars from the adjacent beams need be carried beyond the support only far enough to develop a lap splice. With such an arrangement the compressive reinforcement is equal in amount to one half of the tensile reinforcement. In either case, the section of the beam at the support is that of an inverted rectangular beam reinforced for compression and it may be analyzed according to the principles of Art. 93.

Since the negative moment decreases very rapidly, and only a short section is under maximum stress, a higher compressive stress is allowed in the concrete at the support than at mid-span.

The bond stress on the tension rods at the support may be computed in the same manner as for the tension rods at the end of a simply supported beam. A slight excess of the computed stress over the allowable is of no consequence, since the actual

stress is undoubtedly less than the theoretical, due to the stiffening action of the bent-up portion of the rods.

The points at which the horizontal rods may be bent up will depend upon the variation in positive bending moment along the beam. The location of these points may be determined as in Art. 78. It is also necessary to determine the points at which the upper rods may be bent down, that is, the points at which the inclined rods must intersect the uppermost portion of the beam in order to furnish sufficient reinforcement for the negative moment existing at those sections. It is safe to assume the curve of negative moment as a straight line from the support to the point of zero moment, and determine the location of the bending points accordingly.

If the two sets of values, one for positive and one for negative moment, are such that the bending of the rods is impossible without exceeding the limitations imposed, a greater number of rods must be used at the center of the span, or additional rods placed over the support, and the design governed accordingly. In bending rods, care should be taken to keep the center of gravity of the remaining rods in a vertical plane through the axis of the beam. To accomplish this, the rods should be bent in pairs, or multiples of two, except when an odd number of rods is to be bent. In this case, bend first one from the middle and then the remaining ones in pairs, or bend three rods at one point.

It is usually desirable to provide for diagonal tension as much as possible with bent-up rods. On account of the restrictions placed upon the bending of the rods, it is necessary in most cases to add stirrups or some other form of web reinforcement fully to provide for the inclined stresses. The analysis is similar to that for simply supported beams, the spacing of the stirrups being computed for the regions over which they are required.

In all computations relating to diagonal tension, shear, and bond, the average value of j for the shape of beam under consideration may be used, and revision made later for any discrepancy. This value for rectangular beams reinforced for compression may be taken as .85 and for T-beams as .92. It is necessary to consider the section where the condition under investigation exists.

It is sometimes necessary that beams framing into columns or girders should be calculated as simple beams where a moment coefficient of $\frac{1}{8}wl^2$ is used. Some negative moment will actually exist at the supports due to the monolithic nature of the construction.[4] It is good practice to provide a small amount of steel at the top over the supports to prevent the formation of cracks, the amount of steel being left to the judgment of the designer.

97. Design of a Typical Floor Beam.[5] A fully continuous floor slab 4 in. in thickness is reinforced with $\frac{3}{8}$-in. round bars spaced $4\frac{1}{2}$ in. center to center. The slab is designed for a uniform live load of 100 lb. per sq. ft. and is supported on beams 9 ft.-0 in. center to center. The beams are continuous over girders 14 in. in width and 21 ft.-0 in. center to center. Use a 2000-lb. concrete. Design the beam.

The unit stresses allowable for a concrete of this strength are:

$f_c = 800$, except at supports where 900 is permissible.

$u = 100$ (deformed bars)

v (no web reinforcement) $= 40$

v (proper web reinforcement) $= 120$

$n = 15 \quad f_s = 16{,}000$

Assume the weight of the stem below the slab to be approximately 10 per cent of the total known load. Any error greater than 1 per cent of the total load should be corrected. The other limitations as to spacing of rods and ratio of depth to breadth of beam should be taken as outlined in the preceding articles.

Design of Beam at Center.

Weight of slab per square foot $= \frac{4}{12} \times 150 = 50$ lb.

Load on beam from slab $= (100 + 50) \times 9 = 1350$ lb. per ft.

Assume weight of stem $= 150$ lb. per ft.

Total load per linear foot $= 1500$ lb.

Maximum shear $= V = 1500 \times 10.5 = 15{,}750$ lb.

Maximum moment $= \frac{1}{12} \times 1500 \times 21^2 \times 12 = 663{,}000$ in.-lb.

The positive and negative moments are assumed equal.

As stated in Art. 96, the required beam section is determined by the shear at the support. The value of j to be used in the

[4] The Joint Committee recommends that a moment not less than $\frac{1}{16}wl^2$ shall be provided for at the ends of such uniformly loaded beams.

[5] See Art. 161 for another typical design of a floor beam.

computation of the shearing area necessary depends, therefore, upon the properties of the rectangular section reinforced for compression at the end of the beam. As this is still unknown, a preliminary estimate may be made with $j = .85$ and revision made later if necessary.

$$b'd \text{ required} = \frac{15{,}750}{.85 \times 120} = 154 \text{ sq. in.}$$

In order to keep d within the limits of two to three times b', let $b' = 8$ in. and $d = {}^{154}\!/_{8} = 19.2$ in., which may safely be reduced to 19 in. because of the strengthening action of the girder. This is the value of the required effective depth at the support to provide fully for shearing stresses, and is measured upward from the bottom of the beam (the compressive face) to the center of the tension steel (which is near the top of the slab).

At the point where the girder frames into the column, the tensile steel in the girder and the tensile steel in the beam which frames into the same column must be so arranged as to cross each other without interference, *i.e.*, the distance to the center of the steel from the top of the slab cannot be the same for both beam and girder. Sometimes the beam steel is so placed as to pass above the girder steel, and *vice versa*. The former permits of somewhat easier placing of the steel while the latter gives the girder, which has the larger moment, the advantage of having the maximum possible effective depth at the support. In the present case, the top row of beam steel will be placed above the corresponding girder steel, and the other rows (if required) will alternate. The insulation to the center of the uppermost row of beam bars at the support will be made 1½ in., and the distance center to center of rows, 2 in. The insulation to the center of the lowermost row of beam bars at the center will be made 2 in. and the distance center to center of rows, 2 in.

The effective depth at the center will, with the above arrangement, be ½ in. less than at the support, or 18½ in. At the center, the approximate required area of steel is

$$A_s = \frac{663{,}000}{16{,}000(18.5 - 2)} = 2.5 \text{ sq. in.}$$

Six ¾-in. round bars, furnishing 2.65 sq. in., will be placed in two rows as outlined above. The total height of beam is 21.5 in.,

the cross-section below the slab is 8 × 17.5 in., and the weight of the stem per linear foot is 146 lb., which checks the assumed value.

Review of Beam at Center.

The flange width b cannot exceed ¼ span = ¼ × 21 × 12 = 63 in. or $16t + b' = (16 \times 4) + 8 = 72$ in.

$$p = \frac{2.65}{63 \times 18.5} = .0023 \qquad \frac{t}{d} = \frac{4}{18.5} = .216$$

From Diagram 6, k is found to be .231 and j .925. Since kd is greater than t, the neutral axis is in the stem and the T-beam equations apply.

$$f_s = \frac{663{,}000}{2.65 \times .925 \times 18.5} = 14{,}600 \text{ lb. per sq. in.}$$

$$f_c = 14{,}600 \times \frac{.231}{15(1 - .231)} = 292 \text{ lb. per sq. in.}$$

Design and Review over Support. One-half of the rods will be bent up over the support in order to furnish the required tensile area at that point. The remaining rods will be brought straight through the supporting girder far enough to develop their strength in bond. The upper three rods will be bent from each side, the remaining rods from one side being offset to clear those from the other side (see Fig. 28). With the proposed arrangement, it will be necssary to determine whether the unit stresses in the steel and in the concrete at the support are within the allowable limits. With $\frac{d'}{d} = 3/19 = .158$ and $p = p' = \frac{2.65}{8 \times 19} = .0174$, Diagram 8 gives $k = .414$ and $j = .854$.

$$f_s = \frac{663{,}000}{2.65 \times .854 \times 19} = 15{,}400 \text{ lb. per sq. in.}$$

$$f_c = 15{,}400 \times \frac{.414}{15(1 - .414)} = 727 \text{ lb. per sq. in.}$$

The bond stress along the six rods at the top of the beam at the support is

$$u = \frac{15{,}750}{14.14 \times .854 \times 19} = 69 \text{ lb. per sq. in.}$$

The true shearing area required is

$$b'd = \frac{15{,}750}{.854 \times 120} = 153 \text{ sq. in.}$$

Since the allowable values of f_s, f_c, and u are 16,000, 900, and 100, respectively, and since the shearing area furnished ($8 \times 19 = 152$) is for all practical purposes as great as required, the above design may be considered satisfactory. A somewhat more economical selection of steel could have been made by using two different sizes of bars, but the concrete section could not have

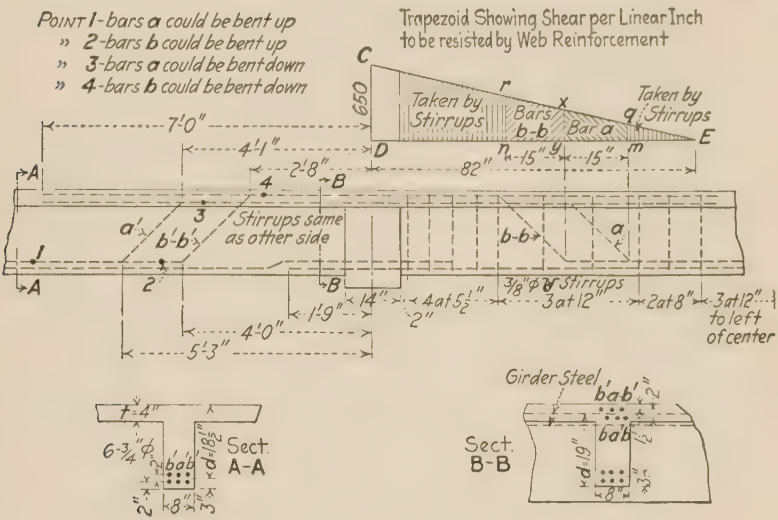


FIG. 28.

been reduced without exceeding materially the allowable unit stress in shear. The comparatively low actual value of f_c therefore cannot be avoided in the present case, and does not indicate a lack of economy in the design.

Design for Diagonal Tension. Diagram 1 may be employed to find the points at which the horizontal rods may be bent up. One rod or one-sixth of the total steel area will be bent up at one point, and two additional rods at another point, leaving 50 per cent of the reinforcement at the bottom. The single rod may be bent

$$.34 \times 21 \times 12 = 86 \text{ in.}$$

from the support, and the other two

$$.21 \times 21 \times 12 = 53 \text{ in.}$$

from the support.

The points at which the upper horizontal rods over the support, two at one point and one at another, may be bent down are determined from a consideration of the negative bending moment. Assuming that this moment becomes zero at a point one-third of the span from the center of the supporting girder, the two rods may be bent down at a distance of

$$\frac{2}{6} \times \frac{21 \times 12}{3} = 28 \text{ in. from the center of the girder.}$$

The single rod may be bent down at a distance of

$$\frac{3}{6} \times \frac{21 \times 12}{3} = 42 \text{ in. from the center of the girder.}$$

The rods will be bent as shown in Fig. 28, both of the above sets of values being taken into account in addition to the fact that the maximum allowable spacing of inclined bars equals $\frac{45}{45 + 10}d = 15$ in. if these bars are to be considered as resisting diagonal tension.

The distance from the support beyond which web reinforcement is no longer required, that is, where the unit shear becomes equal to 40 lb. per sq. in., is

$$x_1 = \frac{21}{2} - \frac{40 \times 8 \times .925 \times 18.5}{1500} = 6.85 \text{ ft.} = 82 \text{ in.}$$

The concrete can provide for a total shear of $V_c = 40 \times 8 \times .854 \times 19 = 5200$ lb. The total shear at the support to be resisted by the web reinforcement is $V' = 15{,}750 - 5200 = 10{,}550$ lb. The diagonal tension to be provided for by the web reinforcement may be represented by the triangle CDE (Fig. 28); the length of the base DE is 82 in. and the altitude CD is $\frac{10{,}550}{.854 \times 19} = 650$ lb. per lin. in.

The area $mqrn$ is provided for by the inclined rods, for, by specification, bar a can provide for diagonal tension for a distance equal to 15 in., measured from the point of bending, and .7 of the area $mqxy$ is less than the tensile strength of the bar; likewise

the two bars b provide fully for area $rxyn$, the distance yn being 15 in.

The remaining portions to the left and right of $mqrn$ must be taken care of by other web reinforcement, vertical stirrups being chosen for this case. The spacing of $\frac{3}{8}$-in. round U-stirrups at the support is

$$s = \frac{2 \times .1104 \times 16{,}000 \times .854 \times 19}{10{,}550} = 5.5 \text{ in.}$$

A spacing of $5\frac{1}{2}$ in. will be used at the support and continued to the point n.

The required spacing at the point m is

$$s = \frac{2 \times .1104 \times 16{,}000 \times .925 \times 18.5}{10{,}550 - \frac{63}{12} \times 1500} = 22 \text{ in.}$$

The maximum allowable spacing, $.45d = 8$ in., will be used throughout the length m-E. Good practice requires that stirrups be placed over the remaining portions of the beam, at a spacing of from 12 to 15 in., merely to assist in binding together the web and flange.

The first stirrup will be placed 2 in. from the edge of the girder. The bars bent up over the support will be continued to the one-third point of the adjacent span, that is, $21 \times \frac{12}{3} = 84$ in. from the center of the support, before being cut off, in order fully to provide for the negative bending moment. The length of embedment of the lower (compressive) rods at the support is

$$l_1 = \frac{15 \times 727}{4 \times 100} \times \frac{3}{4} = 21 \text{ in.}$$

By continuing each rod the required distance from the center of the girder, the full strength of the six rods can be utilized, as assumed in the design.

TABLE I.—AREAS, PERIMETERS, AND WEIGHTS OF RODS

Size (inches)	Round rods			Square rods		
	Area (sq. in.)	Perimeter (inches)	Weight (lb. per ft.)	Area (sq. in.)	Perimeter (inches)	Weight (lb. per ft.)
1/4	**0.0491**	**0.785**	**0.17**	0.0625	1.00	0.21
5/16	0.0767	0.982	0.26	0.0977	1.25	0.33
3/8	**0.1104**	**1.178**	**0.38**	0.1406	1.50	0.48
7/16	0.1503	1.374	0.51	0.1914	1.75	0.65
1/2	**0.1963**	**1.571**	**0.67**	**0.2500**	**2.00**	**0.85**
9/16	0.2485	1.767	0.85	0.3164	2.25	1.08
5/8	**0.3068**	**1.964**	**1.04**	0.3906	2.50	1.33
11/16	0.3712	2.160	1.26	0.4727	2.75	1.61
3/4	**0.4418**	**2.356**	**1.50**	0.5625	3.00	1.91
13/16	0.5185	2.553	1.76	0.6602	3.25	2.25
7/8	**0.6013**	**2.749**	**2.04**	0.7656	3.50	2.60
15/16	0.6903	2.945	2.35	0.8789	3.75	2.99
1	**0.7854**	**3.142**	**2.67**	**1.0000**	**4.00**	**3.40**
1 1/8	0.9940	3.534	3.38	**1.2656**	**4.50**	**4.30**
1 1/4	1.2272	3.927	4.17	**1.5625**	**5.00**	**5.31**

TABLE II.—SECTIONAL AREA OF ROUND RODS IN SQUARE INCHES

Size of rod	Number of rods												
	2	3	4	5	6	7	8	9	10	11	12	13	14
3/8	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10	1.21	1.32	1.44	1.55
1/2	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5/8	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
3/4	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7/8	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
1	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
1 1/8	1.98	2.98	3.98	4.97	5.96	6.96	7.95	8.95	9.94	10.94	11.93	12.92	13.92
1 1/4	2.45	3.68	4.91	6.14	7.36	8.59	9.82	11.04	12.27	13.50	14.73	15.95	17.18

SECTIONAL AREA OF SQUARE RODS IN SQUARE INCHES

Size of rod	Number of rods												
	2	3	4	5	6	7	8	9	10	11	12	13	14
3/8	0.28	0.42	0.56	0.70	0.84	0.98	1.12	1.27	1.41	1.55	1.69	1.83	1.97
1/2	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5/8	0.78	1.17	1.56	1.95	2.34	2.73	3.12	3.52	3.91	4.30	4.69	5.08	5.47
3/4	1.12	1.68	2.25	2.81	3.38	3.94	4.50	5.06	5.62	6.19	6.75	7.31	7.88
7/8	1.53	2.29	3.06	3.83	4.59	5.36	6.12	6.89	7.66	8.42	9.19	9.95	10.72
1	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
1 1/8	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	16.72
1 1/4	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87

TABLE III.—AREA OF SLAB BARS

SECTIONAL AREA OF STEEL PER FOOT OF SLAB FOR VARIOUS SPACINGS

Spacing (inches)	Size of round bar							Size of square bar			
	1/4 in.	3/8 in.	1/2 in.	5/8 in.	3/4 in.	7/8 in.	1 in.	1/2 in.	1 in.	1 1/8 in.	1 1/4 in.
3	0.20	0.44	0.78	1.23	1.77	2.40	3.14	1.00	4.00	5.06	6.25
3 1/2	0.17	0.38	0.67	1.05	1.51	2.06	2.69	0.86	3.43	4.34	5.36
4	0.15	0.33	0.59	0.92	1.32	1.80	2.36	0.75	3.00	3.80	4.69
4 1/2	0.13	0.29	0.52	0.82	1.18	1.60	2.09	0.67	2.67	3.37	4.17
5	0.12	0.26	0.47	0.74	1.06	1.44	1.88	0.60	2.40	3.04	3.75
5 1/2	0.11	0.24	0.43	0.67	0.96	1.31	1.71	0.55	2.18	2.76	3.41
6	0.10	0.22	0.39	0.61	0.88	1.20	1.57	0.50	2.00	2.53	3.12
6 1/2		0.20	0.36	0.57	0.82	1.11	1.45	0.46	1.85	2.34	2.89
7		0.19	0.34	0.53	0.76	1.03	1.35	0.43	1.71	2.17	2.68
7 1/2		0.18	0.31	0.49	0.71	0.96	1.26	0.40	1.60	2.02	2.50
8		0.17	0.29	0.46	0.66	0.90	1.18	0.37	1.50	1.89	2.34
9		0.15	0.26	0.41	0.59	0.80	1.05	0.33	1.33	1.69	2.08
10		0.13	0.24	0.37	0.53	0.72	0.94	0.30	1.20	1.52	1.87
12		0.11	0.20	0.31	0.44	0.60	0.78	0.25	1.00	1.27	1.56

TABLE IV.—RECTANGULAR BEAMS AND SLABS

DESIGN

$$k = \frac{n}{n + r} \qquad j = 1 - \frac{k}{3} \qquad p = \frac{n}{2r(n + r)} \qquad K = \tfrac{1}{2}f_c kj \text{ or } pf_s j$$

n = 12						n = 15			
k	j	p	K	f_s	f_c	k	j	p	K
0.300	0.900	0.0054	67.5	14,000	500	0.348	0.884	0.0062	76.7
0.320	0.893	0.0063	78.6		550	0.372	0.876	0.0073	89.5
0.340	0.888	0.0073	90.6		600	0.391	0.870	0.0084	102.0
0.358	0.881	0.0083	102.5		650	0.410	0.863	0.0095	114.8
0.375	0.875	0.0094	114.8		700	0.428	0.857	0.0107	128.3
0.391	0.870	0.0105	127.6		750	0.446	0.851	0.0120	142.3
0.407	0.864	0.0116	140.4		800	0.462	0.846	0.0132	156.3
0.435	0.855	0.0140	167.5		900	0.491	0.836	0.0158	184.8
0.462	0.846	0.0165	195.3		1000				
0.273	0.909	0.0043	62.0	16,000	500	0.319	0.894	0.0050	71.3
0.292	0.903	0.0050	72.2		550	0.339	0.887	0.0058	82.3
0.310	0.897	0.0058	83.2		600	0.358	0.881	0.0067	94.4
0.328	0.891	0.0067	95.0		650	0.379	0.874	0.0077	107.7
0.344	0.885	0.0075	106.2		700	0.397	0.868	0.0087	120.6
0.360	0.880	0.0084	118.8		750	0.414	0.862	0.0097	133.8
0.375	0.875	0.0094	131.3		800	0.429	0.857	0.0107	146.7
0.403	0.866	0.0113	156.5		900	0.458	0.847	0.0129	174.4
0.429	0.857	0.0140	183.7		1000				
0.250	0.917	0.0035	57.3	18,000	500	0.294	0.902	0.0041	66.3
0.268	0.911	0.0041	67.2		550	0.314	0.895	0.0048	77.4
0.286	0.905	0.0048	77.5		600	0.333	0.889	0.0056	88.9
0.302	0.899	0.0055	88.4		650	0.351	0.883	0.0063	100.8
0.318	0.894	0.0062	99.6		700	0.368	0.877	0.0072	113.1
0.333	0.889	0.0069	111.1		750	0.385	0.872	0.0080	125.7
0.348	0.884	0.0077	123.0		800	0.400	0.867	0.0089	138.7
0.375	0.875	0.0094	147.7		900	0.429	0.857	0.0107	165.3
0.400	0.867	0.0111	173.3		1000				
0.230	0.923	0.0029	53.1	20,000	500	0.272	0.909	0.0034	61.8
0.248	0.917	0.0034	62.4		550	0.292	0.903	0.0040	72.2
0.264	0.912	0.0040	72.2		600	0.311	0.897	0.0047	83.7
0.280	0.907	0.0046	82.4		650	0.328	0.891	0.0053	94.4
0.295	0.902	0.0052	93.3		700	0.344	0.885	0.0060	106.2
0.310	0.897	0.0058	104.3		750	0.359	0.880	0.0067	117.9
0.324	0.892	0.0065	115.6		800	0.374	0.875	0.0075	130.9
0.351	0.883	0.0079	139.5		900				
0.375	0.875	0.0094	164.1		1000				

TABLE V.—VALUES OF k AND j FOR RECTANGULAR BEAMS AND SLABS

REVIEW

$$k = \sqrt{2pn + (pn)^2} - pn \qquad j = 1 - \tfrac{1}{3}k$$

p	$n = 12$		$n = 15$		p	$n = 12$		$n = 15$	
	k	j	k	j		k	j	k	j
0.0010	0.145	0.952	0.158	0.947	0.0090	0.370	0.877	0.402	0.866
0.0012	0.155	0.948	0.169	0.944	0.0092	0.373	0.876	0.405	0.865
0.0014	0.166	0.945	0.181	0.940	0.0094	0.376	0.875	0.407	0.864
0.0016	0.177	0.941	0.192	0.936	0.0096	0.379	0.874	0.411	0.863
0.0018	0.186	0.938	0.202	0.933	0.0098	0.381	0.873	0.414	0.862
0.0020	0.196	0.935	0.217	0.928	0.0100	0.385	0.872	0.418	0.861
0.0022	0.204	0.932	0.222	0.926	0.0102	0.387	0.871	0.420	0.860
0.0024	0.212	0.929	0.231	0.923	0.0104	0.391	0.870	0.423	0.859
0.0026	0.220	0.927	0.240	0.920	0.0106	0.394	0.869	0.426	0.858
0.0028	0.227	0.924	0.248	0.917	0.0108	0.396	0.868	0.429	0.857
0.0030	0.235	0.922	0.258	0.914	0.0110	0.398	0.867	0.432	0.856
0.0032	0.241	0.920	0.263	0.912	0.0112	0.402	0.866	0.434	0.855
0.0034	0.248	0.917	0.271	0.910	0.0114	0.404	0.865	0.437	0.854
0.0036	0.254	0.915	0.277	0.908	0.0116	0.407	0.864	0.440	0.853
0.0038	0.260	0.913	0.284	0.905	0.0118	0.410	0.863	0.443	0.852
0.0040	0.266	0.911	0.292	0.903	0.0120	0.412	0.863	0.446	0.851
0.0042	0.270	0.910	0.297	0.901	0.0122	0.415	0.862	0.448	0.851
0.0044	0.276	0.908	0.303	0.899	0.0124	0.417	0.861	0.451	0.850
0.0046	0.281	0.906	0.309	0.897	0.0126	0.419	0.860	0.454	0.849
0.0048	0.286	0.904	0.315	0.895	0.0128	0.422	0.859	0.457	0.848
0.0050	0.291	0.903	0.320	0.893	0.0130	0.424	0.859	0.459	0.847
0.0052	0.295	0.901	0.324	0.892	0.0132	0.427	0.858	0.461	0.846
0.0054	0.300	0.900	0.329	0.891	0.0134	0.429	0.857	0.464	0.845
0.0056	0.304	0.899	0.333	0.889	0.0136	0.432	0.856	0.466	0.845
0.0058	0.309	0.897	0.337	0.888	0.0138	0.434	0.855	0.468	0.844
0.0060	0.314	0.895	0.344	0.885	0.0140	0.436	0.855	0.471	0.843
0.0062	0.317	0.894	0.348	0.884	0.0142	0.437	0.854	0.473	0.843
0.0064	0.322	0.893	0.352	0.883	0.0144	0.440	0.853	0.475	0.842
0.0066	0.325	0.892	0.356	0.881	0.0146	0.442	0.853	0.477	0.841
0.0068	0.330	0.890	0.360	0.880	0.0148	0.444	0.852	0.479	0.840
0.0070	0.334	0.889	0.365	0.878	0.0150	0.446	0.851	0.481	0.840
0.0072	0.338	0.887	0.369	0.877	0.0152	0.449	0.850	0.483	0.839
0.0074	0.342	0.886	0.372	0.876	0.0154	0.451	0.850	0.485	0.838
0.0076	0.345	0.885	0.376	0.875	0.0156	0.453	0.849	0.487	0.838
0.0078	0.349	0.884	0.380	0.873	0.0158	0.455	0.848	0.489	0.837
0.0080	0.353	0.882	0.384	0.872	0.0160	0.457	0.848	0.493	0.836
0.0082	0.356	0.881	0.387	0.871	0.0170	0.467	0.845	0.502	0.833
0.0084	0.360	0.880	0.390	0.870	0.0180	0.476	0.841	0.513	0.829
0.0086	0.363	0.879	0.394	0.869	0.0190	0.485	0.838	0.522	0.826
0.0088	0.366	0.878	0.398	0.867	0.0200	0.493	0.836	0.531	0.823

DIAGRAM 1

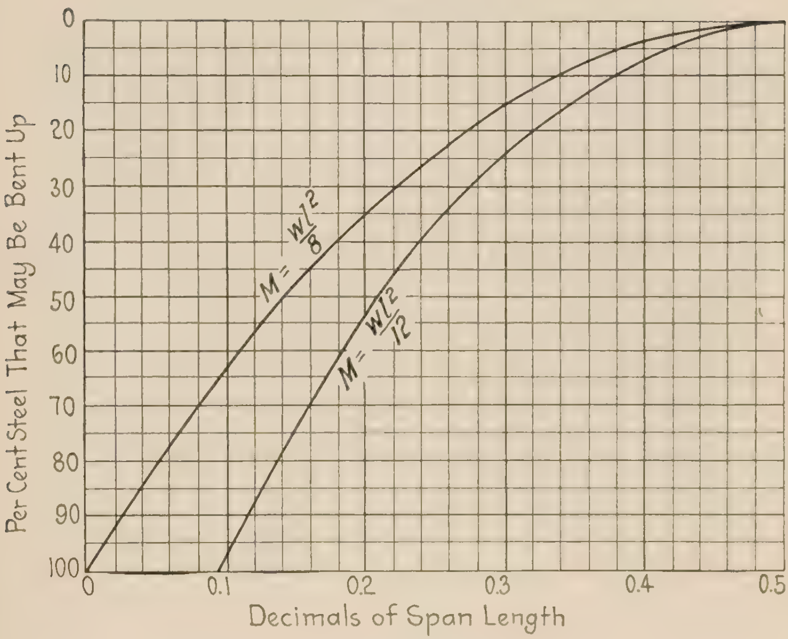


DIAGRAM 2

T-Beam Design

Based on $f_s = 16000$ **$n = 12$**

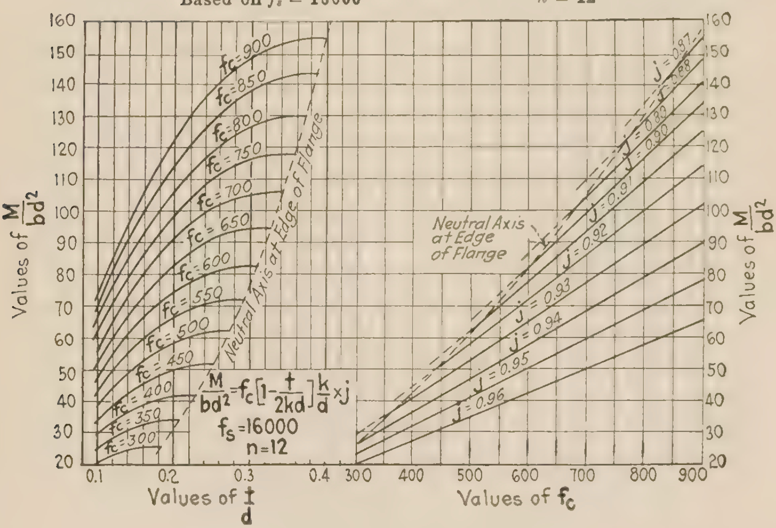


DIAGRAM 3

T-Beam Design

Based on $f_s = 16000$ $n = 15$

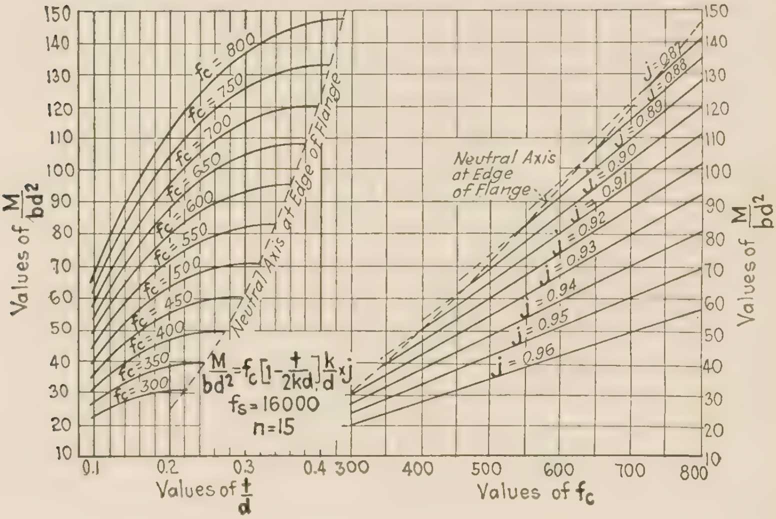


DIAGRAM 4

T-Beam Design

Based on $f_s = 18000$ $n = 15$

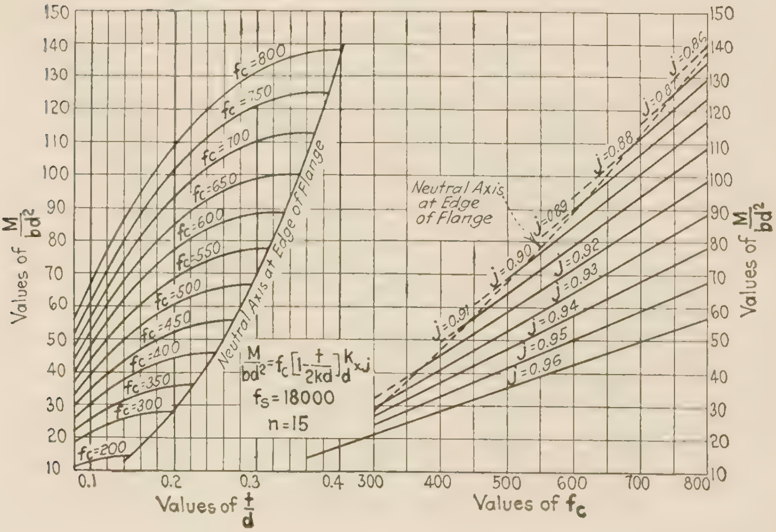


DIAGRAM 5

T-Beam Review

Values of k and j for various percentages of steel. Based on $n = 12$

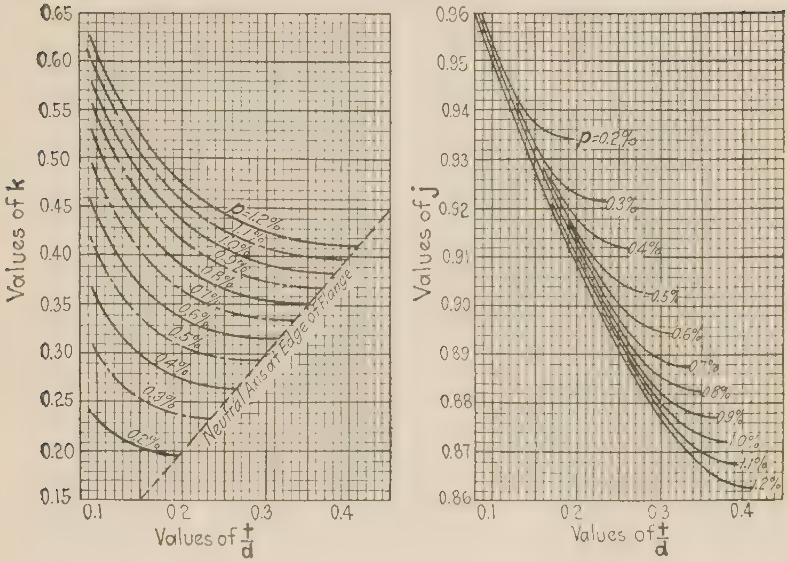


DIAGRAM 6

T-Beam Review

Values of k and j for various percentages of steel. Based on $n = 15$

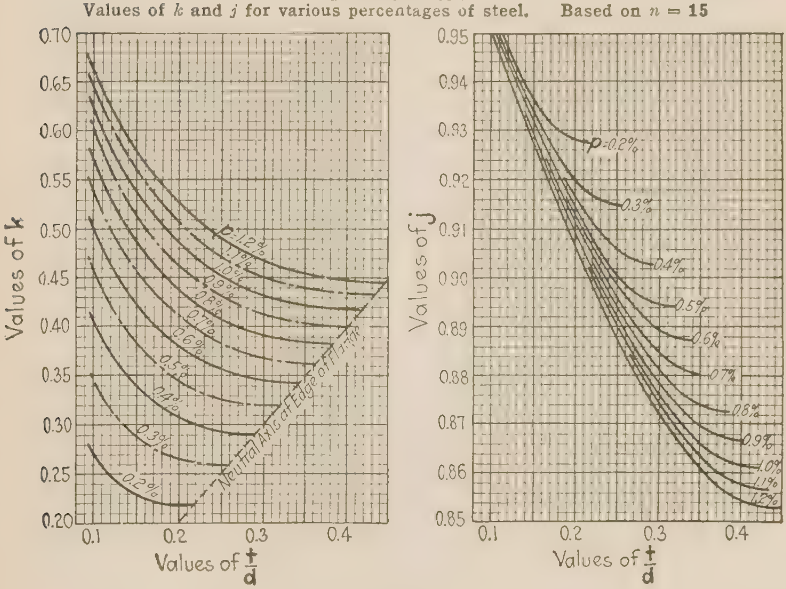
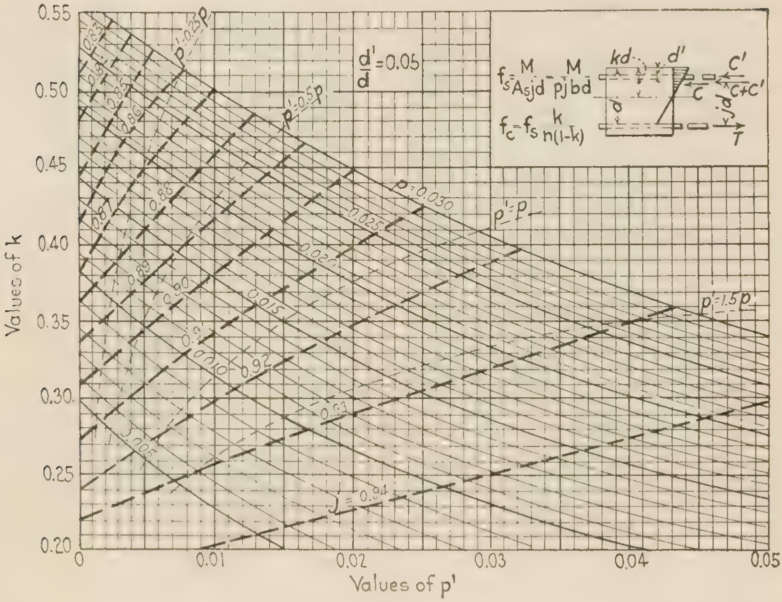
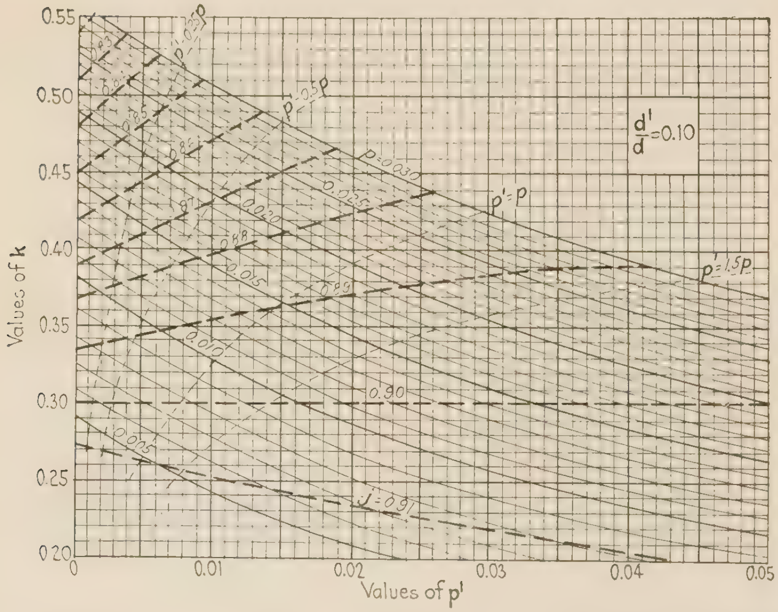


DIAGRAM 7[10]

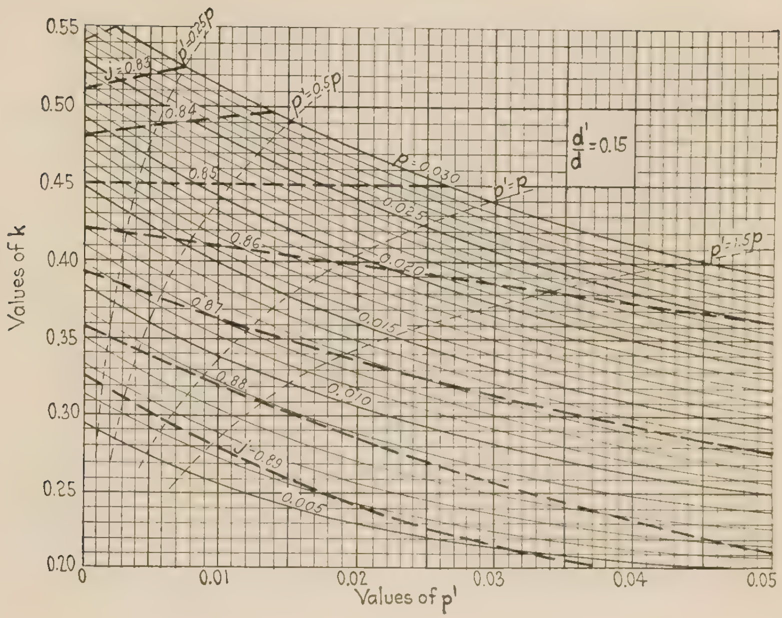
Review of Rectangular Beams with Steel in Top and Bottom
Based on $n = 12$





[10] Diagrams 7 and 8 taken by permission of the authors from HOOL and JOHNSON "Concrete Engineer's Handbook."

DIAGRAM 7 (*Continued*)



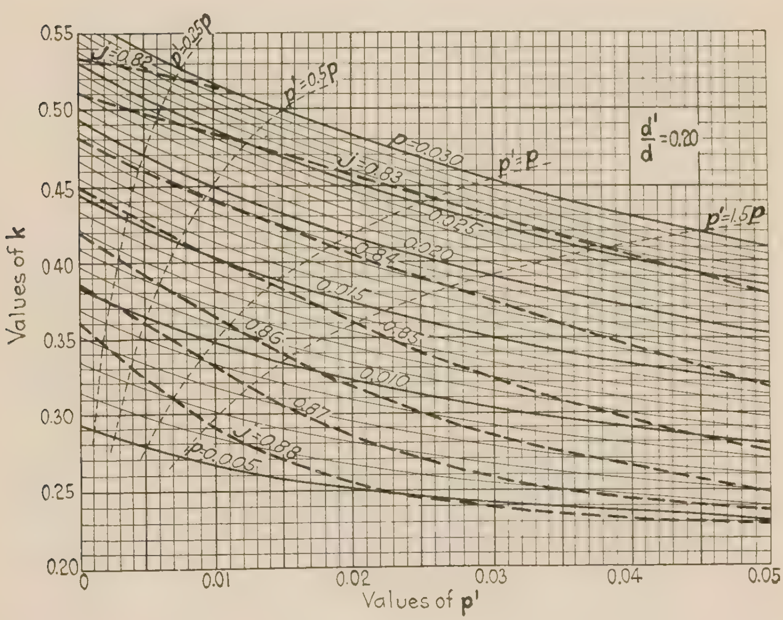
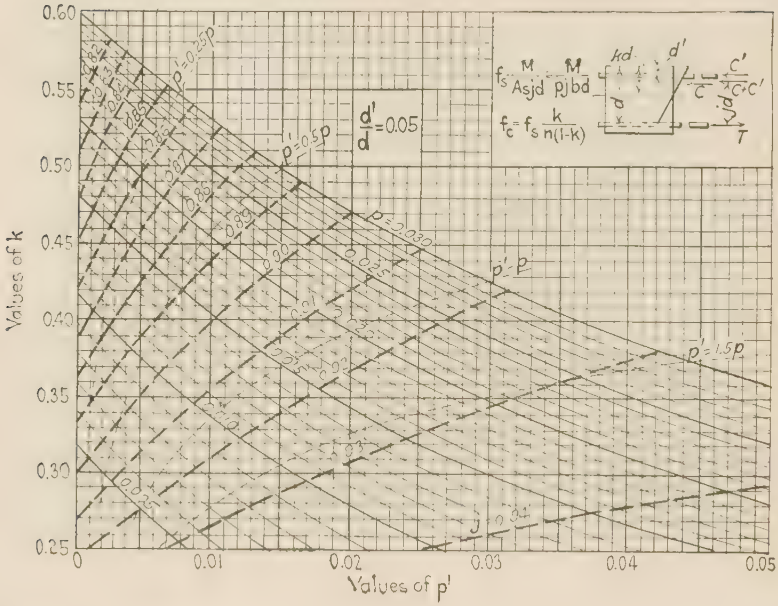


DIAGRAM 8

Review of Rectangular Beams with Steel in Top and Bottom
Based on $n = 15$



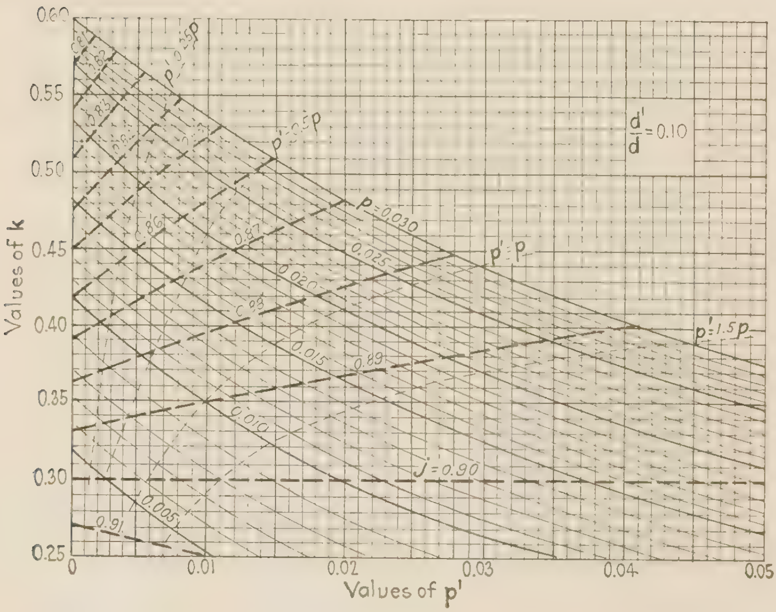
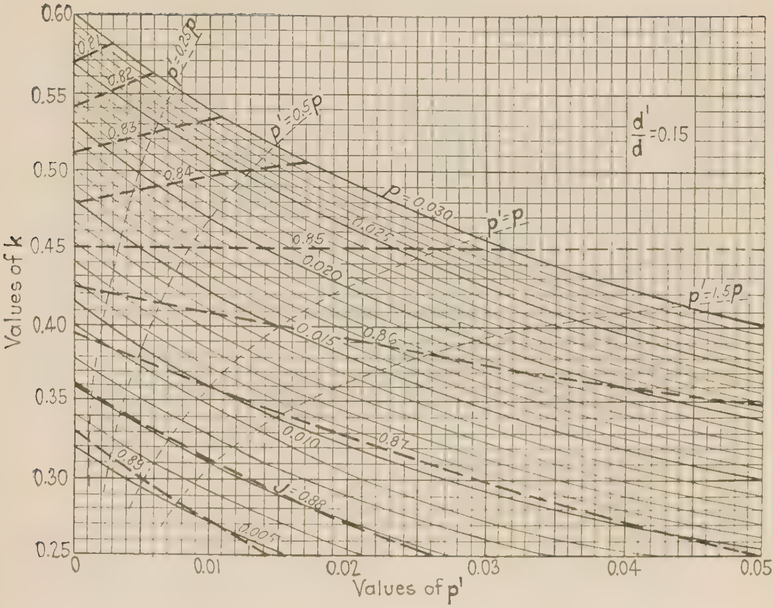


DIAGRAM 8 (*Continued*)



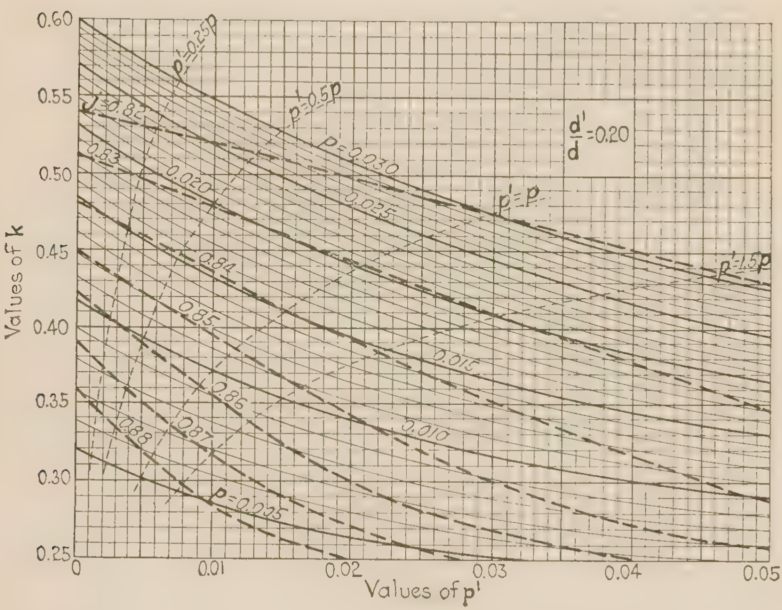
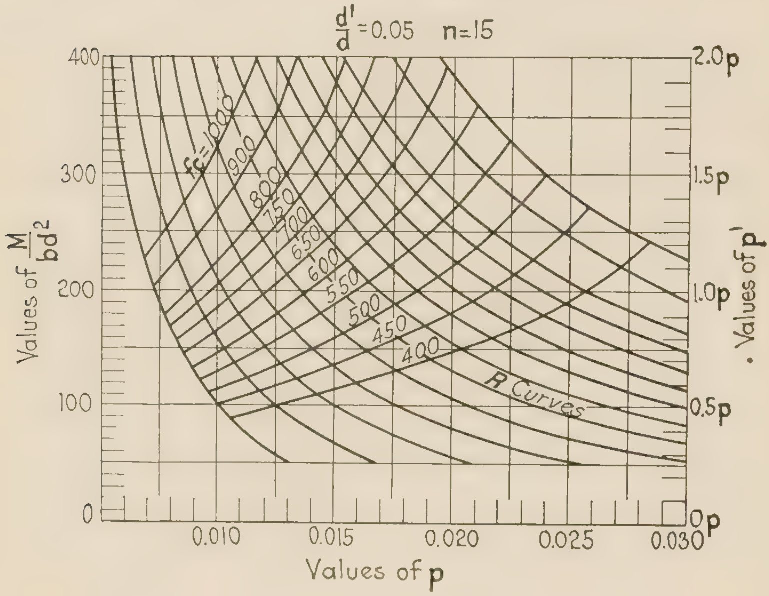


DIAGRAM 9

Review of Rectangular Beams with Steel in Top and Bottom
f_c curves. Based on $n = 15$



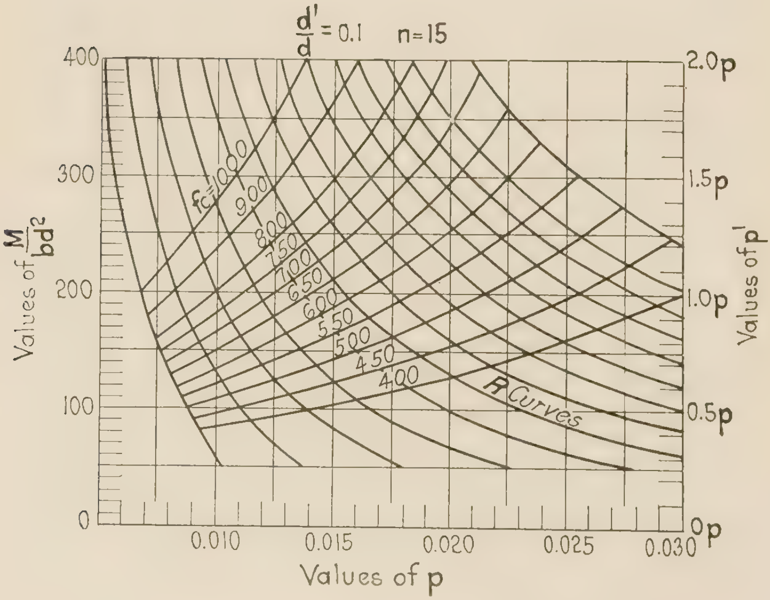
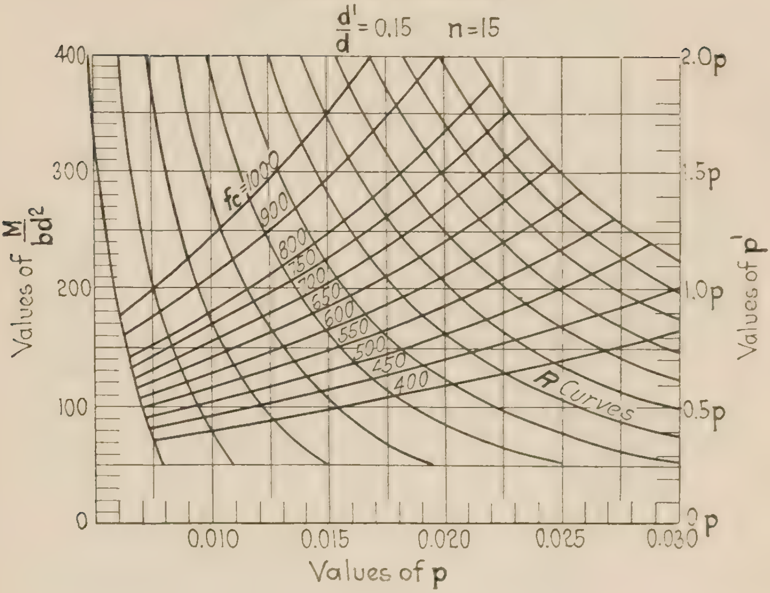


DIAGRAM 9 *Continued*



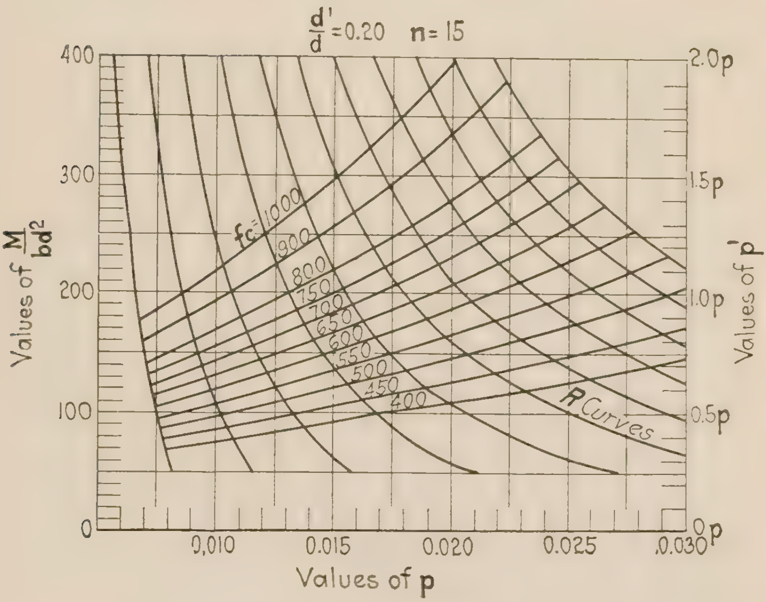
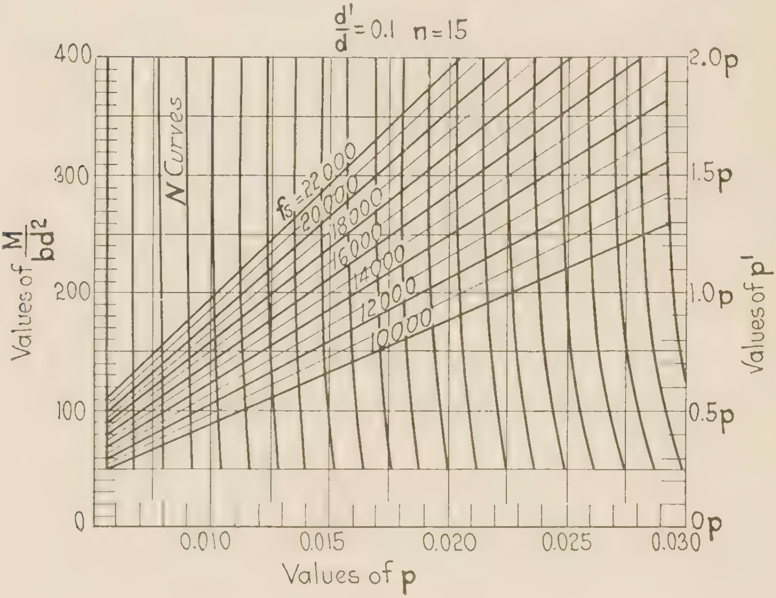
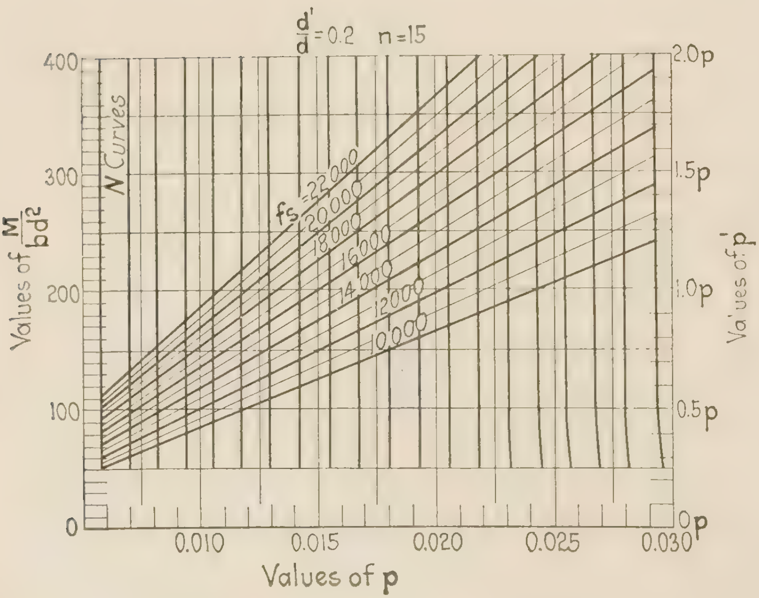


DIAGRAM 10

Review of Rectangular Beams with Steel in Top and Bottom

f_s curves. Based on $n = 15$





CHAPTER IV

FLEXURE AND DIRECT STRESS

98. Where a beam is acted upon by forces normal to its axis, the resultant stresses are due to bending, so that the deductions of Chap. III are applicable. Then, too, columns sustaining a load applied at the center of the section are subject to direct stress only, so the method of design given in Chap. V may be used.

There are, however, cases when none of the above analyses is possible. The more usual of these are.

1. A beam subject to inclined forces, or a beam acting as a strut between its supports.

2. A column sustaining an eccentric load, or receiving lateral pressure from horizontal or inclined forces.

3. An arch ring, where the arch thrust acts other than parallel to, and along the axis of, the ring.

In all of the cases above, the resultant stress is a combination of that produced by flexure and by the direct stress acting along the axis of the member.

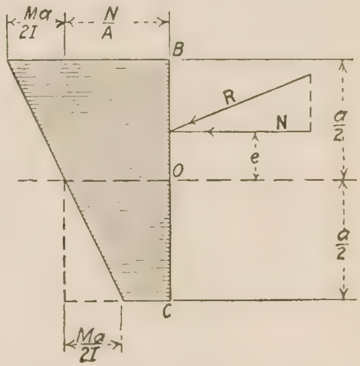


FIG. 29.

Let Fig. 29 represent a plain concrete section BC. The resultant of all the forces R is applied at distance e from the gravity axis of the member. If the resultant R were applied at the point O, the intensity of stress over the whole section BC, whose area is A, would be uniform and equal to $\frac{N}{A}$. Since, however, the resultant R is not applied at the center of the section O, it produces a moment M about the point O equal to Ne; that is,

the force N applied at a distance e from the axis may be replaced by an equal force N applied at O and a couple whose moment is Ne. The intensity of the stress at the extreme fibers of the section produced by this moment is $M \times \frac{a}{2} \div I$, in which I is the moment of inertia of the section about an axis O perpendicular to the plane of the paper. The total intensity of the compression at the edge B is then

$$f_c = \frac{N}{A} + \frac{Ma}{2I}$$

and at the edge C,

$$f'_c = \frac{N}{A} - \frac{Ma}{2I}.$$

If the stress f'_c is a negative quantity, it shows that the stress produced by the flexure is greater than that produced by the direct action of N, and the resultant stress at the edge C is tension.

In a reinforced concrete member it is presupposed that the bond between the steel and the concrete remains intact under stress. Therefore, the steel in the compression side of a member subject to combined flexure and direct stress can withstand a stress only sufficient to make it deform equally with the concrete,

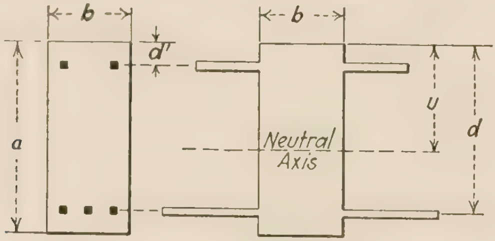


Fig. 30.

or n times the stress in the concrete. This steel might then be replaced by n times the amount of concrete at the same distance from the axis of the section. Such a section is known as the transformed section.

The following additional notation will be used. The face of the member most highly stressed will be called the "compressive surface," and the opposite face, the "tension surface."

R = resultant of all forces on the section.

N = resultant of all forces acting normal to the section, that is, the normal component of R.

e = eccentric distance of N.

M = bending moment = Ne.

$$p = \frac{A_s}{ba}, \quad p' = \frac{A'_s}{ba}, \quad p_o = \frac{A_s + A'_s}{ba}$$

u = distance from compressive surface to neutral axis of transformed section.

A_t = area of transformed section.

I_c = moment of inertia of concrete about neutral axis.

I_s = moment of inertia of steel about neutral axis.

By referring to Fig. 30 it may be seen that

$$A_t = ba + (n - 1)(A_s + A'_s)$$

$$I = I_c + (n - 1)I_s$$

$$u = \frac{\frac{a}{2} + p(n - 1)d + p'(n - 1)d'}{1 + p(n - 1) + p'(n - 1)}$$

$$I_c = \frac{1}{3}b[u^3 + (a - u)^3]$$

Neglecting I_s about the axis of the bars,

$$I_s = A_s(d - u)^2 + A'_s(u - d')^2$$

If the reinforcement is symmetrical, then $u = \frac{a}{2}$, and

$$I_c = \frac{1}{12}ba^3 \text{ and } I_s = 2A'_s\left(\frac{a}{2} - d'\right)^2$$

If the eccentricity $\frac{e}{a}$ is within certain limits, then compression exists over the whole section. For greater eccentricities there will be tension over a part of the section. If it be assumed that the concrete takes no tension, the analyses for these two cases are quite different. The value of $\frac{e}{a}$ which results in zero stress on the tension surface is dependent upon the relative amounts of steel and concrete, and the ratio of the moduli of elasticity of the two materials.

99. *Case I.—Compression over the Whole Section, Fig.* 31. The maximum unit stress in the concrete may be computed as though the member were homogeneous and is

$$f_c = \frac{N}{A_t} + \frac{Mu}{I} \tag{38}$$

For sections with symmetrical reinforcement, since $u = \frac{a}{2}$ and $A_s = A'_s$

$$f_c = \frac{N}{ba + 2(n-1)A'_s} + \frac{Ma}{2[I_c + (n-1)I_s]} \tag{39}$$

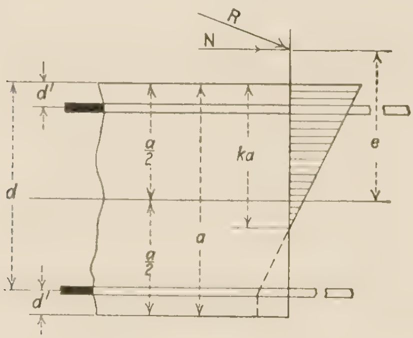


FIG. 31.

On the compressive side of the member the unit flexural stress in the plane of the reinforcement is $\frac{M(u - d')}{I}$, and on the other side $\frac{M(d - u)}{I}$. The maximum unit stress in the steel is then

$$f'_s = n\left(\frac{N}{A_t}\right) + \frac{nM(u - d')}{I}$$

which is less than nf_c and is therefore always within the limits of a reasonable value for f'_s, provided f_c has a safe value. Since $f_s = n\left(\frac{N}{A_t}\right) - \frac{nM\,(d - u)}{I}$, it will always be less than f'_s.

Equation (39) may be written

$$f_c = \frac{N}{ba + p_o ba(n-1)} + \frac{Ne\left(\frac{a}{2}\right)}{\frac{1}{12}ba^3 + p_o ba\left(\frac{a}{2} - d'\right)^2 (n-1)}$$

$$= \frac{N}{ba}\left[\frac{1}{1 + (n - 1)p_o} + \frac{\frac{ea}{2}}{\frac{a^2}{12} + p_o\left(\frac{a}{2} - d'\right)^2 (n-1)}\right]$$

For given values of n and $\frac{d'}{a}$ it may be simplified further to

$$f_c = \frac{N}{ba}\left[\frac{1}{1 + (n - 1)p_o} + \frac{e}{a} \times \frac{6}{1 + Zp_o}\right] \qquad (40)$$

where Z is a constant depending only upon the values of n and $\frac{d'}{a}\cdot$ By allowing the expression within the brackets to be known as K, Diagrams 11 to 16 have been plotted from equation (40) for different values of n and $\frac{d'}{a}$. By entering these diagrams with p_o and $\frac{e}{a}$ as arguments, the value of K may be obtained for use in the equation

$$f_c = \frac{NK}{ba} \qquad (41)$$

100. *Case II.—Tension over Part of the Section, Fig.* 32. When the second term of equation (38) is greater than the first, it indicates tension over part of the section. Unless this tension is so small that the concrete can take its proportionate part, the analysis of Case I is not applicable. With any appreciable tension on the tension surface of the member, it is usual to neglect the tension taken by the concrete and assume the full stress to be taken by the steel.

By reference to Fig. 32

$$f'_s = nf_c\left(1 - \frac{d'}{ka}\right) \qquad (42)$$

$$\text{and } f_s = nf_c\left(\frac{d}{ka} - 1\right) \qquad (43)$$

Since the resultant fiber stress $= N$

$$\tfrac{1}{2}f_cbka + f'_sA'_s - f_sA_s = N$$

With symmetrical reinforcement, and by using the values of f'_s and f_s from equations (42) and (43),

$$N = \frac{f_cba}{2} \times \frac{k^2 + 2np_ok - np_o}{k} \qquad (44)$$

and since the moment of the stresses about the gravity axis $= M$

$$\frac{1}{2} f_c bka\left(\frac{a}{2} - \frac{ka}{3}\right) + f'_s A'_s\left(\frac{a}{2} - d'\right) + f_s A_s\left(d - \frac{a}{2}\right) = M$$

With symmetrical reinforcement, and by eliminating f'_s and f_s as before,

$$\frac{M}{ba^2 f_c} = \frac{np_o(a - 2d')^2}{4ka^2} + \frac{k}{12}(3 - 2k) \qquad (45)$$

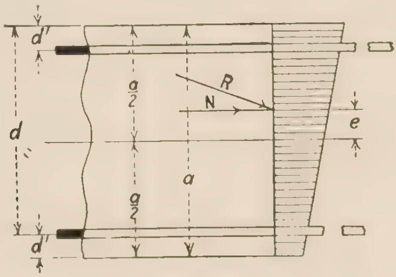


FIG. 32.

The position of the neutral axis must be determined before equation (45) can be used. Since $M = Ne$, equation (44) may be multiplied by e and this value substituted for M in equation (45). The following equation results:

$$k^3 - 3\left(\frac{1}{2} - \frac{e}{a}\right)k^2 + 6np_o\frac{e}{a}k = 3np_o\left[\frac{e}{a} + \frac{(a - 2d')^2}{2a^2}\right] \qquad (46)$$

For constant values of n and $\frac{d'}{a}$ values of k may be substituted in equation (46), and diagrams plotted for various values of p_o and $\frac{e}{a}$. Diagrams 17 to 19 and 21 to 23 based on equation (46) give the values of k. When k is known, equation (45) may be used, and by keeping n and $\frac{d'}{a}$ constant, and substituting values of $\frac{M}{ba^2 f_c}$, diagrams may be plotted for various values of p_o and $\frac{e}{a}$.

The first term of the right-hand side of equation (45) may be written

$$\frac{np_o}{4k}\frac{(a - 2d')^2}{a^2}$$

With the quantity $p_o\frac{(a - 2d')^2}{a^2}$ a constant, the value of $M \div ba^2 f_c$ will be the same for any value of k. Therefore, only one

diagram for $\frac{M}{ba^2f_c}$ is necessary for each value of n, and Diagrams 20 and 24 have been plotted for $\frac{d'}{a} = .10$. For $\frac{d'}{a} = .05$, divide the value of p_o by .790, and for $\frac{d'}{a} = .15$, divide the value of p_o by 1.306 before entering Diagrams 20 and 24.

101. *Illustrative Problems.*

I. An arch rib is 12 in. thick and is reinforced with ⅝-in. round bars spaced 5 in. center to center, placed 1¾ in. from each surface of the concrete. The maximum thrust on a unit section is 50,000 lb., and since this thrust does not act along the gravity axis of the section, it produces a bending moment of 75,000 in.-lb. Assume $n = 15$. Determine the maximum unit stress in the concrete.

$$p_o = \frac{2 \times .307}{5 \times 12} = .0102$$

$$\frac{e}{a} = \frac{75{,}000}{50{,}000 \times 12} = .125$$

$$\frac{d'}{a} = \frac{1\frac{3}{4}}{12} = .15$$

From Diagram 16 $\quad K = 1.52$

$$f_c = \frac{NK}{ba} = \frac{50{,}000 \times 1.52}{12 \times 12} = 530 \text{ lb. per sq. in.}$$

II. Suppose that the moment produced in Problem I were 150,000 in.-lb.

Then $\quad \frac{e}{a} = \frac{150{,}000}{50{,}000 \times 12} = .25$

From Diagram 23 $\quad k = .82$

$$p_o \div 1.306 = .0078$$

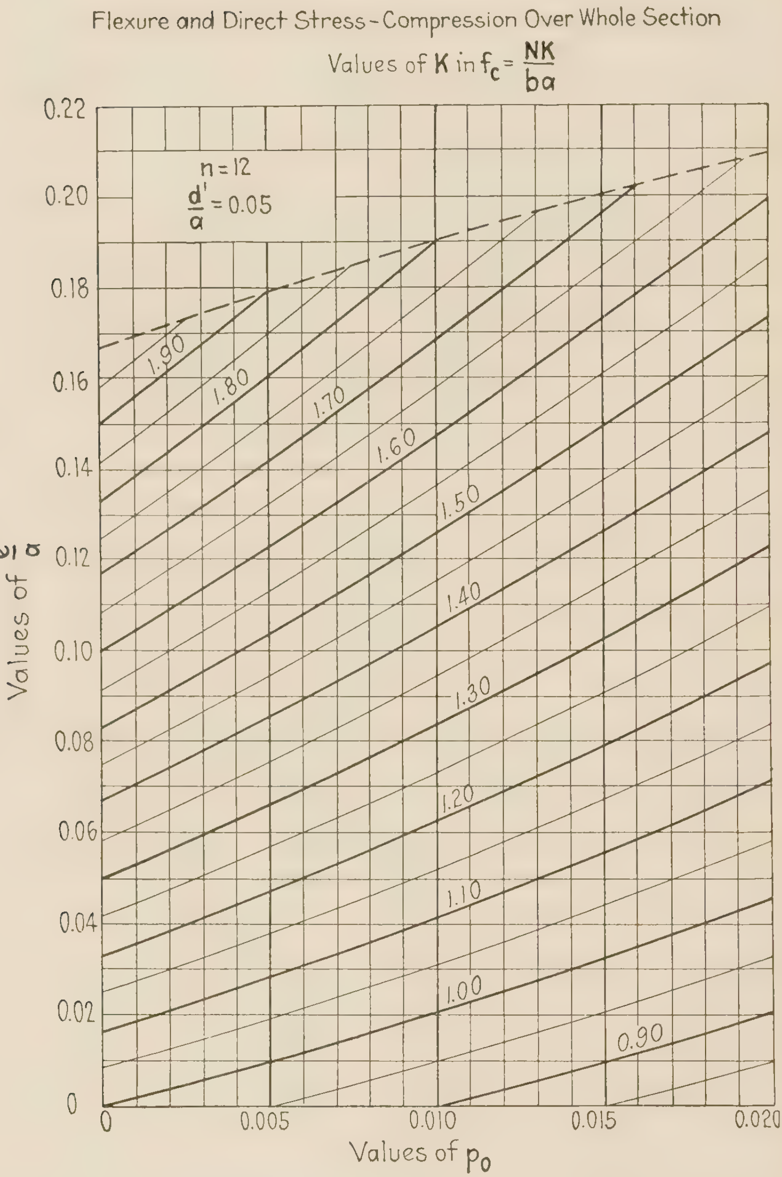
From Diagram 24 $\quad \frac{M}{ba^2f_c} = .116$

$$f_c = 750 \text{ lb. per sq. in.}$$

By equation (42) $\quad f'_s = 15 \times 750\left(1 - \frac{1.75}{.82 \times 12}\right) = 9300$ lb. per sq. in.

By equation (43) $\quad f_s = 15 \times 750\left(\frac{10.25}{.82 \times 12} - 1\right) = 450$ lb. per sq. in.

DIAGRAM 11



Flexure and Direct Stress - Compression Over Whole Section
Values of K in $f_c = \frac{NK}{ba}$
n = 12
$\frac{d'}{a} = 0.05$
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
Values of $\frac{e}{a}$
0
0.005
0.010
0.015
0.020
Values of p_0

DIAGRAM 12

Flexure and Direct Stress - Compression Over Whole Section

Values of K in $f_c = \frac{NK}{ba}$

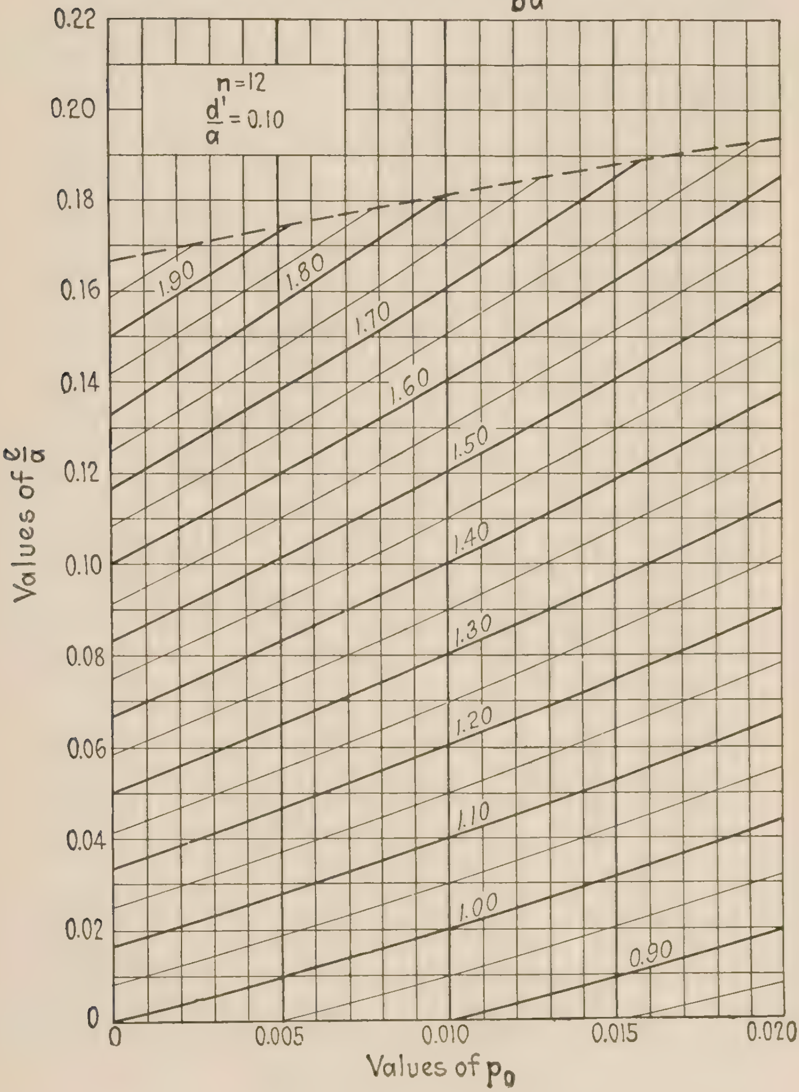


DIAGRAM 13

Flexure and Direct Stress-Compression Over Whole Section

Values of K in $f_c = \frac{NK}{ba}$

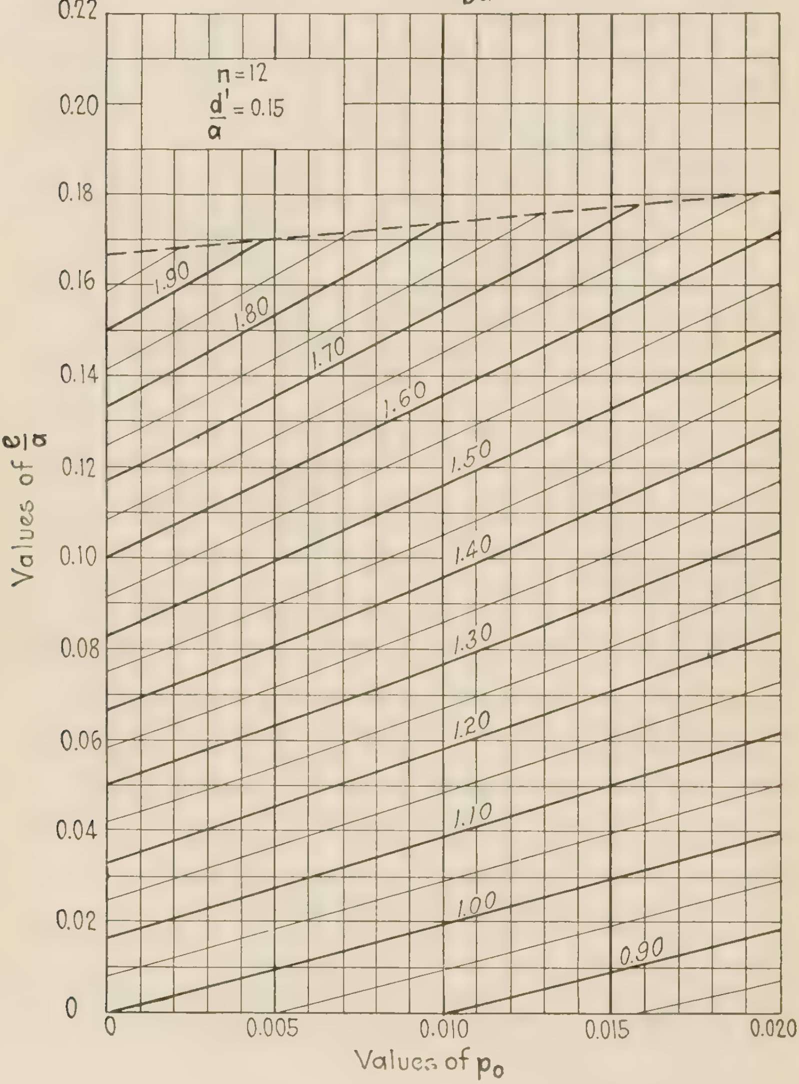


DIAGRAM 14

Flexure and Direct Stress - Compression Over Whole Section

Values of K in $f_c = \frac{NK}{ba}$

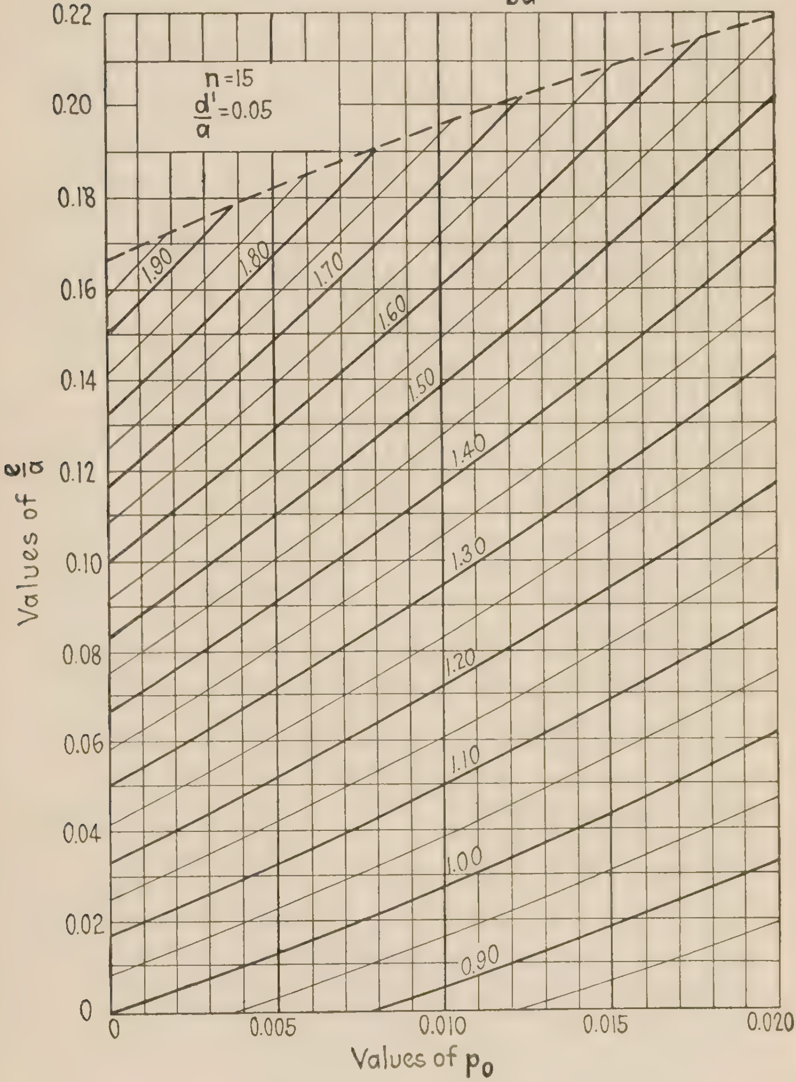


DIAGRAM 15

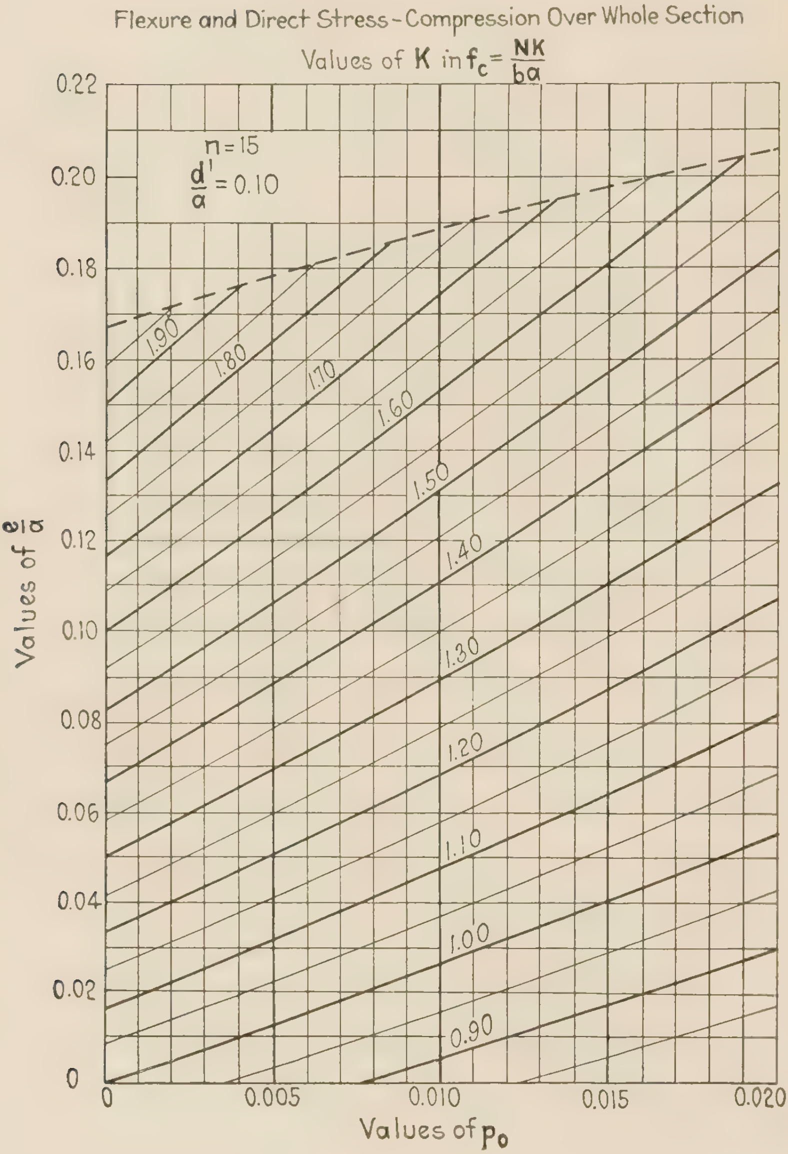

Flexure and Direct Stress - Compression Over Whole Section
Values of K in $f_c = \frac{NK}{ba}$
n = 15
$\frac{d'}{a} = 0.10$
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
Values of $\frac{e}{a}$
0
0.005
0.010
0.015
0.020
Values of p_0

DIAGRAM 16

Flexure and Direct Stress - Compression Over Whole Section

Values of K in $f'_c = \frac{NK}{ba}$

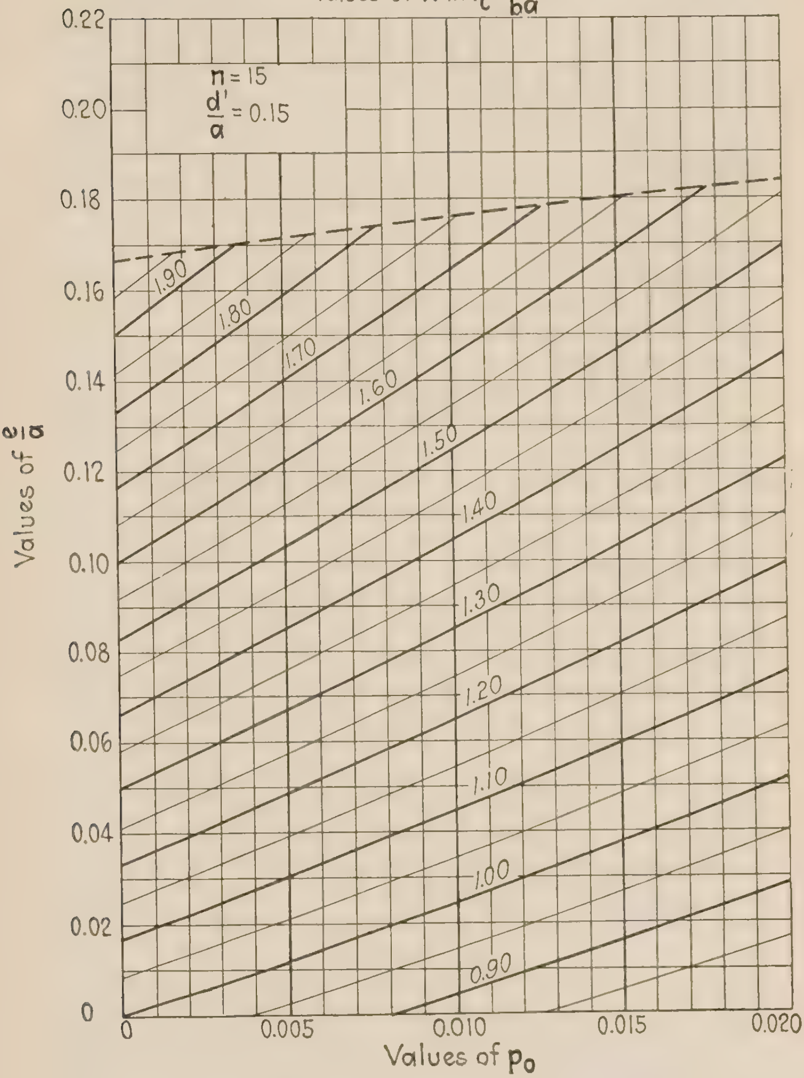


Diagram 17

Flexure and Direct Stress—Tension Over Part of the Section

Values of **k**

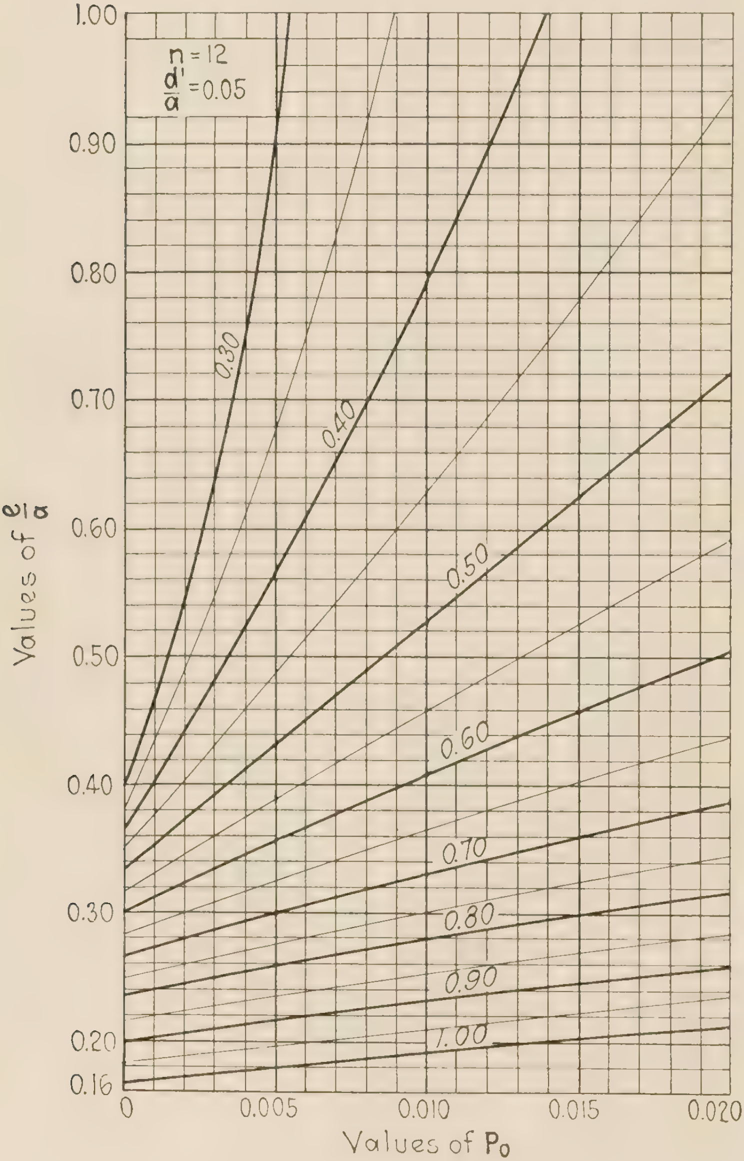
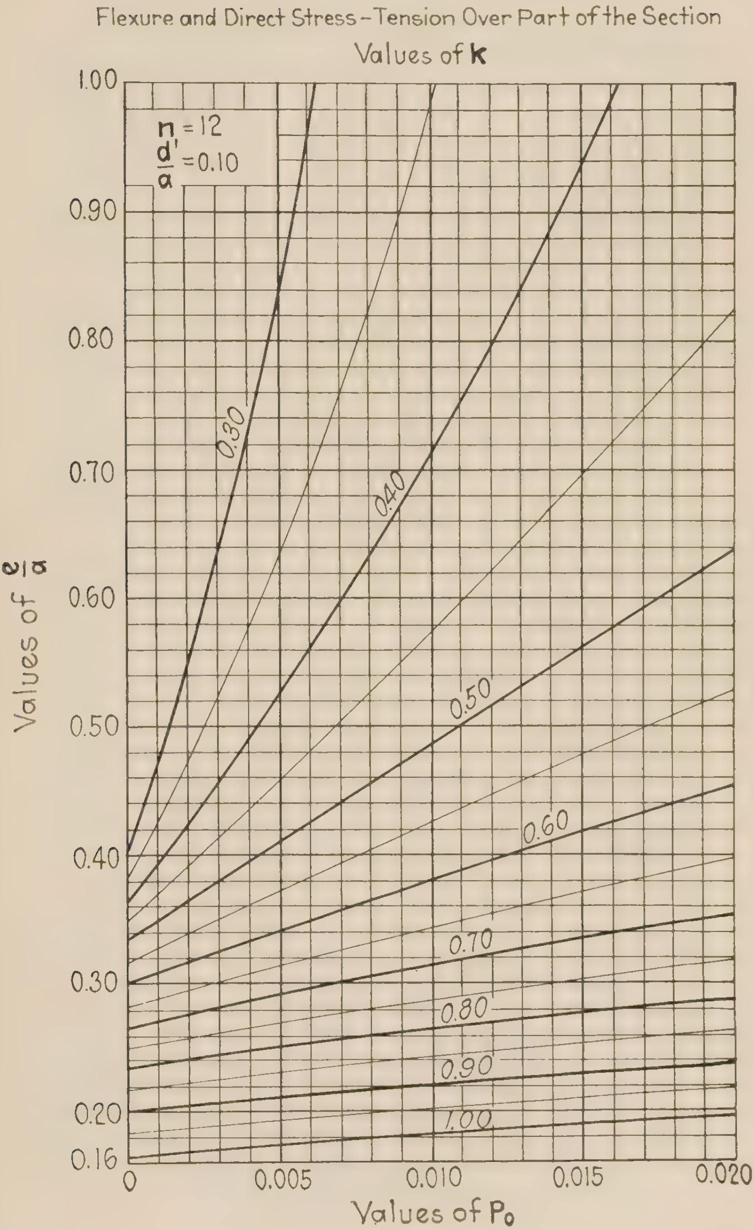


DIAGRAM 18



Flexure and Direct Stress-Tension Over Part of the Section
Values of k
n = 12
d'/a = 0.10
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
Values of e/a
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.16
0
0.005
0.010
0.015
0.020
Values of Po

DIAGRAM 19

Flexure and Direct Stress–Tension Over Part of the Section

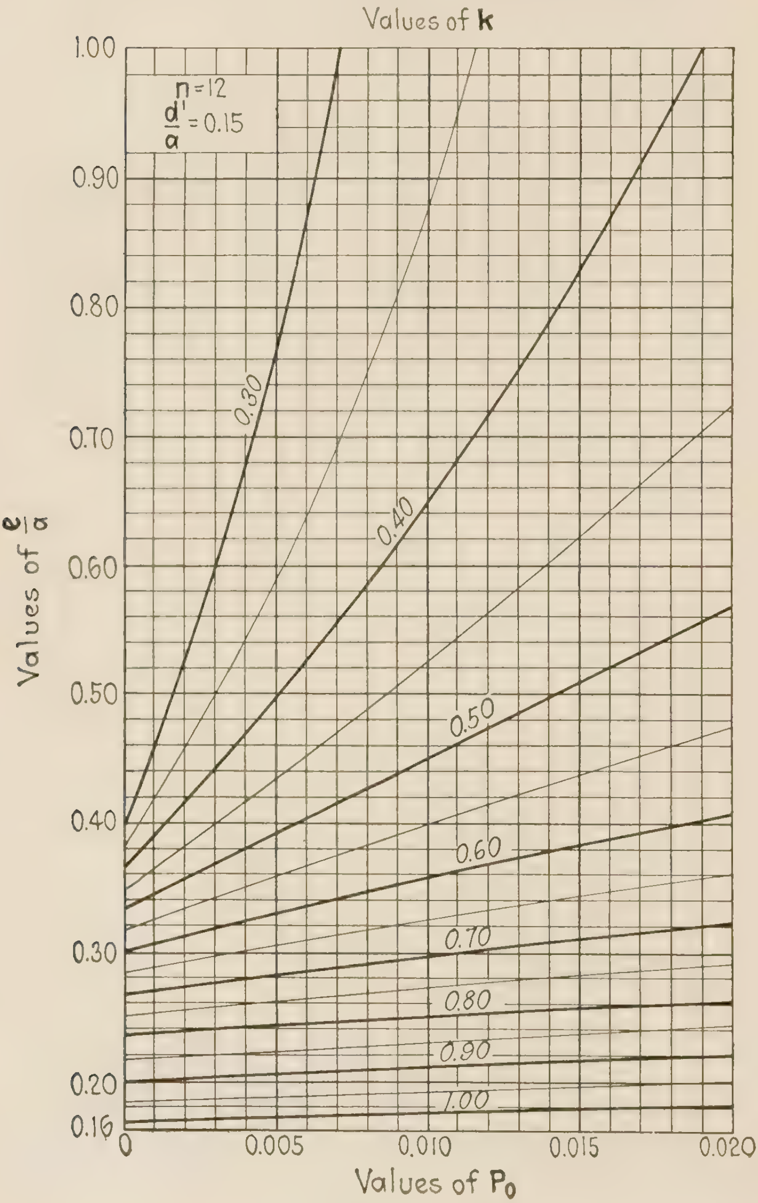
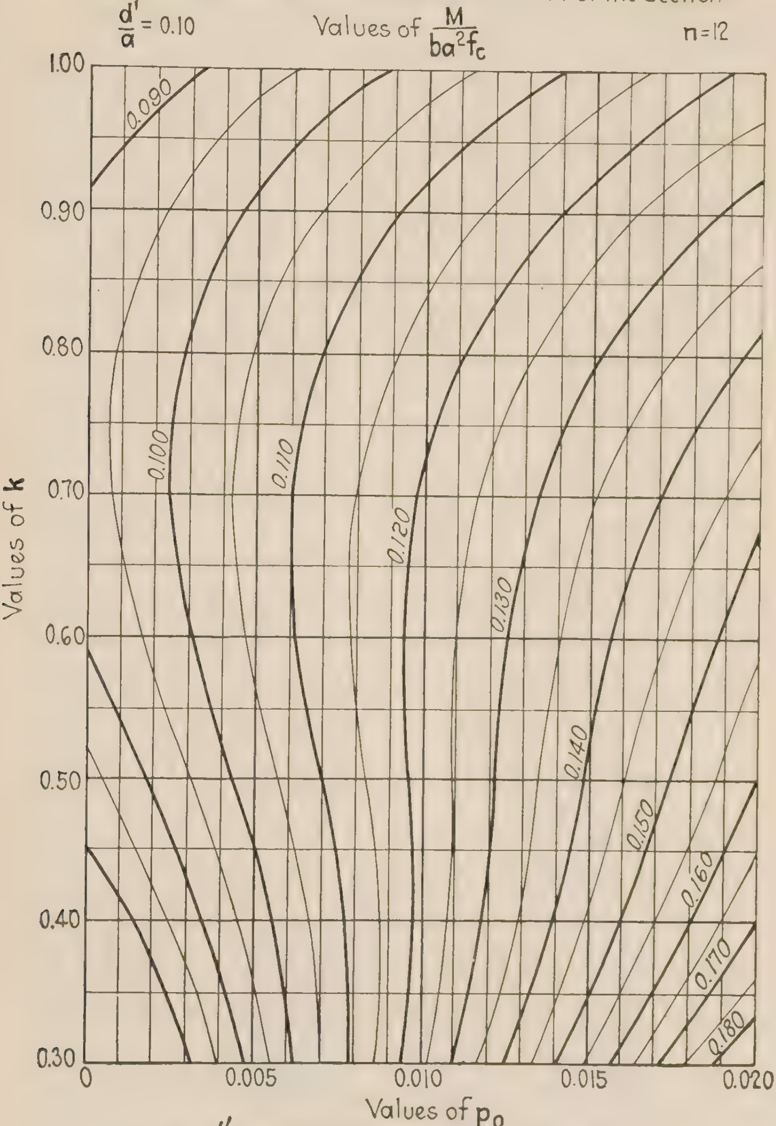


DIAGRAM 20

Flexure and Direct Stress—Tension Over Part of the Section



For $\frac{d'}{a} = 0.05$ *divide value of* p_o *by 0.790*

For $\frac{d'}{a} = 0.15$ *divide value of* p_o *by 1.306*

DIAGRAM 21

Flexure and Direct Stress—Tension Over Part of the Section

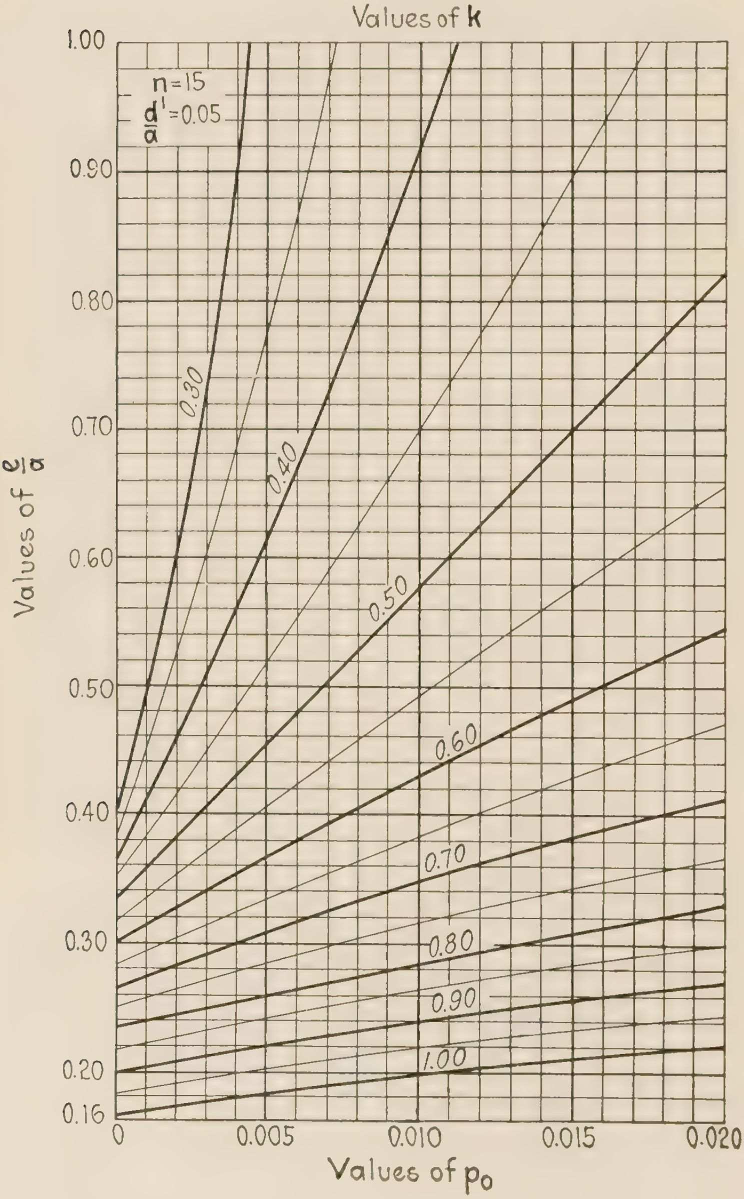


DIAGRAM 22

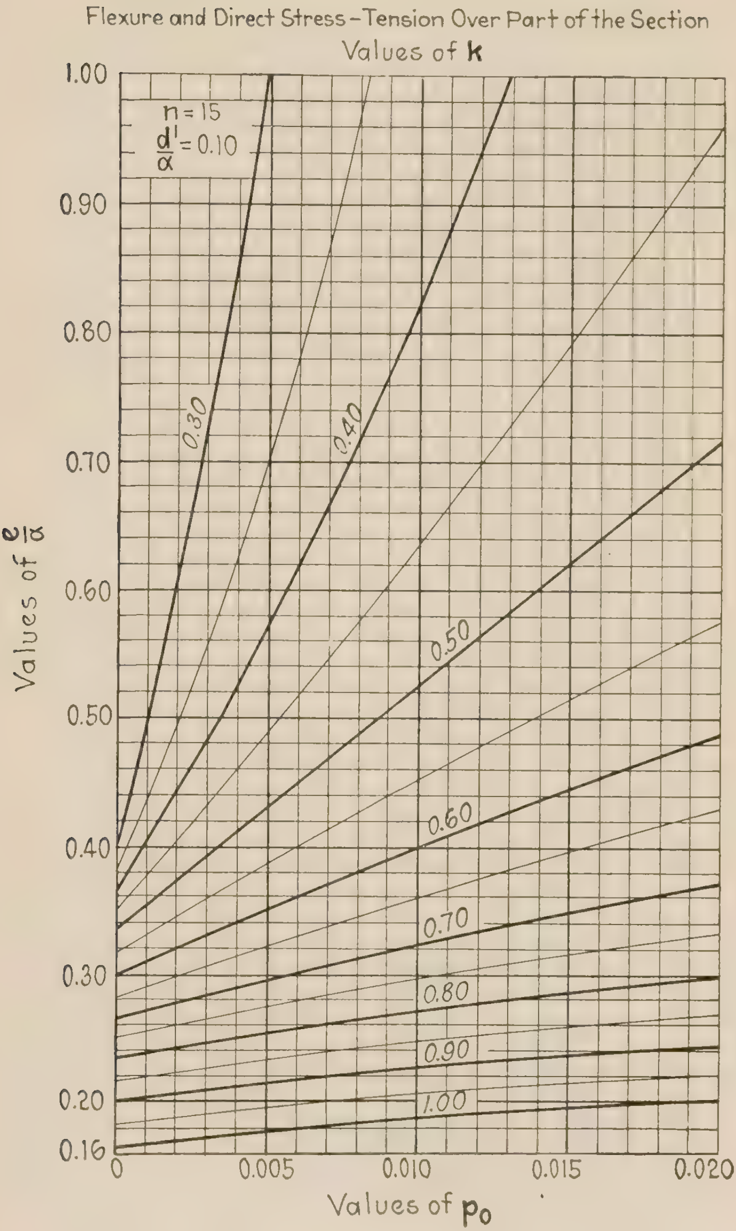

Flexure and Direct Stress-Tension Over Part of the Section
Values of k
n = 15
d'/α = 0.10
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.16
Values of e/α
0
0.005
0.010
0.015
0.020
Values of p_0

DIAGRAM 23

Flexure and Direct Stress—Tension Over Part of the Section

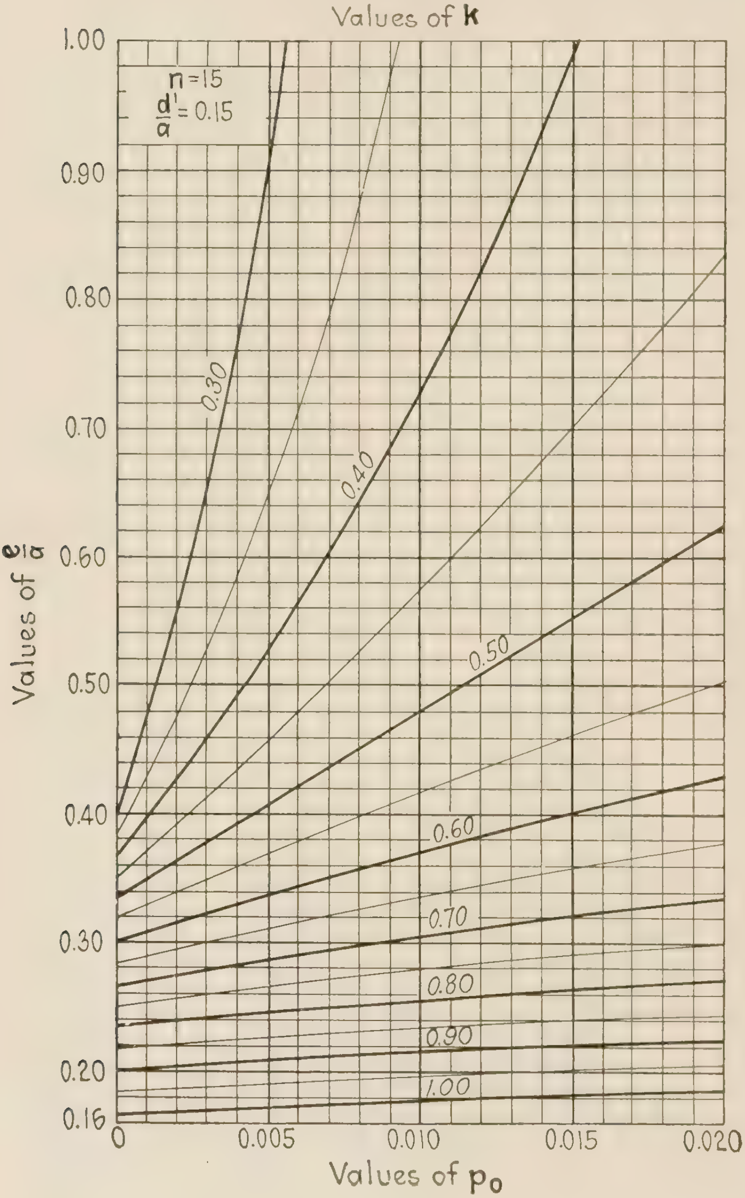
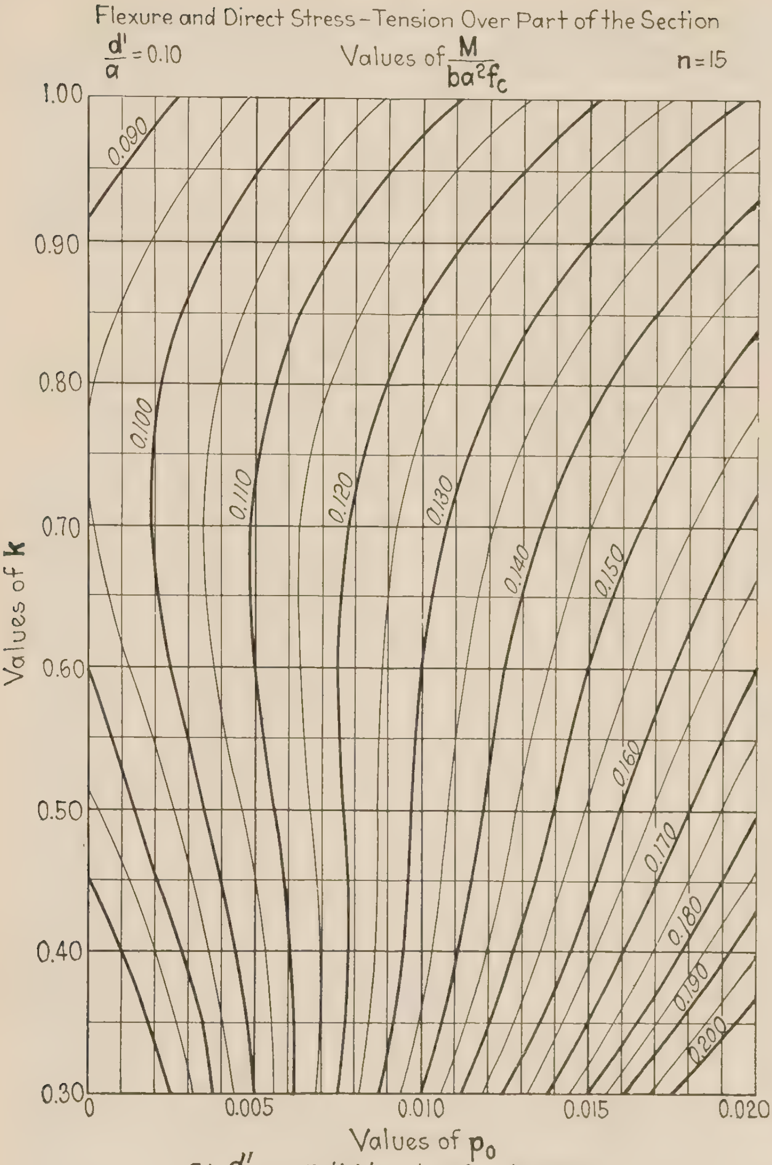


DIAGRAM 24



For $\frac{d'}{a} = 0.05$ *divide value of* p_0 *by 0.790*

For $\frac{d'}{a} = 0.15$ *divide value of* p_0 *by 1.306*

CHAPTER V

COLUMNS

102. Concrete columns may be divided into five classes, namely:

1. Plain concrete columns.
2. Columns reinforced with longitudinal rods only.
3. Columns reinforced with hoops or spirally wound metal.
4. Columns reinforced with both longitudinal rods and hoops.
5. Columns reinforced with structural steel shapes.

It is not the general practice to allow the construction of plain concrete columns. Concrete compression members whose unsupported length is less than four times the least dimension are usually referred to as piers. Compression members, whose ratio of length to width is greater than four, are usually reinforced.

Columns reinforced with hoops or spirals, but without longitudinal reinforcement, are uncommon, and are not considered good design, as without the longitudinal rods it is difficult to assure a well-centered spiral core, and a column so constructed lacks the added stiffness gained from the longitudinal steel.

Columns reinforced with structural steel shapes are generally not strictly reinforced concrete columns, but steel columns encased in concrete. Where the structural shapes are so placed that they are the most effective, there is still a loss in theoretical strength over that of a column with the same amount of steel placed in the form of longitudinal rods. Where the relative section of the steel is large, the action of the concrete is uncertain, and it should not be considered as adding to the strength of the column.

Columns reinforced with longitudinal rods only, and columns reinforced with both longitudinal rods and hoops are practically the only types recognized today as good reinforced concrete construction. The remainder of this chapter refers to these two types only. They are illustrated in Fig. 33.

103. Unsupported Length and Limiting Dimensions. The reinforced concrete column, as it is commonly used in ordinary construction, may be classified as a short column. In specifications it is usual to establish a ratio of length to diameter, or of length to least radius of gyration, above which the column can no longer be considered as a short column. Tests have shown that as long as the ratio of length to diameter is less than 20 or

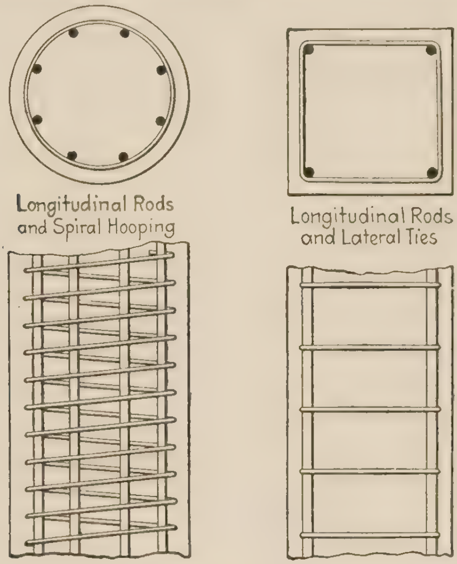


FIG. 33.

the ratio of length to least radius of gyration is less than 60, there is little variation in the actual strength of columns of the same cross-section for variations in length. In practice it is usual to specify either that the ratio of length to least diameter shall not exceed 15, or that the ratio of the length to least radius of gyration shall not exceed 40.[1] Good practice does not allow

[1] For ratios of length to least radius of gyration greater than 40, the Joint Committee recommends that the permissible working load on the core of a spirally reinforced column shall not be greater than

$$P\left(1.33 - \frac{h}{120R}\right)$$

in which P is the total safe load on a column of the same section whose h/R is less than 40, R the least radius of gyration of the column core, and h, the unsupported length of the column.

the construction of columns having a diameter or shorter side of less than 12 in., when these columns are the main supports of the floor or roof above. The difficulty of making uniform deposition of the concrete in a form of smaller dimensions is obvious, especially when there are several longitudinal bars with their ties or spirals in the form. In addition, a smaller column has very little reserve strength to withstand possible slight shocks not allowed for in the design, and if at any time it is damaged by fire, the loss in effective section is relatively large. Auxiliary posts which are not continuous from story to story may be of smaller diameter, but in no case should they be less than 6 in.

The unsupported length of a column h is the distance between those points at either end where lateral support is present in at least two directions, making an angle of 90 degrees or nearly 90 degrees with one another. Therefore, it follows that the unsupported length is:

1. In flat slab construction, the clear distance between the floor and the underside of the capital.

2. In beam and slab construction, the clear distance between the floor and the underside of the shallowest beam framing into the column.

3. In floor construction with beams in one direction only, the clear distance between floor slabs.

In cases where the columns are supported between floors by struts or beams, the unsupported length may be considered decreased, provided these struts or beams meet the column at approximately the same elevation and make horizontal angles of approximately 90 degrees with one another. When haunches are used on beam or struts, the unsupported length may be considered to be reduced by two-thirds of the depth of the haunch.

104. Columns with Longitudinal Reinforcement and Lateral Ties. As long as the bond between the steel and the concrete is effective, the two materials will deform equally, and the intensities of stress will be proportional to their moduli of elasticity. That is, since $E_c = \frac{f_c}{\lambda_c}$, and $E_s = \frac{f'_s}{\lambda_s}$ or $\lambda_c = \frac{f_c}{E_c}$ and $\lambda_s = \frac{f'_s}{E_s}$, and since λ_c must equal λ_s, as long as the bond between the two materials remains intact, it follows that $\frac{f'_s}{E_s} = \frac{f_c}{E_c}$ or $f'_s = f_c n$.

Let

A = total effective cross-section of column = $A_c + A'_s$

A_c = area of concrete

A'_s = area of longitudinal steel

p_o = steel ratio $\frac{A'_s}{A}$

f'_s = unit compressive stress in steel

f_c = unit compressive stress in concrete

P = total strength of reinforced column for the stress f_c

$$\text{Then } P = f_cA_c + f'_sA'_s = f_c(A - p_oA) + f_cnp_oA$$
$$= f_cA[1 + (n - 1)p_o]$$
$$= f_c[A + (n - 1)A'_s] \qquad (47)$$

The economy of steel reinforcement is dependent upon the working stresses permissible in the concrete, and the value of n. Excepting unusual conditions, the value of n decreases as the value of f_c increases, so that the steel used for reinforcement is rarely stressed to as high a value as would be allowed in a compressive member composed entirely of steel.

The Joint Committee specifies that the value of f_c, for this type of column, shall not exceed $0.20f'_c$. The amount of longitudinal reinforcement considered in the calculations shall not be more than 2 per cent nor less than 0.5 per cent of the total area of the column. The longitudinal reinforcement shall consist of not less than four bars of minimum diameter of 1/2 in., placed with a clear distance from the face of the column not less than 2 in.

The longitudinal bars are held in alignment during construction by lateral ties as illustrated in Figs. 33 and 34. These ties should be made of wire at least 1/4 in. in diameter,[2] and the vertical distance between ties or sets of ties should not exceed 8 in. When the number of rods in a column exceeds four, the ties should be so detailed as to prevent the outward bending of every bar at the 8-in. interval. The methods of accomplishing this are illustrated in Fig. 34.

[2] There is no rational method of determining the size of wire that should be used for a lateral tie. A safe rule to follow is to use wire of such diameter that the area of its section is not less than 2 per cent of the section of the longitudinal reinforcement held in place by the tie.

The minimum longitudinal reinforcement specified above for a reinforced column of this type is rather light, considering that the only lateral support is a series of light disconnected ties. It is preferable to use not less than four $^5/_8$- or even $^3/_4$-in. rods as reinforcement for a tied column although the greater steel area thus furnished is not theoretically required.

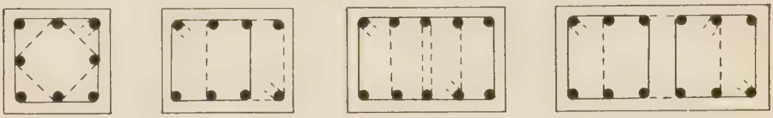


Fig. 34.

105. Columns with Spiral and Longitudinal Reinforcement. Whenever a material is subjected to compression in one direction, there will be an expansion in the direction perpendicular to the compression axis. Where this expansion is resisted, lateral compressive stresses are developed, which tend to neutralize the effect of the longitudinal compressive stress, and thus to increase the resistance against failure. This is the principle involved in the use of spiral or hooped reinforcement. Within the limit of elasticity the hooped reinforcement is much less effective than longitudinal reinforcement. Such reinforcement, however, raises the ultimate strength of the column, because the hooping prevents ultimate failure of the concrete. The concrete continues to compress and to expand laterally, thus increasing the tension in the bands, while final failure occurs upon the excessive stretching or breaking of the hooping. Thus a somewhat higher working stress may be employed on the concrete contained within such hooping than on a concrete not so confined. Tests show that about 1 per cent of closely spaced spiral hooping increases the resistance to ultimate failure sufficiently to allow a reasonable increase in the working stress in the concrete. Although some specifications and building codes allow a smaller amount, not less than one-half of 1 per cent of spiral hooping should be used in a column where increase in the concrete stress is allowed on account of the presence of the spiral.

The Joint Committee specifies that the safe axial load on columns reinforced with longitudinal bars and closely spaced

spirals enclosing a circular core shall be determined by the following formula:

$$P = f_c A[1 + (n - 1)p_o]$$

This formula is the same as equation (47) developed in the previous article for columns with longitudinal reinforcement and lateral ties. In this case, A is the area enclosed within the spiral, the diameter of the effective section (or of the spiral) being taken as the distance center to center of spiral wire. The application of this formula is limited to columns whose ratio of unsupported length to least radius of gyration is less than 40. The value of f_c to be used in the above expression for P is

$$f_c = 300 + (0.10 + 4p_o)f'_c$$

The longitudinal reinforcement shall consist of at least six bars of a minimum diameter of ½ in., and its effective cross-sectional area shall not be less than 1 per cent nor more than 6 per cent of the area enclosed within the spiral.

The spiral reinforcement shall not be less in amount than one-fourth of the volume of the longitudinal reinforcement. It shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical spacer bars. The spacing of spirals shall not be greater than one-sixth of the diameter of the core, and in no case more than 3 in. . . . Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core, which shall have a minimum thickness of 1½ in. in square columns, and 2 in. in round or octagonal columns.

Both the New York and Chicago building codes allow the amount of spiral reinforcement to influence the safe column load.

New York—$P = f_c(A - p_oA) + nf_cp_oA + 2f'_sp'A$, where p' is the percentage of spiral reinforcement and is limited to not less than ½ nor more than 2 per cent; f_c is taken as 500 lb. per sq. in., and f'_s as 20,000 lb. per sq. in. The ratio of unsupported length of column to diameter of core is limited to 15.

Chicago—$P = Af_c(1 + 2.5np')[1 + (n - 1)p_o]$ where f_c and n depend upon the quality of the concrete. The ratio of unsupported length of column to diameter of core is limited to 12.

106. Flexural Stresses in Columns. The previous articles have dealt with columns subject to direct axial load only. While there are many cases where this is the only type of load sustained by the column, there are many more cases where the maximum stress developed in the column is a combination of direct stress and flexure.

Bending moments are produced in columns (*a*) by reactions from eccentrically placed beams; (*b*) by the loads on brackets or cantilevers; (*c*) by the eccentricity of the columns themselves, a condition which often occurs in the wall columns of a building where the sections of the columns are changed at some floor levels while the exterior faces of the columns are kept in line throughout the height of the structure; (*d*) by the application of a direct horizontal force or of a force having a horizontal component; or (*e*) by the transfer from slabs or girders built monolithic with the columns of unbalanced moments due to the loads on the slabs or girders.

With conditions such as are described in (*a*), (*b*), and (*c*), the amount of moment produced in the column is easily determined, for the amount of load and the eccentricity of its center of application are known. A condition such as described in (*d*) is caused by the wind pressure on the walls of a building, but, on account of the massiveness and rigidity of the structure, it is not usual to calculate the wind stresses in the frame of any but high and narrow buildings[3] of reinforced concrete. A moment caused by the direct application of any other type of horizontal force is not common, but in such cases the moment is usually directly determinate. With conditions such as described in (*e*), the column is a component part of a rigid frame made up of columns and slabs or columns and girders. The distribution of moments in rigid frames will be considered in Chap. VI.

107. Eccentric Loads on Columns. An eccentric load applied to a column at any point will produce a maximum moment at the point of application. The distribution of the moment to the column depends upon the height at which the load is applied, and

[3] See "Wind Stresses in the Steel Frames of Office Buildings," by W. M. Wilson and G. A. Maney, *Bulletin* 80, Engineering Experiment Station, University of Illinois.

the end conditions of the column. A bending moment tending to cause tension on the outside face of the column below the point of application of the load is known as a negative moment, and that tending to cause tension on the near side of the column above the load is known as a positive bending moment. When the load is applied at the top or bottom of the column, the bending moment has its maximum possible value, and is equal to Px. For other positions of the load the moment is less, the minimum moment being $\frac{Px}{2}$. The coefficient of Px for different end conditions and different positions of the load may be obtained from

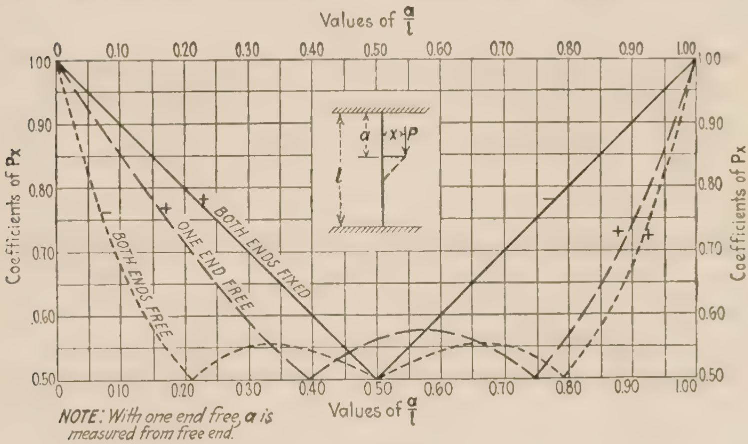


FIG. 35.

Fig. 35. The values of the extreme fiber stresses on either side of the column are obtained by the general method for combined flexure and direct stress explained in Chap. IV. The value of e is obtained by dividing the moment M by the total load (not necessarily P alone) supported by the column at the point where the eccentric load is applied.

108. Unit Stresses in Columns Subject to Flexure. The maximum permissible stress in the concrete may be increased when the combined effect of flexure and direct stress is considered. The maximum stress occurs only on one side of the column and rapidly decreases towards the axis of the column. Tests have shown that a much higher unit stress can be developed on the

extreme fiber in flexure than when the stress is uniformly distributed over the cross-section.

The Joint Committee allows a 20 per cent increase in the compressive unit stress on the concrete within the spiral of a column reinforced with longitudinal bars and spiral hooping, and a 50 per cent increase in the compressive unit stress on the concrete of a column reinforced with longitudinal bars and lateral ties. In the latter type of column, additional longitudinal reinforcement may be added if required, but in no case must the total amount of reinforcement considered in the calculations exceed 4 per cent of the total area of the column. The tensile stress in the steel should not in any case exceed 16,000 lb. per sq. in.

109. Column Tables. Tables VI to XIV may be used to advantage in the proportioning of reinforced concrete columns or in determining the maximum unit stresses in columns subject to flexure.

Tables VI to X are based on the Joint Committee specifications. They give the safe load for reinforced concrete columns with longitudinal bars and spiral hooping. The variables are the effective diameter of the column, the percentage of longitudinal reinforcement and the strength of the concrete.

Table XI gives the area of section, weight per foot, and moment of inertia of circular and octagonal sections. It also gives these same functions for a rectangular section 1 in. in width. These latter values may be used in the calculations of the stresses in square or rectangular columns.

Table XII gives the moment of inertia of the longitudinal reinforcement when the bars composing it are arranged in the form of a circle. The circle is assumed to be 1 in. less in diameter than the diameter of the spiral. While this is not always exactly correct in the column as proportioned, the error will not be great in the majority of cases. The values are given for a steel ratio, $p = \frac{A'_s}{A}$, of 1 per cent. The values of the moment of inertia for other percentages may be obtained by multiplying the value taken from the table by the percentage of steel in the member in question.

Table XIII gives the moment of inertia of single bars about an axis at varying distances from the center of the bars. It may be used in determining the moment of inertia of the reinforcement in columns of square or rectangular section.

Table XIV gives the pitch of the spiral for different diameters of spirals and spiral wire that will furnish certain definite percentages of spiral steel. Since most specifications do not allow a greater pitch than 3 in., no values greater than this are given in the table. Similarly, since a pitch of less than $1\frac{1}{2}$ in. is seldom if ever used on account of the difficulty of making proper deposition of the concrete with the wires more closely spaced, no values less than $1\frac{1}{2}$ in. are included.

110. *Illustrative Problems.* I. A round column reinforced with longitudinal steel and spiral hooping has an unsupported length of 20 ft.-0 in. and sustains a direct axial load of 200,000 lb. The ultimate strength of the concrete is specified as 2500 lb. per sq. in. Design the column.

With the minimum amount of reinforcement allowed by the specifications (1 per cent), from Table VI it appears that a column with an effective diameter of 19 in. can sustain a load of 204,600 lb. Since, however, a column of this size weighs more than 4600 lb., a larger column will be required. Assuming a column with an effective diameter of 20 in. and allowing 2 in. of concrete all around the column as a protection to the steel, the weight of the column (Table XI) is $471.2 \times 20 = 9400$ lb., making the the total load on the base of the column 209,400 lb. Since from Table VI a column with an effective diameter of 20 in. can sustain a load of 226,700 lb., such a column satisfies the requirements for the above conditions. The steel area required is 3.14 sq. in. Four 1-in. round bars furnish exactly this area, but according to the specifications not less than six bars may be used. Eight $\frac{3}{4}$-in. round bars furnish 3.53 sq. in. and are selected for this design.

According to the Joint Committee specifications $\frac{1}{4}$ per cent of spiral reinforcement is sufficient for this column, but following the recommendations of Art. 105, $\frac{1}{2}$ per cent will be used. From Table XIV, $\frac{1}{4}$-in. round with a pitch of $1\frac{7}{8}$ in. is selected.

An inspection of Tables VI to X shows that with greater percentages of steel, the size of the column required is less. With 6 per cent reinforcement, a column with an effective diameter of 12 in. and an outside diameter of 16 in. is sufficient for the above case. The amount of concrete required in the latter column is slightly less than half that required for the column as designed, but, on the other hand, more than twice as much longitudinal steel is required and the weight of the spiral is greater. The 20-24-in. column is the more economical of materials. If space is a major consideration, however, a larger percentage of steel and a consequently smaller column may be true economy.

II. A column reinforced with longitudinal steel and lateral ties is to support the same load under the conditions specified for Problem I. The column is to be of as small section as is possible.

The maximum amount of longitudinal steel that may be included in the calculations for a column of this type is 2 per cent. From equation (47) the approximate area of the column is

$$A = \frac{P}{f_c[1 + (n - 1)p_o]} = \frac{200{,}000}{500(1 + 11 \times .02)} = 328 \text{ sq. in.}$$

A square column of this area has a weight of $\frac{328}{144} \times 150 \times 20 =$ 6800 lb., which requires an additional area of section of 11 sq. in. A column 19 in. square weighs $\frac{361}{144} \times 150 \times 20 = 7500$ lb., and, with 2 per cent of longitudinal reinforcement, is capable of sustaining a load of $361 \times 500 \times 1.22 = 220{,}200$ lb. The reinforcement required is $.02 \times 361 = 7.22$ sq. in. Eight 1-in. square bars with an area of 8 sq. in. are chosen. Two sets of ties are required and, according to the rule for size of ties given in the footnote on page 175, the wire of the tie must have an area of section of $.02 \times 4 = .08$ sq. in. Five-sixteenths wire, although furnishing an area slightly less than .08 sq. in., may be used since it more than satisfies the rule for the amount of longitudinal reinforcement actually required.

III. In addition to the direct load of 200,000 lb., the column of Problem I is to support on a bracket an additional load of 20,000 lb. The center of bearing of this load is 8 in. from the outside face of the column and the center of the bracket is 12 ft.-0 in. from the base of the column.

The total load supported by the column is 200,000 + 20,000 + 9400 = 229,400 lb. The actual value of p_o for the column as designed above is $\frac{3.53}{314.2} = .0112$, the allowable value of $f_c = 300 + (.10 + 4 \times .0112)2500 = 662$, and the safe load, $P = 662 \times 314.2(1 + 11 \times .0112) = 233{,}600$ lb., so that the column as designed above can safely sustain the additional load if it is considered as an axial load. The distance from the point of application of the eccentric load to the axis of the column is 8 + 12 in. = 20 in. If the load were applied at either the top or the bottom of the column, the moment produced by this load would be 20,000 × 20 = 400,000 in.-lb. Since, however, the point of application is 12 ft.-0 in. from the base of the column, from Fig. 35 (considering both ends fixed), the actual moment produced at the point of application of the load is .60 × 400,000 = 240,000 in.-lb. At this point in the column the total load supported is 5600 lb. less than the load at the base of the column.

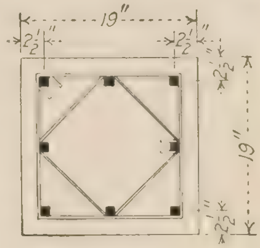


Fig. 36.

From Tables XI and XII the moment of inertia of the column section is determined as 7854 + 1.12 × 1560 = 9600 in.^{4}, and from equation (38)

$$f_c = \frac{223{,}800}{314.2(1 + 11 \times .0112)} + \frac{240{,}000 \times 10}{9600} = 886 \text{ lb.}$$

The allowable unit stress is 662 × 1.20 = 794 lb.

Increasing the reinforcement to eight $\frac{7}{8}$-in. round bars

$$f_c = \frac{223{,}800}{314.2(1 + 11 \times .0153)} + \frac{240{,}000 \times 10}{10{,}240} = 844 \text{ lb.}$$

while the allowable unit stress is 1.20 [300 + (1 + 4 × .0153) 2500] = 844 lb.

IV. Suppose that 120,000 lb. of the load delivered to the column of Problem II comes from a smaller column 14 × 19 in. in section, and that the 19-in. outside faces of both columns are in the same vertical plane. In such a case, the moment at the top of the square column is 120,000 × $2\frac{1}{2}$ = 300,000 in.-lb.

With the reinforcement arranged as in Fig. 36, the moment of inertia of the section as obtained with the aid of Tables XI and XIII is $19 \times 572 + 11(6 \times 49) = 14{,}100$ in.^{4} and the maximum unit stress at the top of the column is

$$\frac{200{,}000}{361(1 + 11 \times .0221)} + \frac{300{,}000 \times 9\frac{1}{2}}{14{,}100} = 650 \text{ lb.}$$

while the allowable unit stress is 50 per cent greater than 500 lb. or 750 lb. (Note that in this latter computation the actual percentage of the steel in the column was used since the specifications allow steel up to 4 per cent to be used in computations involving flexural stresses.)

TABLE VI.—COLUMNS WITH LONGITUDINAL BARS AND SPIRAL HOOPING

$P = f_c A[1 + (n - 1)p_o]$ $\qquad$ $f_c = 300 + (0.10 + 4p_o)f'_c$

Loads in thousand pounds. d = effective diameter, center to center of spiral wire

d	$p_o = 0.01$				$p_o = 0.015$			
	f'_c			A_s in sq. in.	f'_c			A_s in sq. in.
	2000	2500	3000		2000	2500	3000	
12	74.8	81.6	88.8	1.13	84.9	92.3	100.2	1.70
13	87.8	95.7	104.2	1.33	99.6	108.2	117.5	1.99
14	101.8	111.1	120.8	1.54	115.5	125.5	136.3	2.31
15	116.9	127.5	138.7	1.77	132.6	144.1	156.5	2.65
16	132.9	145.1	157.8	2.01	150.8	163.9	178.0	3.02
17	150.1	163.7	178.1	2.27	170.3	185.1	200.9	3.41
18	168.3	183.6	199.7	2.54	190.9	207.5	225.3	3.82
19	187.5	204.6	222.5	2.84	212.7	231.2	251.0	4.25
20	207.7	226.7	246.5	3.14	235.7	256.2	278.1	4.71
21	229.0	249.9	271.8	3.46	259.8	282.4	306.6	5.19
22	251.3	274.2	298.3	3.80	285.2	310.0	336.5	5.72
23	274.7	299.8	326.1	4.15	311.7	338.8	367.8	6.23
24	299.1	326.4	355.0	4.52	339.4	368.9	400.5	6.78
25	324.6	354.2	385.2	4.91	368.3	400.3	434.5	7.36
26	351.1	383.0	416.7	5.31	398.3	432.9	470.0	7.96
27	378.6	413.1	449.4	5.73	429.5	466.9	506.9	8.58
28	407.2	444.3	483.3	6.16	462.0	502.1	545.1	9.23
29	436.7	476.6	518.4	6.61	495.5	538.6	583.8	9.91
30	467.4	510.0	554.8	7.07	530.3	576.4	624.8	10.60
31	499.0	544.6	592.3	7.55	566.2	615.5	667.2	11.32
32	531.7	580.3	631.2	8.04	603.3	655.9	710.0	12.06
33	565.5	617.1	671.3	8.55	641.6	697.5	755.2	12.83
34	600.3	655.1	712.5	9.08	681.1	740.4	801.8	13.62
35	636.1	694.1	755.1	9.62	721.8	784.6	849.8	14.43
36	673.0	734.4	798.8	10.18	763.6	830.0	899.1	15.27

TABLE VII.—COLUMNS WITH LONGITUDINAL BARS AND SPIRAL HOOPING

$P = f_cA[1 + (n - 1)p_o]$ $f_c = 300 + (0.10 + 4p_o)f'_c$

Loads in thousand pounds. d = effective diameter, center to center of spiral wire

d	$p_o = 0.02$				$p_o = 0.025$			
	f'_c			A_s in sq. in.	f'_c			A_s in sq. in.
	2000	2500	3000		2000	2500	3000	
12	95.6	103.5	112.1	2.26	106.9	115.4	124.6	2.83
13	112.1	121.4	131.5	2.65	125.4	135.4	146.3	3.32
14	130.0	140.8	152.5	3.08	145.5	157.0	169.7	3.85
15	149.3	161.7	175.1	3.53	167.0	180.2	194.8	4.47
16	169.9	184.0	199.2	4.02	190.0	205.0	221.7	5.13
17	191.7	207.7	225.0	4.54	214.5	231.5	250.2	5.67
18	215.0	232.8	252.3	5.09	240.5	259.5	280.5	6.36
19	239.5	259.4	281.1	5.67	268.0	289.2	312.6	7.09
20	265.4	287.5	311.4	6.28	296.9	320.5	346.3	7.85
21	292.6	316.9	343.3	6.93	327.3	353.3	381.9	8.66
22	321.2	347.8	376.8	7.60	359.2	387.8	419.1	9.51
23	351.0	380.2	411.9	8.31	392.6	423.8	458.1	10.38
24	382.2	413.9	448.4	9.05	427.5	461.4	498.8	11.31
25	414.7	449.2	486.5	9.82	463.9	500.7	541.2	12.27
26	448.5	485.8	526.3	10.62	501.8	541.5	585.4	13.38
27	483.7	523.9	567.5	11.45	541.1	584.0	631.3	14.32
28	520.2	563.4	610.3	12.32	581.9	628.1	678.9	15.39
29	558.0	604.3	654.7	13.21	624.2	673.8	728.2	16.51
30	597.2	646.8	700.6	14.14	668.0	721.0	779.3	17.67
31	637.6	690.7	748.1	15.10	713.2	769.8	832.1	18.87
32	679.4	735.9	797.2	16.09	760.0	820.3	886.7	20.11
33	722.6	782.6	847.8	17.11	808.3	872.4	942.9	21.38
34	767.0	830.8	899.9	18.16	858.0	926.1	1001.0	22.70
35	812.8	880.3	953.7	19.24	909.2	981.4	1060.7	24.05
36	859.9	931.3	1008.9	20.36	961.9	1038.2	1122.2	25.45

TABLE VIII.—COLUMNS WITH LONGITUDINAL BARS AND SPIRAL HOOPING

$P = f_c A[1 + (n - 1)p_o]$ $\qquad$ $f_c = 300 + (0.10 + 4p_o)f'_c$

Loads in thousand pounds. d = effective diameter, center to center of spiral wire

d	$p_o = 0.03$				$p_o = 0.035$			
	f'_c			A_s in sq. in.	f'_c			A_s in sq. in.
	2000	2500	3000		2000	2500	3000	
12	118.8	127.8	137.9	3.39	131.4	140.9	151.7	3.96
13	139.6	150.0	161.9	3.98	154.4	165.4	178.0	4.64
14	161.9	174.0	187.7	4.62	178.9	191.9	206.4	5.39
15	185.4	199.7	215.4	5.30	205.6	220.3	236.9	6.18
16	210.9	227.3	245.1	6.03	233.9	250.6	269.7	7.06
17	236.8	256.6	276.8	6.81	264.0	283.0	304.5	7.95
18	265.8	287.6	310.3	7.63	296.0	317.2	341.3	8.91
19	296.3	320.5	345.7	8.51	330.1	353.4	380.3	9.92
20	328.5	355.1	383.0	9.42	365.7	391.6	421.4	11.00
21	362.1	391.6	422.3	10.39	403.2	431.8	462.6	12.12
22	397.7	344.8	463.5	11.40	442.4	473.8	509.9	13.30
23	434.3	469.7	506.6	12.46	483.4	517.9	557.3	14.54
24	473.0	511.4	551.5	13.57	526.4	563.9	606.9	15.83
25	513.5	555.0	598.5	14.72	571.1	611.9	658.4	17.18
26	555.5	600.2	647.3	15.93	617.7	661.9	712.2	18.58
27	599.0	647.3	698.1	17.18	666.0	713.7	768.0	20.04
28	644.6	696.1	750.7	18.47	716.2	767.5	825.9	21.55
29	691.5	746.7	805.3	19.82	768.5	822.3	886.0	23.12
30	740.1	799.1	861.8	21.21	822.3	880.1	846.1	24.74
31	790.5	853.2	920.3	22.64	878.0	939.9	1012.3	26.42
32	842.5	909.2	980.5	24.13	935.5	1001.5	1078.8	28.15
33	896.1	967.0	1042.8	25.66	995.0	1065.1	1147.2	29.94
34	951.4	1026.4	1107.0	27.24	1056.2	1130.7	1217.8	31.78
35	1008.3	1087.7	1173.0	28.86	1119.1	1198.2	1290.5	33.67
36	1066.9	1150.7	1241.0	30.54	1183.9	1267.8	1365.3	35.63

TABLE IX.—COLUMNS WITH LONGITUDINAL BARS AND SPIRAL HOOPING

$P = f_cA[1 + (n - 1)p_o]$ $f_c = 300 + (0.10 + 4p_o)f'_c$

Loads in thousand pounds. d = effective diameter, center to center of spiral wire

d	$p_o = 0.04$				$p_o = 0.045$			
	f'_c			A_s in sq. in.	f'_c			A_s in sq. in.
	2000	2500	3000		2000	2500	3000	
12	144.6	154.9	166.1	4.52	158.6	169.1	181.1	5.09
13	169.8	181.5	194.9	5.31	186.0	198.4	212.6	5.97
14	196.9	210.6	226.2	6.16	215.8	230.2	246.6	6.93
15	226.1	241.8	259.5	7.07	247.8	264.2	282.9	7.95
16	257.2	275.0	295.3	8.04	281.8	290.5	321.9	9.05
17	290.4	310.5	333.4	9.08	318.2	339.3	363.5	10.22
18	325.5	348.1	373.8	10.18	356.7	380.4	407.5	11.45
19	362.7	387.9	416.4	11.34	397.5	423.9	454.2	12.76
20	401.9	429.8	472.3	12.57	440.4	469.7	503.2	14.14
21	443.0	473.9	508.7	13.86	485.6	517.8	554.7	15.59
22	486.3	520.0	558.4	15.20	532.9	568.3	608.9	17.10
23	531.4	568.4	610.3	16.62	582.4	621.1	665.4	18.70
24	578.7	618.8	664.5	18.10	634.2	676.3	724.6	20.36
25	628.0	671.6	721.0	19.64	688.1	733.9	786.4	22.09
26	679.2	726.3	779.9	21.24	744.2	793.7	850.4	23.89
27	732.4	783.3	841.0	22.90	802.6	856.0	917.0	25.77
28	787.7	842.4	904.4	24.63	863.2	920.5	986.2	27.71
29	844.9	903.5	970.2	26.42	925.9	987.5	1057.9	29.72
30	904.2	967.0	1038.2	28.28	990.9	1056.8	1132.1	31.81
31	965.5	1032.6	1108.6	30.19	1058.1	1128.4	1209.0	33.97
32	1028.8	1100.2	1181.3	32.17	1127.4	1202.4	1288.2	36.19
33	1094.1	1170.0	1256.3	34.21	1198.9	1278.7	1369.9	38.49
34	1161.4	1242.0	1333.6	36.32	1272.7	1357.3	1454.2	40.86
35	1230.7	1316.0	1413.2	38.48	1348.7	1438.4	1541.1	43.29
36	1302.1	1392.4	1495.0	40.72	1426.8	1521.7	1630.3	45.81

TABLE X.—COLUMNS WITH LONGITUDINAL BARS AND SPIRAL HOOPING

$P = f_c A[1 + (n - 1)p_o]$ $\quad f_c = 300 + (0.10 + 4p_o)f'_c$

Loads in thousand pounds. d = effective diameter, center to center of spiral wire

d	$p_o = 0.05$				$p_o = 0.06$			
	f'_c			A_s in sq. in.	f'_c			A_s in sq. in.
	2000	2500	3000		2000	2500	3000	
12	173.1	184.1	196.4	5.66	203.9	215.9	229.9	6.79
13	203.0	216.0	231.1	6.64	239.3	253.3	269.5	7.96
14	235.5	250.5	267.8	7.70	277.5	293.8	312.8	9.23
15	270.4	287.6	307.4	8.84	318.6	337.3	359.1	10.60
16	307.6	327.2	349.8	10.01	362.6	383.9	408.8	12.07
17	347.3	369.4	395.0	11.35	409.3	433.3	461.4	13.62
18	389.3	414.1	442.8	12.73	458.9	485.4	517.3	15.27
19	433.8	461.5	493.3	14.18	511.2	541.2	576.3	17.01
20	480.7	511.3	546.6	15.71	566.6	599.8	635.7	18.85
21	529.9	563.7	602.6	17.32	624.6	661.3	704.2	20.78
22	581.6	618.7	661.4	19.01	685.4	725.6	772.7	22.81
23	635.7	676.2	722.9	20.78	749.2	793.2	844.6	24.93
24	692.2	736.3	787.2	22.62	815.7	863.6	919.6	27.14
25	751.0	798.8	854.2	24.54	885.2	937.1	997.6	29.45
26	812.3	864.0	923.8	26.55	957.1	1003.5	1079.2	31.85
27	876.1	931.9	996.2	28.63	1032.5	1093.1	1164.0	34.36
28	942.1	1002.1	1071.4	30.79	1110.0	1175.6	1251.8	36.95
29	1010.6	1075.0	1149.4	33.03	1191.0	1260.9	1342.6	39.63
30	1081.5	1150.4	1229.9	35.34	1274.5	1349.5	1437.0	42.41
31	1155.1	1228.4	1313.3	37.74	1361.1	1440.9	1534.4	45.29
32	1230.5	1308.9	1399.4	40.21	1450.5	1535.2	1634.8	48.25
33	1308.6	1392.0	1488.2	42.77	1542.3	1632.8	1738.6	51.32
34	1389.1	1477.7	1579.8	45.40	1637.3	1733.2	1845.6	54.47
35	1472.0	1565.9	1674.1	48.11	1734.9	1836.6	1955.8	57.73
36	1557.4	1656.6	1771.1	50.89	1835.7	1943.2	2069.2	61.07

TABLE XI.—AREAS, WEIGHTS, AND MOMENTS OF INERTIA

Moments of inertia about the axis A-A

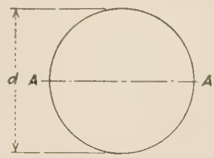
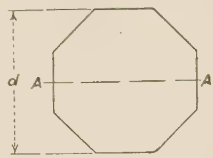
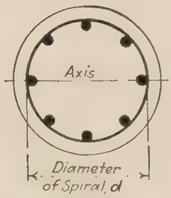
d									
	Area in sq. in.	Weight per ft. in lb.	I in in.^{4}	Area in sq. in.	Weight per ft. in lb.	I in in.^{4}	Area in sq. in.	Weight per ft. in lb.	I in in.^{4}
12	113.1	117.8	1,018	119.3	124.3	1,136	12.0	12.5	144
13	132.7	138.2	1,402	140.0	145.8	1,565	13.0	13.5	183
14	153.9	160.3	1,886	162.4	169.2	2,105	14.0	14.6	229
15	176.7	184.1	2,485	186.4	194.2	2,775	15.0	15.6	281
16	201.1	209.5	3,217	212.1	220.9	3,591	16.0	16.7	341
17	227.0	236.5	4,100	239.4	249.4	4,577	17.0	17.7	409
18	254.5	265.1	5,153	268.4	279.6	5,753	18.0	18.8	486
19	283.5	295.3	6,397	299.1	311.6	7,142	19.0	19.8	572
20	314.2	327.3	7,854	331.4	345.2	8,768	20.0	20.8	667
21	346.4	360.8	9,547	365.3	380.5	10,658	21.0	21.9	772
22	380.1	395.9	11,499	401.0	417.7	12,837	22.0	22.9	887
23	415.5	432.8	13,737	438.2	456.5	15,335	23.0	24.0	1,014
24	452.4	471.2	16,286	477.2	497.1	18,181	24.0	25.0	1,152
25	490.9	511.4	19,175	517.8	539.4	21,406	25.0	26.1	1,302
26	530.9	553.0	22,432	560.0	583.3	25,042	26.0	27.1	1,465
27	572.6	596.5	26,087	603.9	629.1	29,123	27.0	28.1	1,640
28	615.8	641.5	30,172	649.5	676.6	33,683	28.0	29.2	1,829
29	660.5	688.0	34,719	696.7	725.7	38,759	29.0	30.2	2,032
30	706.9	736.3	39,761	745.6	776.7	44,388	30.0	31.2	2,250
31	754.8	786.2	45,333	796.1	829.3	50,609	31.0	32.3	2,483
32	804.2	837.7	51,472	848.3	883.6	57,462	32.0	33.3	2,731
33	855.3	890.9	58,214	902.2	939.8	64,988	33.0	34.4	2,995
34	907.9	945.7	65,597	957.7	997.6	73,231	34.0	35.4	3,275
35	962.1	1002.2	73,662	1014.8	1057.1	82,234	35.0	36.5	3,573
36	1017.9	1060.3	82,448	1073.6	1118.3	92,043	36.0	37 5	3,880
37	1075.2	1120.0	91,998	1134.1	1181.3	102,704	37.0	38.5	4,221
38	1134.1	1181.3	102,354	1196.3	1246.1	114,265	38.0	39.6	4,573
39	1194.6	1244.4	113,561	1260.0	1312.5	126,777	39.0	40.6	4,943
40	1256.6	1308.9	125,664	1325.5	1380.7	140,288	40.0	41.7	5,333

TABLE XII.—MOMENT OF INERTIA OF COLUMN VERTICALS

Arranged in a circle of diameter 1 in. less than the diameter of the spiral

Values of $(n - 1)I_s$ in inches⁴ for $p_o = \frac{A'_s}{A} = 0.01$



Effective diameter of column d	Diameter of circle	$\frac{E_s}{E_c} = n$		
		15	12	10
12	11	239	188	154
13	12	334	263	215
14	13	454	357	292
15	14	606	476	389
16	15	792	622	509
17	16	1,016	799	654
18	17	1,287	1,011	827
19	18	1,607	1,263	1,033
20	19	1,985	1,560	1,276
21	20	2,425	1,905	1,559
22	21	2,933	2,305	1,886
23	22	3,519	2,765	2,282
24	23	4,191	3,293	2,694
25	24	4,948	3,888	3,181
26	25	5,807	4,562	3,733
27	26	6,774	5,322	4,355
28	27	7,856	6,173	5,050
29	28	9,062	7,120	5,825
30	29	10,404	8,174	6,688
31	30	11,888	9,341	7,642
32	31	13,525	10,626	8,694
33	32	15,327	12,043	9,853
34	33	17,294	13,588	11,117
35	34	19,463	15,293	12,512
36	35	21,821	17,145	14,028

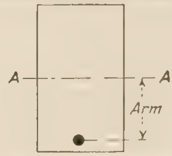
The bars are assumed transformed into a continuous cylinder having the same sectional area as the bars.

Then $I_s = [A'_s(d - 1)^2] \div 8$.

For other values of p_o multiply the value taken from the table by p_o.

TABLE XIII.—MOMENTS OF INERTIA OF BARS IN INCHES[4]

For various distances from an axis A-A



Arm in in.	Round bars, inches					Square bars, inches			
	½	⅝	¾	⅞	1	½	1	1⅛	1¼
2	1	1	2	2	3	1	4	5	6
2½	1	2	3	4	5	2	6	8	10
3	2	3	4	5	7	2	9	12	14
3½	2	4	5	7	10	3	12	16	19
4	3	5	7	10	13	4	16	20	25
4½	4	6	9	12	16	5	20	26	32
5	5	8	11	15	20	6	25	32	39
5½	6	9	13	18	24	8	30	38	47
6	7	11	16	22	28	9	36	46	56
6½	8	13	19	25	33	11	42	54	66
7	10	15	22	29	39	12	49	62	77
7½	11	17	25	34	44	14	56	71	88
8	13	20	28	39	50	16	64	81	100
8½	14	22	32	43	57	18	72	92	113
9	16	25	36	49	64	20	81	103	127
9½	18	28	40	54	71	23	90	114	141
10	20	31	44	60	79	25	100	127	156
10½	22	34	49	66	87	28	110	149	172
11	24	37	53	73	95	30	121	153	189
11½	26	41	58	80	104	33	132	168	207
12	28	44	64	87	113	36	144	182	225
13	33	52	75	102	133	42	169	214	264
14	38	60	87	118	154	49	196	248	306
15	44	69	99	135	177	56	225	285	352
16	50	79	113	154	201	64	256	324	400
17	57	89	128	174	227	72	289	366	452
18	64	99	143	195	255	81	324	410	506
19	71	111	160	217	284	90	361	457	564
20	79	128	177	241	314	100	400	506	625
21	87	135	195	265	346	110	441	558	689
22	95	148	214	291	380	121	484	613	756
23	104	162	234	318	416	132	529	670	827
24	113	177	254	346	452	144	576	729	900
25	123	192	276	376	491	156	625	791	977
26	133	207	299	407	531	169	676	856	1056

TABLE XIV.—COLUMN SPIRALS—PITCH OF SPIRAL IN INCHES FOR GIVEN PERCENTAGES AND VARIOUS WIRE GAUGES

Based on American Steel and Wire Co., Standard Gauges

Diameter of spiral in in.	Gauge of wire and practical equivalent bar size														
	No. 3 ¼ in. φ			No. 0 5/16 in. φ			No. 3/0 ⅜ in. φ				No. 5/0 7/16 in. φ			No. 7/0 ½ in. φ	
	Percentage of spiral reinforcement														
	½	¾	1	½	¾	1	½	¾	1	1½	1	1¼	1½	1	1½
12	3	2	1½			2½				2¼					
13	2⅞	1⅞	1½		3	2¼			3	2⅛			3		
14	2⅝	1¾			2¾	2⅛			3	2			2¾		
15	2⅜	1⅝			2⅝	1⅞			2¾	1⅞		3	2½		
16	2¼	1½			2½	1¾			2½	1¾		2⅞	2⅜		
17	2⅛	1½			2¼	1¾			2⅜	1⅝		2¾	2¼		3
18	2				2⅛	1⅝		3	2¼	1½		2⅝	2⅛		2¾
19	1⅞			3	2	1½		2⅞	2⅛	1½	3	2½	2		2⅝
20	1⅞			3	2	1½		2¾	2		2⅞	2⅜	1⅞		2½
21	1¾			2¾	1⅞			2⅝	2		2¾	2¼	1⅞		2⅜
22	1⅝			2⅝	1¾			2½	1⅞		2⅝	2⅛	1¾		2¼
23	1⅝			2½	1¾			2⅜	1¾		2½	2	1⅝		2⅛
24	1½			2½	1⅝			2¼	1¾		2⅜	1⅞	1⅝		2⅛
25				2⅜	1½			2⅛	1⅝		2⅜	1⅞	1½	3	2
26				2¼	1½		3	2⅛	1⅝		2¼	1¾	1½	2⅞	1⅞
27				2⅛	1½		3	2	1½		2⅛	1¾		2¾	1⅞
28				2⅛			2⅞	2	1½		2	1⅝		2⅝	1¾
29				2			2⅞	1⅞			2	1⅝		2⅝	1¾
30				2			2¾	1¾			1⅞	1½		2½	1⅝
31				1⅞			2⅝	1¾			1⅞	1½		2⅜	1⅝
32				1⅞			2½	1¾			1¾			2⅜	1½
33				1¾			2½	1⅝			1¾			2¼	1½
34				1¾			2⅜	1⅝			1¾			2¼	1½
35				1⅝			2⅜	1⅝			1⅝			2⅛	
36				1⅝			2¼	1½			1⅝			2⅛	

CHAPTER VI

STRESSES IN CONTINUOUS BEAMS AND BUILDING FRAMES

111. The design of a reinforced concrete structure involves an analysis of stress distribution somewhat different from that required for a structure of steel or timber. In the last two types the various elementary members are fabricated or cut separately and joined together in the structure by rivets, bolts, or nails. Such joints often do not establish complete continuity of a beam over a support, and the junction of beams and columns is not necessarily of sufficient rigidity to transfer bending moments from the beams to the columns. In a reinforced concrete structure consisting of slabs, beams, and columns, as much of the concrete as is practical is poured in one continuous operation, and the whole structure is more or less of a monolith. The slabs and beams are, therefore, continuous from span to span and rigidly joined to the columns which support them. Even when these are designed as simple beams, negative moment occurs over the supports and must be provided for. It is, then, desirable to recognize the continuity of the slabs and beams in their design and in some cases to analyze the columns for the bending stresses transferred to them.

112. Moments in Continuous Beams. The calculation of moments, shears, and reactions for continuous beams is based on the theorem of three moments. Considering all supports on the same level, the two fundamental equations are: (See Fig. 37).

For uniform loads

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 = -\tfrac{1}{4}w_1 l_1^3 - \tfrac{1}{4}w_2 l_2^3 \qquad (48)$$

For concentrated loads

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 = -\Sigma P_1 l_1^2(k_1 - k_1^3) - \Sigma P_2 l_2^2(2k_2 - 3k_2^2 + k_2^3) \qquad (49)$$

By using the equation applicable to the particular case, the bending moments at all of the supports may be determined, the reactions computed, and finally, the bending moment at any section of the beam may be obtained. From equation (49),

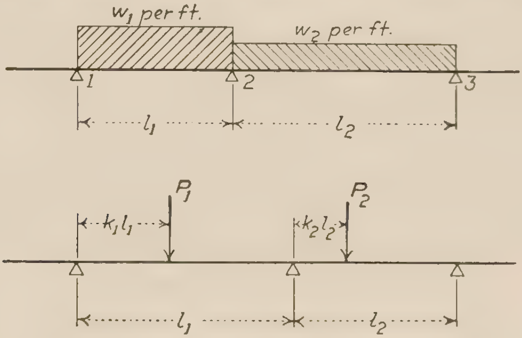


FIG. 37.

influence lines may be plotted for the moment at any section of the beam, and the loading determined which will produce the maximum moment in that section. In the case of uniform load and equal spans, the exact coefficients of wl^2 for the theoretical maximum moments are tabulated below.

Number of spans	Intermediate spans and supports				End spans and second support			
	At Center Positive Moment		At Support Negative Moment		At Center Positive Moment		At Support Negative Moment	
	Dead load	Live load	Dead load	Live load	Dead load	Live load	Dead load	Live load
Two					.070	.095	.125	.125
Three	.025	.075			.080	.100	.100	.117
Four	.036	.081	.071	.107	.071	.098	.107	.120
Five	.046	.086	.079	.111	.072	.099	.105	.120
Six	.043	.084	.086	.116	.072	.099	.106	.120
Seven	.044	.084	.085	.114	.072	.099	.106	.120

These coefficients are for freely supported beams. In a reinforced concrete structure the more or less fixed condition of the supports, and their width, tend to make the actual maximum moments considerably less than those tabulated. Disregarding the case of the two-span beam, the maximum coefficients are:

For intermediate spans

At the center	+0.086
At the support	−0.116

For end spans

At the center	+0.100
At the support	−0.120

These coefficients are all for live load, while those for dead load are much smaller. Taking this into consideration, together with the restraining influence and width of supports, it is recommended that for both positive and negative moments, and for both dead and live loads, the following coefficients be used:[1]

For intermediate spans	$\frac{1}{12}$
For end spans	$\frac{1}{10}$

In the case of a single load or symmetrical concentrated loads, such as often occur in beam and girder floor construction, it is satisfactory to compute the moment as for a simple beam and make a reduction of one-third or one-fifth, due to the continuity of the construction. The moment so determined is used not only as the positive moment at the center of the span but also as the negative moment at the support.

For beams and slabs of two spans only, a coefficient of $\frac{1}{8}$ is recommended for center supports, and $\frac{1}{10}$ for the center of the span.

Beams and slabs of unequal spans or those sustaining unsymmetrical heavy concentrated loads should be analyzed more

[1] The Joint Committee recommends moments at critical sections of beams or slabs cast monolithic with columns or similar supports and carrying uniformly distributed loads as follows:

Supports of intermediate spans,

$$M = \frac{wl^2}{12}$$

Center of intermediate spans,

$$M = \frac{wl^2}{16}$$

Beams in which $\frac{I}{l}$ is less than twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below, which are built into the beam

Center and first interior support,

$$M = \frac{wl^2}{12}$$

exactly, consideration being given to the actual conditions of restraint.

In many reinforced concrete structures, a considerable economy may be effected by making an exact analysis of the moments in the beams and girders. The principal justification for using the more or less arbitrary coefficients given above is the uncertainty of the live load. The simultaneous application of full live load to all portions of the structure is rarely realized. Even the live load on one panel may vary from a maximum to zero. If all such possible (and in many instances probable) variations of the live load are considered, the maximum moment is seldom found to be much less than that determined by the use of the arbitrary coefficients.

Exact methods of analysis may, of course, be made according to the theorem of three moments, but a much more expeditious method is that recently developed by L. H. Nishkian and D. B. Steinman and published in the *Proceedings* of the American Society of Civil Engineers in October, 1925.[2]

113. Bending Moments in Columns. When the slabs or the beams and slabs of a floor system of reinforced concrete are built monolithic with the columns which support them and the floors above, a certain portion of the bending stresses in the floor is distributed to the columns. When a column is symmetrically loaded in all directions, the bending moments from the adjacent beams and slabs act equally and opposite to one another, so no bending results in the column. With unsymmetrical loading, however, a bending moment is produced in the column, and since

Exterior supports,

$$M = \frac{wl^2}{12}$$

Beams in which $\frac{I}{l}$ is equal to, or greater than, twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beam,

Center of span and at first interior support of end span,

$$M = \frac{wl^2}{10}$$

Exterior support,

$$M = \frac{wl^2}{16}$$

[2] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points."

the whole structure acts as one rigid frame, this moment produces proportional moments in other members of the frame. In members widely separated, the effect may be so small as to be negligible. Where the unsymmetrical load is a small part of the total load, the increase in the stress is slight. For example, in the upper floors of a building the bending stresses produced in the exterior columns are relatively large, while in the lower floors the bending stresses in the interior columns are comparatively small. Tests show[3] that reinforced concrete buildings act as rigid frames and that bending stresses in the columns are developed in sufficient magnitude to warrant their consideration in design.

The amount of moment transferred to the columns from the floor depends upon the relative stiffnesses of the members of the frame. The stiffness of a member depends upon its length and cross-section and is defined as the moment of inertia of the cross-section divided by the length. The analysis of rigid frames is developed in the following articles. This analysis is based on the principles of area moments and slope deflections.[4]

114. The Principles of the Area Moment Method. The line AB of Fig. 38a represents a portion of the elastic curve of a member in flexure. An elementary length ds of the member is shown in Fig. 38b. The angle between the radii at the ends of ds is denoted by $d\theta$. The linear deformation of a fiber at a distance c from the neutral surface is $cd\theta$, and the unit deformation of the same fiber is $\frac{cd\theta}{ds}\cdot$ The unit stress in the fiber is $f = \frac{Mc}{I}$, in which M is the resisting moment and I the moment of inertia of the section.

Since the modulus of elasticity is the ratio of unit stress to unit deformation

$$E = \frac{Mc}{I} \div \frac{cd\theta}{ds}$$

[3] See *Bulletins* 64, 84, and 107, Engineering Experiment Station, University of Illinois.

[4] See "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin* 108, Engineering Experiment Station, University of Illinois.

from which $$d\theta = \frac{M}{EI}\,ds$$

In a well-designed beam the curvature and slope are small, so that dx may be substituted for ds without appreciable error.

Then $$d\theta = \frac{M}{EI}\,dx$$

In Fig. 38c an ordinate measured between the curve $A''B''$ and the straight line $A'B'$ at any point between A' and B'

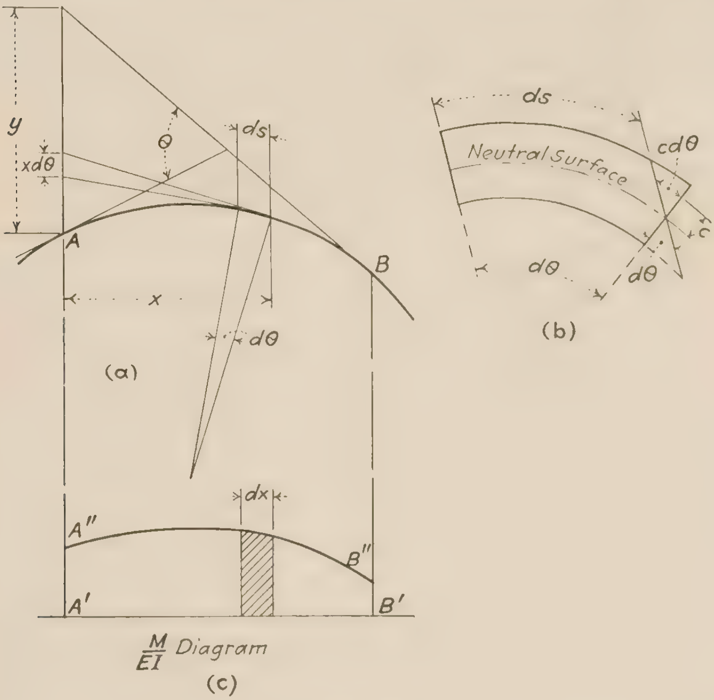


Fig. 38.

represents, to some scale, the moment in the member AB at that point, divided by EI, or $A''B''B'A'$ is the $\frac{M}{EI}$ diagram for the member AB. The area of the diagram for the length dx is $\frac{M}{EI}\,dx$, and the area of the diagram $A''B''B'A'$ is $\int_A^B \frac{M}{EI}\,dx$. But $d\theta = \frac{M}{EI}\,dx$ and the angle between the tangents to the

elastic curve at A and B is $\theta = \int_A^B d\theta = \int_A^B \frac{M}{EI} dx$. Hence,

The change in the slope of the elastic curve between any two points is equal to the area of the $\frac{M}{EI}$ *diagram for the portion of the member between those two points.*

In Fig. 38*a* the tangents at the extremities of the elementary length ds are extended until they intersect the vertical line through A. Since the angles are small, the intercept between these tangents is practically equal to $xd\theta$. The total vertical distance y is the algebraic sum of all the intercepts between the tangents to the curve between A and B. That is

$$y = \int_A^B x d\theta$$

Substituting for $d\theta$, its value as previously determined

$$y = \int_A^B \frac{M}{EI} x dx$$

In the $\frac{M}{EI}$ diagram of Fig. 38*c*, $\frac{M}{EI} dx$ is the area of the diagram for the length dx, and $\frac{M}{EI} dx$ times x is the moment of this area about the point A. The moment of the entire area of the $\frac{M}{EI}$ diagram between the points A and B may be expressed as

$$\int_A^B \frac{M}{EI} x dx$$

which is equal to the expression developed above for y. Hence,

The distance of any point on the elastic curve from a tangent to the curve at any other point measured in a direction normal to the initial position of the member is equal to the moment of the area of the $\frac{M}{EI}$ *diagram, included between the two points, about the first point.*

115. The Fundamental Slope-deflection Equations. (*a*) *Member Restrained at Ends with No Intermediate Load.* The line AB in Fig. 39 represents the elastic curve of a member of constant cross-section which is not acted upon by any external forces or couples except at the ends. The resisting moment at A is represented by M_{AB}, and that at B by M_{BA}. The change in slope of the elastic curve at A from its initial position is represented by θ_A, and that at B by θ_B. The deflection of A from its original position A' is d. The distance of B from the tangent

to the curve at A is $d - l\theta_A$. From the analysis of the previous article, $d - l\theta_A$ may be expressed as the moment of the $\frac{M}{EI}$ diagram[5] for the member AB about the end B.

The area of the $\frac{M}{EI}$ diagram is the algebraic sum of the two triangles *bad* and *bcd*, and the moment of this area about B is equal to the area of the triangle *bad* times the distance of its

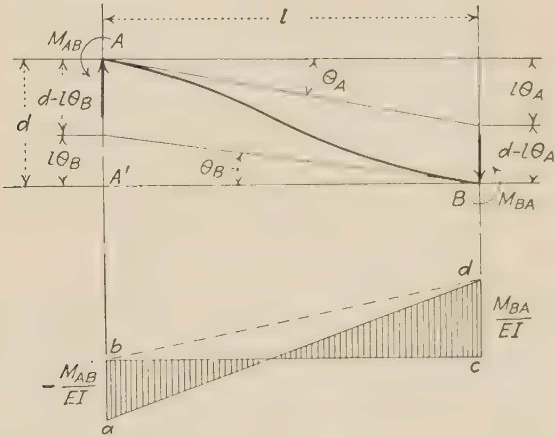


Fig. 39.

centroid from B, plus the area of the triangle *bcd* times the distance of its centroid from B. Hence[6]

$$d - l\theta_A = \frac{l^2}{EI}\left(\frac{M_{BA}}{6} - \frac{M_{AB}}{3}\right) \tag{50}$$

[5] The moment M_{AB} as indicated in Fig. 39 tends to produce tension in the upper fiber of the member AB, while M_{BA} tends to cause compression in the same fiber. The former is considered to be negative and the latter to be positive moment.

[6] In this analysis: (1) The moment of an external force or couple is positive if it tends to cause clockwise rotation; (2) when the tangent to the elastic curve of a member has been turned through a clockwise direction measured from its initial position, the change in a slope, or angular deformation is positive; (3) when the line joining the ends of a member is rotated, the movement of one end of the member with respect to the other end is known as the deflection and is positive when such rotation is in a clockwise direction from the initial position of the member; (4) the resisting moment or moment of the internal stresses on a section is positive when the internal or resisting couple acts in a clockwise direction upon the portion of the member considered.

The change in slope of the member from B to A is $\theta_B - \theta_A$, which, from Art. 113, is equal to the area of the $\frac{M}{EI}$ diagram for the member AB, or

$$\theta_B - \theta_A = \frac{l}{2EI}(M_{BA} - M_{AB}) \tag{51}$$

Combining equations (50) and (51)

$$M_{AB} = \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3d}{l}\right)$$

and

$$M_{BA} = \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3d}{l}\right)$$

Substituting S for $\frac{I}{l}$ and R for $\frac{d}{l}$

$$M_{AB} = 2ES(2\theta_A + \theta_B - 3R) \tag{52}$$

and

$$M_{BA} = 2ES(2\theta_B + \theta_A - 3R) \tag{53}$$

Equations (52) and (53) are the fundamental equations for the moments at the ends of a member carrying no transverse load, in terms of the relative change of slope and deflection of its ends.

(*b*) *Member Restrained at Ends with Any System of Intermediate Loads.* The line AB of Fig. 40*a* represents the elastic curve of the member of Fig. 39. With intermediate loads acting upon the member as shown, the $\frac{M}{EI}$ diagram is the algebraic sum of the $\frac{M}{EI}$ diagram of Fig. 39, and the simple beam $\frac{M}{EI}$ diagram of Fig. 40*b*. This is shown in Fig. 40*c*. Denoting the area of the simple beam moment diagram by F, the distance of its centroid from B by $\bar{x}$, and proceeding as in the analysis for the member without intermediate loads

$$d - l\theta_A = \frac{l^2}{EI}\left(\frac{M_{BA}}{6} - \frac{M_{AB}}{3}\right) - \frac{F\bar{x}}{EI} \tag{54}$$

and

$$\theta_B - \theta_A = \frac{l}{2EI}(M_{BA} - M_{AB}) - \frac{F}{EI} \tag{55}$$

Combining equations (54) and (55) and substituting S for $\frac{I}{l}$, and R for $\frac{d}{l}$ as before

$$M_{AB} = 2ES(2\theta_A + \theta_B - 3R) - \frac{2F}{l^2}(3\bar{x} - l) \tag{56}$$

and

$$M_{BA} = 2ES(2\theta_B + \theta_A - 3R) + \frac{2F}{l^2}(2l - 3\bar{x}) \tag{57}$$

Equations (56) and (57) are identical with equations (52) and (53) except that each contains an additional term due to the effect of the intermediate loads. In a fixed beam with supports on the same level, θ_A, θ_B, and R are zero, and this additional term

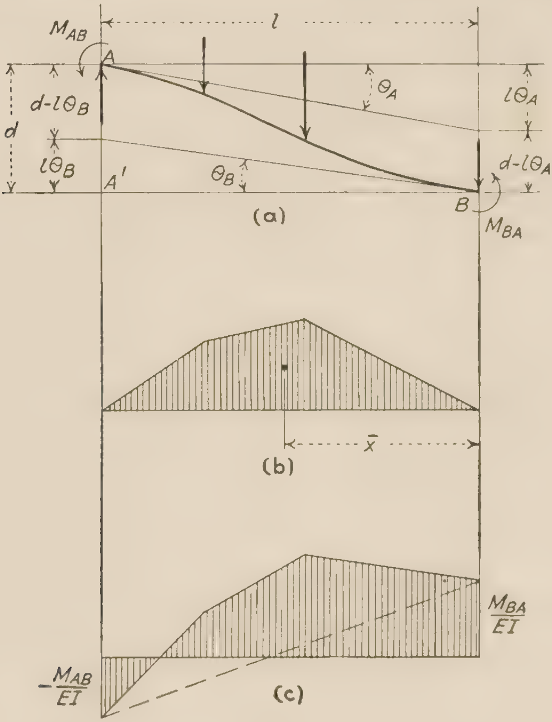


FIG. 40.

is the resisting moment acting on the end of a fixed beam. Expressing the resisting moment at the end of a fixed beam with level supports by C, with subscripts similar to those used for M, equations (56) and (57) may be written in the forms

$$M_{AB} = 2ES(2\theta_A + \theta_B - 3R) - C_{AB} \tag{58}$$

and

$$M_{BA} = 2ES(2\theta_B + \theta_A - 3R) + C_{BA} \tag{59}$$

The sign of the constant C is determined by considering that the sign of the resisting moment at the end of the member is opposite to that of the moment of the external loads.

Equations (58) and (59) are the general slope deflection equations which apply to any condition of loading and restraint.

A Member Restrained at One End and Hinged at the Other. If a member be considered restrained at the end A and hinged at the end B, the resisting moment at the latter end is zero. With $M_{BA} = 0$, equations (58) and (59) may be combined so as to eliminate θ_B, giving

$$M_{AB} = 3ES(\theta_A - R) - \left(C_{AB} + \frac{C_{BA}}{2}\right) \tag{60}$$

Similarly, if the member is restrained at the end B and hinged at the end A

$$M_{BA} = 3ES(\theta_B - R) + \left(C_{AB} + \frac{C_{BA}}{2}\right) \tag{61}$$

Equations (60) and (61) are special forms of equations (58) and (59), applicable only to members having one end hinged.

116. The Effect of the Restraint at One End of a Member upon the Moment at the Other End. In Fig. 41, the member AB represents any member in flexure. The end A is acted upon by a couple M_{AB} such that the tangent to the elastic curve at A makes an angle θ_A with AB. The magnitude of the moment at A depends not only upon the magnitude of θ_A, the moment of inertia of the section, and the length AB, but also upon the degree of restraint at B.

(a) With the end B hinged as shown in Fig. 41a and both R and $\left(C_{AB} - \frac{C_{BA}}{2}\right)$ equal to zero, equation (60) gives

$$M_{AB} = 3ES\theta_A \tag{62}$$

(b) With the end B fixed as shown in Fig. 41b and R, θ_B, and C_{AB} equal to zero, equation (58) gives

$$M_{AB} = 4ES\theta_A \tag{63}$$

(c) With equal restraint at ends A and B as shown in Fig. 41c, $\theta_A = -\theta_B$ and substitution in equation (58) gives

$$M_{AB} = 2ES\theta_A \tag{64}$$

(*d*) With restraint at B of such character that a point of inflection occurs at the mid-point of AB as shown in Fig. 41*d*, $o_A = \theta_B$, and from equation (58)

$$M_{AB} = 6ES\theta_A \tag{65}$$

In the case shown in Fig. 41*a*, $M_{BA} = 0$, while substitution in equation (59) gives for the condition shown in Fig. 41*b*

$$M_{BA} = 2ES\theta_A \tag{66}$$

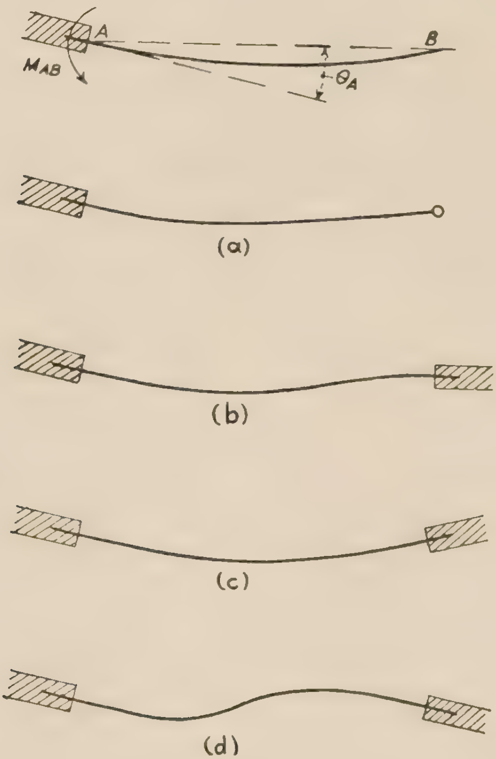


FIG. 41.

for that in Fig. 41*c*

$$M_{BA} = -2ES\theta_A \tag{67}$$

and for that in Fig. 41*d*

$$M_{BA} = 6ES\theta_A \tag{68}$$

For a member subject to intermediate loads or forces, equations (62) to (68) are modified by the addition of a term due to the

effect of these loads. Equation (62) for the case shown in Fig. 41*a* becomes

$$M_{AB} = 3ES\theta_A - \left(C_{AB} + \frac{C_{BA}}{2}\right) \tag{69}$$

Similarly, the other equations for the several conditions of end restraint illustrated in Fig. 41 are modified as follows:

(*c*) $$M_{AB} = 4ES\theta_A - C_{AB} \tag{70}$$

(*d*) $$M_{AB} = 2ES\theta_A - C_{AB} \tag{71}$$

(*b*) $$M_{AB} = 6ES\theta_A - C_{AB} \tag{72}$$

(*b*) $$M_{BA} = 2ES\theta_A + C_{BA} \tag{73}$$

(*c*) $$M_{BA} = -2ES\theta_A + C_{BA} \tag{74}$$

(*d*) $$M_{BA} = 6ES\theta_A + C_{BA} \tag{75}$$

Equations (62) to (75) as developed above cover the conditions of end restraint that are usually encountered. They apply to members with level supports only. If the supports are not on the same level or if the effect of the lowering of one of the supports is desired, equations (58) to (61) must be employed.

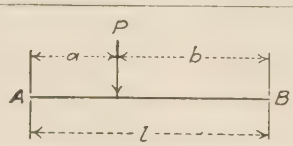
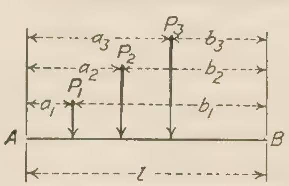
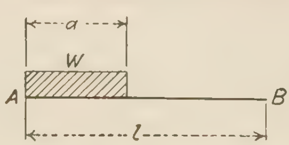
Type of Loading	C_{AB}	C_{BA}
	$\frac{Pab^2}{l^2}$	$\frac{Pba^2}{l^2}$
	$\frac{1}{l^2}\Sigma Pab^2$	$\frac{1}{l^2}\Sigma Pba^2$
	$\frac{Wa}{12l^2}(3a^2-8al+6l^2)$	$\frac{Wa^2}{12l^2}(4l-3a)$

CHART 1.

The more common conditions of unsymmetrical loading are given in the table above together with the values of C_{AB} and C_{BA}.

For any other type of loading, similar values may be obtained by determining the area of the simple beam moment diagram due to these loads, and the distance of the center of gravity of this area from the restrained end of the member. Making the proper substitutions in the last terms of equations (56) and (57), the desired values of C_{AB} and C_{BA} are determined. For a symmetrical load, the last terms of equations (56) and (57) each reduce to $\frac{F}{l}$. The values of $C_{AB} = C_{BA} = \frac{F}{l}$, for the more common conditions of loading are given in the following table. The method

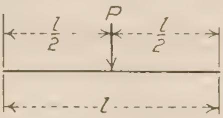
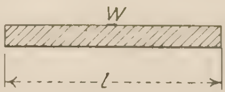
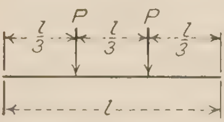
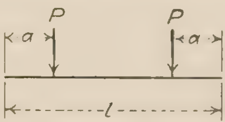
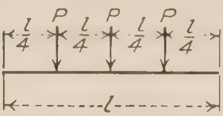
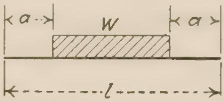
Type of Loading	C_{AB}	Type of Loading	C_{AB}
	$\frac{1}{8}Pl$		$\frac{1}{12}Wl$
	$\frac{2}{9}Pl$		$\frac{Pa}{l}(l-a)$
	$\frac{5}{16}Pl$		$\frac{W}{12l}(l^2+2al-2a^2)$

CHART 2.

of determining such values for other conditions of loading is obvious.

117. Application of the Slope-deflection Methods to Simple Cases. (*a*) Applying the equations of the preceding article to a beam of two spans l_1 and l_2, respectively, resting freely upon its supports and sustaining a uniform load w over the span l_1 (see Fig. 42):

From equation (69),

$$M_{BA} = 3ES_1\theta_B + \frac{w_1l_1^2}{12} + \frac{w_1l_1^2}{24}$$

and $$M_{BC} = 3ES_2\theta_B$$

Since there is equilibrium at the joint B,

$$M_{BA} + M_{BC} = 0$$

or $$(3ES_1 + 3ES_2)\,\theta_B + \frac{3w_1l_1^2}{24} = 0$$

and $$\theta_B = -\frac{1}{3ES_1 + 3ES_2}\,\frac{w_1l_1^2}{8}$$

Substituting the value of θ_B in equation (69),

$$M_{BC} = -M_{BA} = -\frac{S_2}{S_1 + S_2}\left(\frac{w_1l_1^2}{8}\right)$$

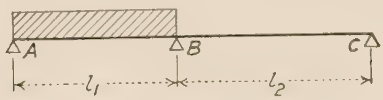


FIG. 42.

If the load covers the span l_2 instead of the span l_1, a similar analysis gives

$$M_{BA} = -M_{BC} = \frac{S_1}{S_1 + S_2}\left(\frac{w_2l_2^2}{8}\right)$$

For both spans sustaining loads w_1 and w_2, respectively,

$$M_{BC} = -M_{BA} = -\left[\frac{S_2(w_1l_1^2) + S_1(w_2l_2^2)}{8(S_1 + S_2)}\right]$$

which for equal loads, equal spans, and equal moments of inertia becomes $-\frac{1}{8}wl^2$, the negative moment over the center support of a beam of two equal spans resting freely on its supports.

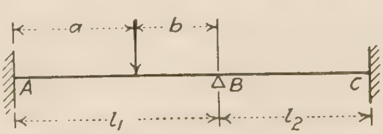


FIG. 43.

(*b*) In Fig. 43, a beam of two spans fixed at both ends rests freely on the intermediate support.

From equations (63) and (70),

$$M_{BC} = 4ES_2\theta_B$$

and $$M_{BA} = 4ES_1\theta_B + \frac{Pba^2}{l^2}$$

from which $$\theta_B = -\frac{1}{4ES_1 + 4ES_2}\left(\frac{Pba^2}{l^2}\right)$$

and $$M_{BC} = -M_{BA} = -\frac{S_2}{S_1 + S_2}\left(\frac{Pba^2}{l^2}\right)$$

118. Building Frames. Reinforced concrete building frames are composed of columns and slabs or of columns, beams, girders, and slabs. In the latter case the girders and columns or the beams and the columns may be considered to form a more or less rigid frame. The columns may be of the same size throughout the structure or their cross-sections may vary with the load that they must sustain. Similarly, uniformity or variation may be found in the beams and girders of a structure.

Consider Fig. 44 to illustrate a portion of a building frame. The point A is the junction of the members shown. The degree of rigidity of A depends upon the relative stiffnesses of the members intersecting in the joint and upon the degree of restraint

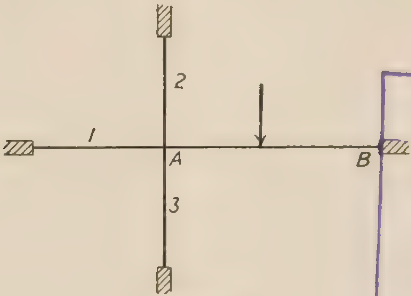


FIG. 44.

imposed upon the further ends of the several members. Also the moment in AB at A and the moment in members 1, 2, and 3 caused by the load on AB depend upon these same considerations. If any or all of the members 1 to 3 are infinitely rigid, the point A is fixed, while if none of the members are at all rigid, the point A may be considered hinged. In the latter case, the moment in AB at A is zero and no moment is transferred to the other members. In a building frame of reinforced concrete, each member of the frame is restrained to some extent at each end, due to the rigid connection existing between it and the other members of the frame. This restraint causes negative moments in the ends of the beams, tending to produce rotation at these points, and results in flexural stresses in the intersecting members.

Figure 45*a* shows one span of a building frame sustaining uniform load. The deformations caused in the various members of the frame by this loading are indicated by the broken lines. The deformations in the members immediately adjacent to *C* and *D* are much greater than those in members further removed from these points. If loads were added on *BC* and *DE*, there would be practically no deformations in the columns at *C* and

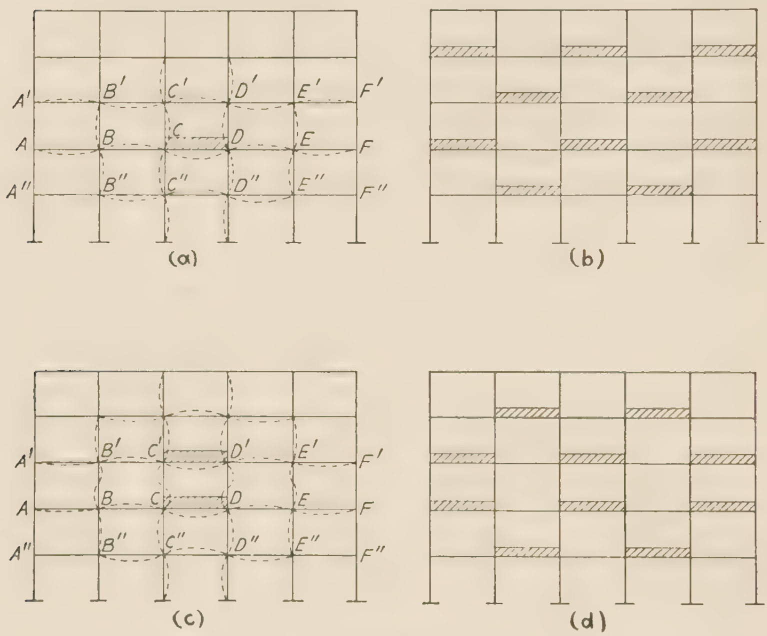


Fig. 45.

D, and *CD* would be practically fixed. On the other hand, if loads were placed on *AB* and *EF*, the deformations of the columns at *C* and *D* would be increased. Still greater deformations of the columns at *C* and *D* could be obtained by the loading shown in Fig. 45*b*.

Another loading producing large stresses in the columns is that shown in Fig. 45*c*. This type of loading develops a point of contraflexure in the center of the columns between *C′D′* and *CD*. A still further increase in stress occurs with the type of loading shown in Fig. 45*d*.

119. Moments in Beam and Girder Building Frames. In Fig. 46, which is a portion of the frame of Fig. 45, it is possible, by assuming various conditions of restraint at the terminals B, C', E, D'', and C'', to approximate any condition of loading. It will be assumed that the three girders have equal cross-sections and lengths, that the upper columns $C'C$ and DD' are equal in stiffness, and that the lower columns CC'' and DD'' are also equal in this respect. The stiffness of the girder is S_1, that of the column above C is S_2, and that below C is S_3.

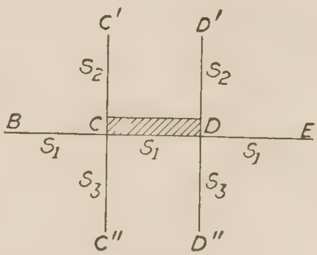


FIG. 46.

(*a*) *All Terminals Hinged.*

From equation (71)

$$M_{CD} = 2ES_1\theta_C - C_{CD}$$

From equation (62)

$$M_{CB} = 3ES_1\theta_C$$
$$M_{CC'} = 3ES_2\theta_C$$
$$M_{CC''} = 3ES_3\theta_C$$

For equilibrium, the sum of all the moments about the joint C must be zero, *i.e.*

$$M_{CD} + M_{CB} + M_{CC'} + M_{CC''} = 0$$

Substituting the values for the several moments

$$\theta_C = C_{CD}\left(\frac{1}{5S_1 + 3S_2 + 3S_3}\right)$$

Substituting the value of θ_C in the moment equations

$$M_{CB} = C_{CD}\left(\frac{3S_1}{5S_1 + 3S_2 + 3S_3}\right)$$

$$M_{CC'} = C_{CD}\left(\frac{3S_2}{5S_1 + 3S_2 + 3S_3}\right)$$

$$M_{CC''} = C_{CD}\left(\frac{3S_3}{5S_1 + 3S_2 + 3S_3}\right)$$

and since $\qquad M_{CD} = -(M_{CB} + M_{CC'} + M_{CC''})$

$$M_{CD} = -C_{CD}\left(\frac{3S_1 + 3S_2 + 3S_3}{5S_1 + 3S_2 + 3S_3}\right)$$

(*b*) *All Terminals Fixed.*

From equation (71)

$$M_{CD} = 2ES_1\theta_C - C_{CD}$$

From equation (63)

$$M_{CB} = 4ES_1\theta_C$$

$$M_{CC'} = 4ES_2\theta_C$$

$$M_{CC''} = 4ES_3\theta_C$$

As before,

$$\theta_C = C_{CD}\left(\frac{1}{6S_1 + 4S_2 + 4S_3}\right)$$

$$M_{CB} = C_{CD}\left(\frac{2S_1}{3S_1 + 2S_2 + 2S_3}\right)$$

$$M_{CC'} = C_{CD}\left(\frac{2S_2}{3S_1 + 2S_2 + 2S_3}\right)$$

$$M_{CC''} = C_{CD}\left(\frac{2S_3}{3S_1 + 2S_2 + 2S_3}\right)$$

and

$$M_{CD} = -C_{CD}\left(\frac{2S_1 + 2S_2 + 2S_3}{3S_1 + 2S_2 + 2S_3}\right)$$

In building frames the actual conditions of restraint at the ends of the various members are usually neither hinged nor fixed. The conditions of restraint may approximate either the hinged or fixed state or be similar to one of those illustrated in Figs. 41*c* or 41*d*. The value of the moment in the girder itself varies but little for the different conditions of end restraint. The coefficient of the terms S_2 and S_3 is the same in both numerator and denominator, and that of S_1 in the denominator is never less than that in the numerator. Therefore, the actual moment is never greater than that in a continuous girder with restrained ends, while in all cases except those involving girders large in comparison with the columns, it will be nearly equal to the moment as determined for a continuous girder with restrained ends.

The conditions of loading and restraint producing the maximum probable moment in the columns will be discussed in the following articles.

120. Interior Columns. Unless the column spacings are very irregular, no moment will be developed in the interior columns by the dead load of the structure. In the usual case of bays equal or nearly so, the dead load is symmetrical with respect to the columns and the only moment that can be developed in

the columns is that caused by the possible eccentricity of the live load.

An inspection of Fig. 45 shows that the maximum moment in an interior column such as $C'C$ occurs under a loading of the type shown in either (*b*) or (*d*). Either one of these loadings is extremely unlikely. Loadings of the type, however, shown in either (*a*) or (*c*) produce the same effect and are much more probable.[7]

Considering the joint C of Fig. 45*a*, any one of the joints at B, C', E, D'', and C'' may have a condition of restraint varying between the hinged and fixed state. The maximum moment in the column CC' would be developed if C' and D' are fixed and the remaining joints hinged. The actual condition of restraint at the joints, however, is not hinged but more closely approaches the fixed state. With all terminals considered fixed, the moment

$$M_{CC'} = C_{CD}\left(\frac{4S_2}{6S_1 + 4S_2 + 4S_3}\right) \tag{76}$$

In the lower column the maximum moment is developed with C'' and D'' fixed and the other joints hinged, but for the same reasons as given above, all terminals will be considered fixed and

$$M_{CC''} = C_{CD}\left(\frac{4S_3}{6S_1 + 4S_2 + 4S_3}\right) \tag{77}$$

Similarly, considering the joint of Fig. 45*c*, the maximum moment in the column CC' occurs when all the joints except C' and D' are fixed, while the condition of restraint of these two joints is similar to that shown in Fig. 41*d*. In this case

$$M_{CC'} = C_{CD}\left(\frac{6S_2}{6S_1 + 6S_2 + 4S_3}\right) \tag{78}$$

Likewise, with the loading on $C''D''$ instead of on $C'D'$, C'' and D'' being in a condition of restraint similar to that shown in Fig. 41*d*, while the other joints are fixed,

$$M_{CC''} = C_{CD}\left(\frac{6S_3}{6S_1 + 4S_2 + 6S_3}\right) \tag{79}$$

[7] F. E. Richart, in "A Study of Bending Moments in Columns," *Proceedings* of American Concrete Institute, vol. 20, p. 495, states that a loading such as (*c*) produces about 80 per cent of the moment in column $C'C$ as would be produced by a loading such as in (*d*).

The upper columns are never larger in cross-section than those below them, so that with equal story heights, S_3 is never less than S_2, but it is often greater. Therefore, the maximum probable moment in an interior column is never likely to be greater than would be obtained from equation (79). Conditions of restraint varying from the conditions assumed in the development of equation (79) would cause a slight variation in the moment in the column, but the conditions as assumed are usually actually realized, so the above equation is recommended for general use.

The basement columns of a building frame present a special case, in that the basement floor is usually not an integral part of the structure, and does not transfer its load to the frame. The maximum probable moment is developed when the basement columns are fixed, all other joints of the frame being likewise assumed as fixed. The value of the moment is then given by equation (77).

Another special case occurs in the upper tier of columns. Any eccentric moment in the roof girder must be transferred to the adjacent girder and the supporting column, there being no upper column to aid in absorbing such moment. If the live load on the roof is nearly as great as that on the floor, so that with the type of loading shown in Fig. 45*c* a point of inflection may be assumed at the center of the columns, the moment at the top of the roof column is

$$M'_{CD}\left(\frac{6S_3}{6S_{1'} + 6S_3}\right) \tag{80}$$

in which M'_{CD} is the moment in the roof girder, and $S_{1'}$ the stiffness of the roof girder.

Usually the moment in the roof girder is so much smaller than the corresponding moment in the floor girder that the maximum moment in the roof column occurs at the bottom of the column. In such a case, the roof load being considerably less than the floor load, it is not reasonable to expect that a loading similar to that of Fig. 45*c* will cause a point of inflection at the center of the columns. If the top of such a column is considered fixed, however, it would seem that the moment computed according to this assumption is great enough to provide for the actual stresses

developed. Therefore, the moment at the bottom of the column may be taken as

$$M_{CD}\left(\frac{4S_2}{6S_1 + 4S_2 + 4S_3}\right) \tag{81}$$

The critical cases are summarized in Fig. 47. When there is no live load on certain panels, while the remainder of the structure is

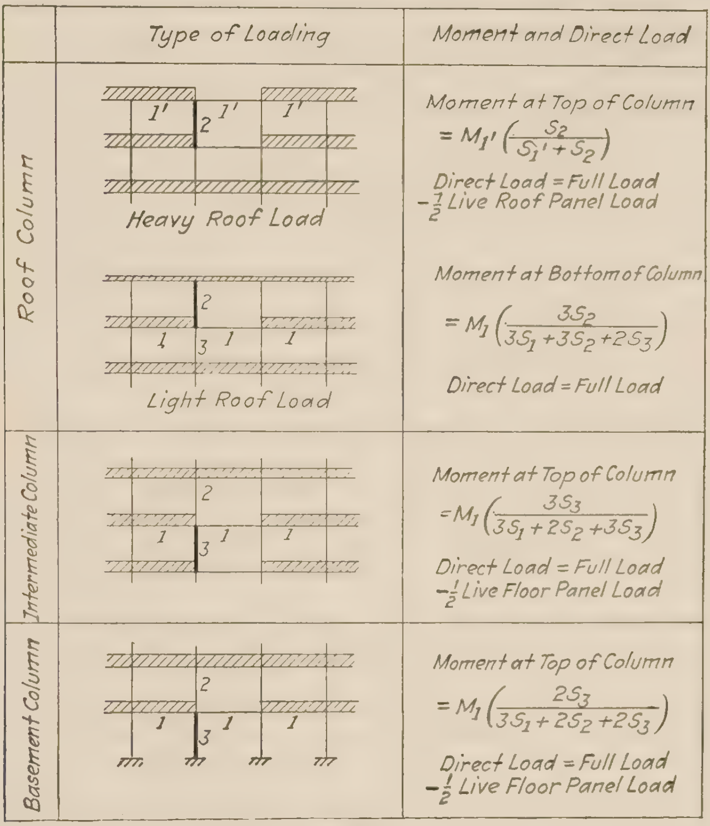


FIG. 47.

sustaining its full live load, the effect on the columns adjacent to the unloaded panel or panels is similar to the effect caused by the loading of these panels only. The type of loading indicated in Fig. 47 produces larger total stresses in the columns than any of the loadings of Fig. 45, since each column must sustain all or nearly all of its design dead and live load. The M_1 of equations

shown in the figure is the value of the live load moment in the girder as given in the table on page 207 or computed for some other type of symmetrical load.

The values of the moments in the columns as determined by the application of the equations of Fig. 47 depend upon the relative stiffnesses of the columns themselves and the relation of the stiffnesses of girders to those of the columns. The sections of the columns seldom change rapidly and are often constant for several tiers.

Figure 48 has been plotted for various ratios of the average stiffness of the columns to the stiffness of the girder. It is plotted for the type of loading shown in Fig. 45*c*, which produces the largest probable moment in an interior column.

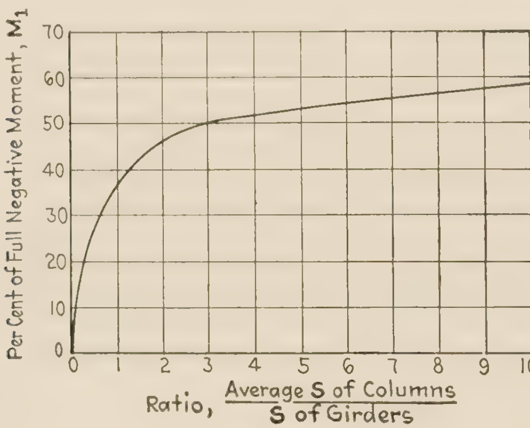


FIG. 48.

121. Exterior Columns. Both dead and live floor and roof loads are applied eccentrically to the exterior columns. A portion of the unbalanced moment thus produced may often be balanced by carrying the spandrel beams on the outside portion of the exterior columns. It is, however, very often impossible fully to compensate for the unbalanced moment due to the dead load alone by this type of construction. In any case, it is necessary to determine the amount of the moment in the column caused by the eccentricity of the roof and floor loads.

For the dead load, it is reasonable to assume that the further ends of the columns are fixed, and that the inner end of the girder

is in the same condition of restraint as the outer end. With these assumptions, the moment in the top of the lower column is (see Fig. 49)

$$M_{AB}\left(\frac{4S_3}{2S_1 + 4S_2 + 4S_3}\right) \tag{82}$$

At the roof level, the moment in the top of the upper column is

$$M_{A'B'}\left(\frac{4S_2}{2S_{1'} + 4S_2}\right) \tag{83}$$

If the full live load is assumed over all portions of the structure, the live load moment in the column is determined in the same manner as the dead load moment, that is, from equation (82). A greater moment in the column is, however, developed when the loading is similar to that of Fig. 45c. With this type of loading, a point of inflection occurs at the center of the columns, and the moment at the top of the column is

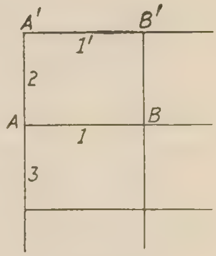
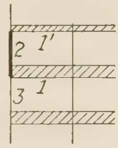
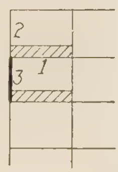
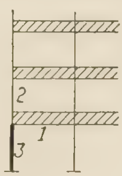


FIG. 49.

$$M_{AB}\left(\frac{6S_3}{2S_1 + 4S_2 + 6S_3}\right) \tag{84}$$

The stress caused by this moment, together with the corresponding direct load stress, is often the maximum stress occurring in a column near the roof. In a column of the lower stories, the decrease in the direct load with this type of loading so reduces the direct load stress that the maximum stress in the column usually occurs under full live load. In such cases equation (82) is applicable for both dead and live loads.

The critical cases are summarized in Fig. 50. The value of M_1 may be taken from the table on page 207 for the type of load sustained by the girder, the dead load being used in the determination of the moment in the column due to the dead load on the girder, and the live load for the corresponding live load determination. Figure 51 is a graphical representation of the two equations given for an intermediate exterior column in Fig. 50. The lower curve should in all cases be used to determine the dead load moment, while either may be used for the determination of the live load moment depending upon the type of loading assumed.

Type of Loading		Moment and Direct Load
Roof Column		Moment at Top of Column Live and Dead Loads $= M_{1'}\left(\frac{2S_2}{S_{1'}+2S_2}\right)$
Column Below Roof Column		Moment at Top of Column Dead Load $= M_1\left(\frac{2S_3}{S_1+2S_2+2S_3}\right)$ $= M_1\left(\frac{3S_3}{S_1+2S_2+3S_3}\right)$ Direct Live Load = Full Live Load $-\frac{1}{2}$ Live Roof Panel Load.[4]
Basement Column		Moment at Top of Column Live and Dead Loads $= M_1\left(\frac{2S_3}{S_1+2S_2+2S_3}\right)$

[4] In some instances this type of loading may produce the maximum stress in a column further from the roof. In such a case the decrease in the direct load will of course be greater.

Fig. 50.

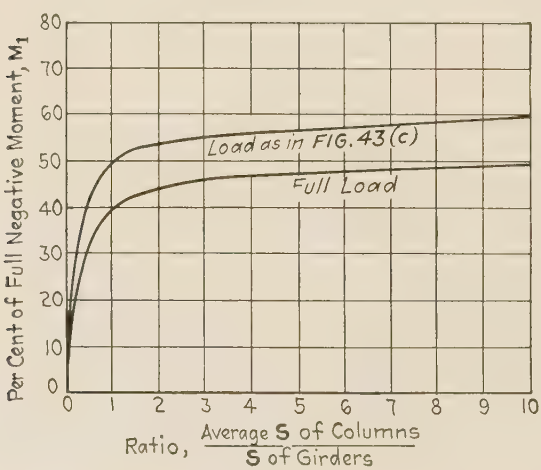


Fig. 51.

122. Moment of Inertia of Sections. The moment of inertia of a section subject to compressive stresses only has been used in Chaps. IV and V. It is obtained by making use of the so-called transformed section as explained in Chap. IV, and the whole area of the section is included. In beams and slabs, however, where both tensile and compressive stresses occur on the section, and since in regions of high tensile stress cracks often appear in the concrete, it becomes a question whether the tension zone of the concrete should be included in the calculations.

In the usual beam of a building frame, the effective section at the support is that of a rectangular beam reinforced for compression, while the section at mid-span is a T-beam. Some designers use the former and others the latter. Some omit the concrete in the tension zone while others include it. In some instances, a mean or average of the two sections with or without the concrete in tension is used. It is certainly true that the flange of a T-beam, although not effective at the support, does increase the stiffness of the member, and this should be given some consideration in the calculations. The stiffness of the member is the result desired, and in a beam of this type there is no actual section whose moment of inertia definitely determines the stiffness of the member for all conditions of stress and loading. It is recommended that the full concrete section (steel not included) at mid-span be used in the calculations for the moment of inertia. The value so obtained is usually somewhat greater than that computed for the doubly reinforced rectangular section at the support, and less than that for the section at the center with the steel included. For the columns, since all, or nearly all, of the steel and concrete sections are in compression, the moment of inertia of the entire steel and concrete sections should be used.

123. *Illustrative Problem.*

Figure 52 shows a section of the concrete frame of a two-story (and basement) building, for which it is required to design the columns. The distance center to center of columns in the direction perpendicular to the section is 23 ft.-0 in. The roof has been designed for a live load of 40 lb. per sq. ft. and an additional dead load of 35 lb. per sq. ft. to allow for cinder concrete surfacing to provide for drainage. The live load on the floors is 200 lb. per

sq. ft. and allowance has been made for 1 in. of surface finish. The floors and roof have been designed for a 2000-lb. concrete. The columns are to be designed for a 2500-lb. concrete.

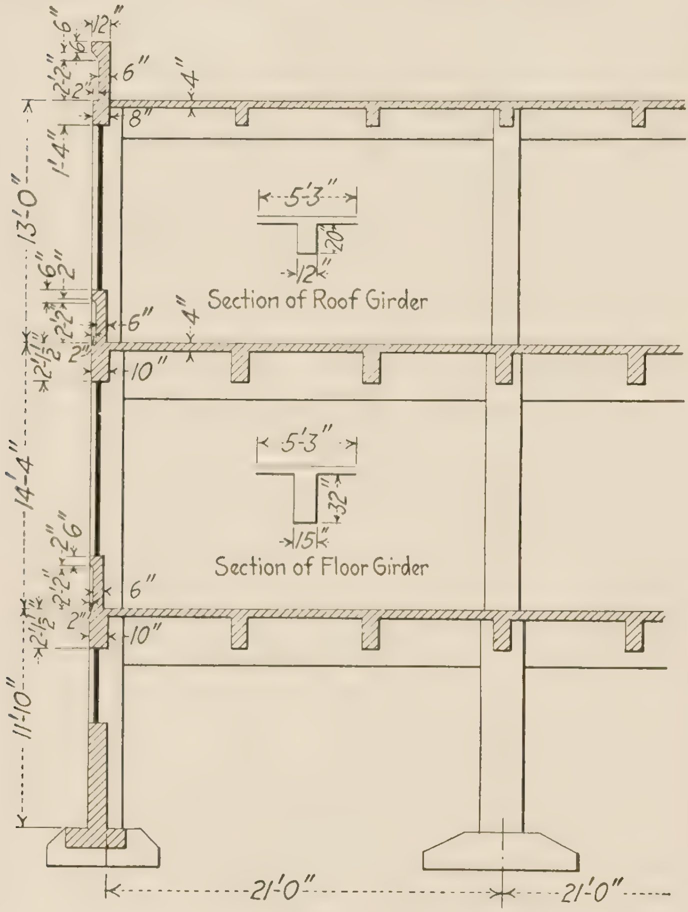


FIG. 52.

The reinforcement in the roof girder is six 1-in. round bars so placed in two rows that at the center $d = 21$ in. Then $p = .0036$, $k = .29$, and $kd = 6.1$ in. The moment of inertia of the roof girder is

$$I = \frac{1}{12} \times 63 \times 4^3 + 63 \times \overline{4.1}^2 + \frac{(\overline{2.1}^3 + \overline{17.9}^3)12}{3} = 27{,}900$$

and

$$S = \frac{27{,}900}{252} = 109$$

The reinforcement in the floor girder is eight 1-in. round bars, $d = 32\frac{1}{2}$ in., $p = .0031$, $k = .32$, $kd = 10.4$ in., $I = 86{,}300$, and $S = 342$.

The bending moments in the girders for which they have been designed both at the center and at the support are:

Roof Girder	
Dead load	1,042,000 in.-lb.
Live load	361,000 in.-lb.
Floor Girder	
Dead load	1,131,000 in.-lb.
Live load	1,803,000 in.-lb.

Interior Columns

The computations necessary for the design of the interior columns are given on page 222. The dead load supported by the upper interior column consists of the weight of the cinder concrete, the floor slab, the stem of three roof beams, and the stem of one girder. The dead load brought to the columns at each floor is obtained in a similar manner. The weights of the columns themselves are obtained with the aid of Table XI, the height being taken as the distance from the floor or top of footing to the bottom of the beam in the roof or floor above. The cross-sections of the columns required to sustain the direct load may be selected from Tables VI to X. Usually the effect of the flexure will require a slight increase in either the steel or concrete section. The areas of the effective concrete section used in determining p_o in column (5) may be taken from Table XI and the size and pitch of the spiral reinforcement selected from Table XIV. In the column supporting the first floor, $\frac{5}{16}$-in. spiral with a pitch of $2\frac{1}{2}$ in. could have been used, but the $\frac{3}{8}$-in. spiral with a pitch of 3 in. was selected for the sake of uniformity. The longitudinal steel in the roof column had to be increased from six $\frac{1}{2}$-in. round bars to eight $\frac{3}{4}$-in. round bars in order to provide for the stresses caused by flexure. Similarly the steel in the column supporting the second floor had to be increased from eight 1-in. round bars to

INTERIOR COLUMNS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Column supporting	Loads	Effective diameter, inches	A_s	p_o	I	S	Moment in column	$f_c = \frac{P}{A} + \frac{Mu}{I}$	Allowable $f_c = 1.2[300 + (.10 + 4p_o)]f_c$
Roof	Dead 53,200 Live 19,300 72,500 Col. 2,500 75,000	12	8-¾ in. ϕ ¼ in. spiral 2 in. pitch	$\frac{3.53}{113.1} = .0312$	3,217 587 3,804	$\frac{3,804}{140} = 27$	Top $361,000\left(\frac{27}{109 + 27}\right) = 72,000$ Bottom $1,803,000\left(\frac{81}{1026 + 81 + 192}\right) = 113,000$	$\frac{75,000}{113.1(1 + 11 \times .0312)} + \frac{113,000 \times 6}{1605} = 920$	1030
Second floor	75,000 Dead 55,500 Live 96,600 227,100 Col. 4,800 231,900	18	10-1 in. ϕ ⅜ in. spiral 3 in. pitch	$\frac{7.85}{254.5} = .0308$	11,499 3,114 14,613	$\frac{14,613}{146} = 100$	Top $1,803,000\left(\frac{300}{1026 + 54 + 300}\right) = 392,000$	$\frac{178,800}{254.5(1 + 11 \times .0308)} + \frac{392,000 \times 9}{8267} = 950$	1030
First floor	231,900 Dead 55,500 Live 96,600 384,000 Col. 6,200 390,200	24	12-1 in. ϕ ⅜ in. spiral 3 in. pitch	$\frac{9.43}{452.4} = .0208$	30,172 6,849 37,021	$\frac{37,021}{116} = 319$	Top $1,803,000\left(\frac{638}{1026 + 200 + 638}\right) = 618,000$	$\frac{335,700}{452.4(1 + 11 \times .0208)} + \frac{618,000 \times 12}{23,125} = 920$	910

ten 1-in. round bars. The values of I in column (6) are obtained from Tables XI and XII. This value is not the value to be used in the equation of column (9). The latter includes only the I_c of the concrete within the spiral. The values of the moments in column (8) are computed from the equations of Fig. 47.

Exterior Columns

The computations necessary for the design of the exterior columns are given on page 224.

The dead load supported by the roof column consists of one-half of the dead load supported by the interior column plus one-half the weight of the stem of one roof beam and the weight of the parapet wall. The dead load brought to the columns at each floor is obtained in a similar manner.

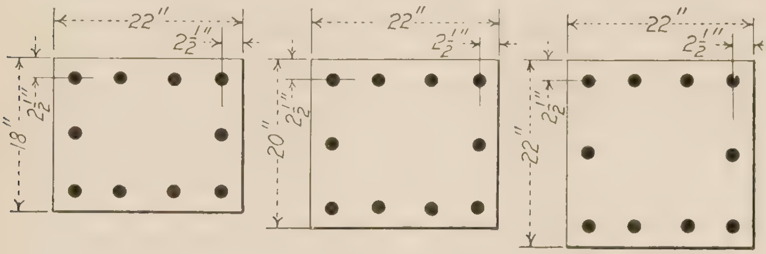


Fig. 53.

The width of the column parallel to the wall is made the same for all columns. Their lengths are the same as those of the corresponding interior columns. The moments of inertia in column (5) are computed with the aid of Tables XI and XIII.

The outside faces of the columns and beams are in the same vertical plane. This causes a compensating moment to be developed in each column. In the roof column this is due to the eccentricity of the beam only. The center line of the beam is $\frac{18 - 8}{2} = 5$ in. from the axis of the column. The load brought to the column by the two wall beams framing into the column is 19,300 lb., and the moment caused by the eccentricity is 97,000 in.-lb. This moment acts in the opposite direction from that brought to the column by the roof girder, and the moment

EXTERIOR COLUMNS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Column supporting	Loads	Cross-section, inches	A_s	$I = I_c + (n-1)I_s$	S	Moment in column	$\frac{e}{a}$	$\frac{d'}{a}$	p_o	k	$\frac{M}{ba^2f_c}$	f_c
Roof	Floor Dead 27,800 Live 9,700 Wall 6,900 44,400 Col. 4,800 49,200	18 × 22	10-1 in. φ Ties 2-5/16 in. φ	$22 \times 486 + 8 \times 11 \times 33 =$ 13,596	$\frac{13,596}{40} =$ 97	From girder $1,403,000\left(\frac{194}{109 + 194}\right) =$ 898,000 From eccentricity $19,300 \times \frac{18 - 8}{2} =$ 97,000 Total 801,000	$\frac{801,000}{44,400 \times 18} = 1.00$	$\frac{2\frac{1}{2}}{18} =$ .14	$\frac{7.85}{18 \times 22} =$ .0198	.41	.150	750
Second floor	49,200 Floor Dead 30,400 Live 48,300 Wall 9,200 137,100 Col. 5,600 142,700	20 × 22	10-1 in. φ Ties 2-5/16 in. φ	$22 \times 667 + 8 \times 11 \times 44 =$ 18,546	$\frac{18,546}{146} =$ 127	From girder Dead $1,131,000\left(\frac{254}{342 + 194 + 254}\right) =$ 364,000 Live $1,803,000\left(\frac{381}{342 + 194 + 381}\right) =$ 749,000 From eccentricity 226,000 Total 887,000	$\frac{887,000}{(137,100 - 9700)20} =$ .35	$\frac{2\frac{1}{2}}{20} =$ .12	$\frac{7.85}{20 \times 22} =$ .0178	.67	.137	740
First floor	142,700 Floor Dead 30,400 Live 48,300 Wall 9,200 230,600 4,700 235,300	22 × 22	10-1 in. φ Ties 2-5/16 in. φ	$22 \times 887 + 8 \times 11 \times 57 =$ 24,530	$\frac{24,530}{116} =$ 211	From girder $2,934,000\left(\frac{422}{342 + 254 + 422}\right) =$ 1,216,000 From eccentricity 355,000 Total 861,000	$\frac{861,000}{230,600 \times 22} = .170$	$\frac{2\frac{1}{2}}{22} =$ .11	$\frac{7.85}{22 \times 22} =$ .0162	K 1.62		770

for which the column must be investigated is their difference. In the lower columns, in addition to the moment produced by the eccentricity of the load on the beam (35,400 lb.), there is an additional moment on account of the eccentricity of the columns with respect to each other, and the sum of these two moments must be subtracted from the moment brought to the column by the girder. The latter are obtained by applying the equations of Fig. 50. The remainder of the solution, as given in columns (8) to (13) is completed with the aid of the diagrams of Chap. IV. The cross-sections of the columns are shown in Fig. 53.

CHAPTER VII

FOUNDATIONS

124. Definitions and Essential Requirements. The foundation of a structure has been defined as that part of it which is usually placed below the surface of the ground, and which distributes the load upon the earth beneath it. Two essential requirements in the design of foundations are that the settlement of the structure shall be as small as possible, and that settlement, if any, shall be uniform throughout the structure. The first requirement may be provided for by distributing the load over an area large enough so that the safe bearing power of the soil will not be exceeded. Uniform settlement may be secured by designing the foundations so that the soil pressure over the entire base of the structure is uniform. Failure to provide for the equalizing of the unit foundation pressures is the principal cause of the cracks which disfigure so many buildings. A large number of buildings have been designed with massive bearing piers carrying nearly the whole weight from the floors. Between these piers, smaller piers or columns carrying very little load have been placed. Often the area of the base of all foundations has been the same. The result has been a settlement of the heavier piers, a shearing of window caps and lintels, and many unsightly cracks. From this cause the Cooper Institute Building in New York became so dangerous as to require radical and expensive repairs. Since it is essentially the dead load that causes the greatest amount of settlement, footings should be proportioned for equal unit pressures under dead load, or in some cases, dead load plus partial live load.

In a reinforced concrete building the required bearing area is furnished by widening the base of the columns or wall. The widened portion is called the footing. Building footings may be divided into three main classes: (1) wall footings, (2) single column footings, (3) multiple column footings. Since the stresses

in footings are mainly compressive, concrete, either plain or reinforced, is particularly adapted to such use.

125. Bearing Capacity of Soils. The sustaining power of earth depends mainly upon the composition, the amount of moisture contained, and the degree of confinement in the mass. Sand, if securely confined, or artificially protected against the possibility of lateral displacement, can sustain heavy loads with negligible compression. The supporting power of clays is extremely variable. Certain deposits are known to be compact and hard, and have a high supporting power, while others are plastic and easily compressed. The chief characteristics which render clay more or less unstable as a foundation material are its property of retaining water which is once admitted, and its tendency to soften gradually as the amount of water increases. The depth of foundation is an important factor in determining the allowable pressure on a clay bed; the greater the depth the less likelihood of lateral displacement of the clay, and a more nearly constant moisture content. When clay is mixed with other materials, such as coarse sand or gravel, its supporting power is materially increased, being greater in proportion as the other materials are in excess, up to the point of forming a cemented mass in which the clay is just sufficient in quantity to act as a cement in binding the other materials together. The allowable pressure on solid rock is usually governed by the strength of the masonry rather than by that of the rock itself.

No definite values can be given to the safe bearing capacity of different classes of soils because of the many variables which of necessity are considered. Unless the bearing capacity of the material at a given site is already known, it should be determined by direct tests, if this is at all feasible.[1] In the absence of any

[1] The factor of safety to be allowed in determining the safe bearing power of the soil will vary from about 1–5, depending upon the superstructure and the character of the load coming to the foundation. The blue clay that underlies the State Capitol Building at Albany, N. Y., was found by careful and elaborate tests to sustain a load of 6 tons per sq. ft. It was decided to adopt 2 tons as the safe load to be used in the design of the foundations. The foundations of the Congressional Library at Washington, D. C., were designed to limit the actual soil pressure to 2½ tons per sq. ft., although the yellow clay supporting them was found capable of carrying a total load of 13½ tons per sq. ft.

satisfactory test, the following limiting values taken from the Building Code of the National Board of Fire Underwriters may be used as a guide in selecting the bearing capacity of any given foundation bed.

Soft clay, 1 ton per sq. ft.

Clay and sand together, wet and springy, 2 tons per sq. ft.

Loam clay and fine sand, firm and dry, 3 tons per sq. ft.

Very firm coarse sand, stiff gravel, or hard clay, 4 tons per sq. ft.

126. Factors Affecting the Design of Concrete Footings. In the ordinary type of footing the load from the wall or column is transmitted vertically through the wall or column and supported by the upward pressure of the soil. Uniformity of this upward pressure is assumed in design, and is essential to a theoretically satisfactory footing. The probability of obtaining such distribution depends among other things upon the material in the foundation bed. An elastic compressible soil of considerable thickness will permit the footing to assume the shape essential to a uniform distribution of the load. A rock bed will not permit this bending, and as a result, the projecting portions of the footing may resist but a small part of the load.[2]

[2] Jacoby and Davis, in "Foundations of Bridges and Buildings," discuss the distribution of the pressure on the base of the footing as follows: "There is some question regarding the error involved in the assumption that the pressure from the footing is uniformly distributed on the ground. Taking the case of the single column square footing, it is evident that the base of the footing will assume a saucer-like shape, and as a consequence the pressure will be a maximum at the center and a minimum at the outside. The law governing the variation in pressure will depend on the relative deflections of different points on the base of the footing, as well as on the modulus of compressibility of the soil and the thickness of the compressible stratum. Where the modulus is low and the thickness considerable, the slight difference in total deformation at different points will cause but a slight difference in pressure. Where the soil is compressible but inelastic, or soft and subject to lateral flow, a fairly uniform distribution of pressure quickly obtains. Where the material has a high modulus of compressibility, as in shale or rock, the footing should be designed for stiffness as well as for strength or else the surface of the material should be shaped to fit the curve taken by the base of the footing when fully loaded, otherwise the pressure will be unevenly distributed."

Assuming uniform pressure distribution as an actuality, the manner in which this pressure is resisted is a more or less uncertain factor, especially in the column footings, where bowl-shaped deformation occurs. In order to obtain information which would permit a rational treatment of the problem, a series of tests was made at the Engineering Experiment Station of the University of Illinois, the results of which were published in *Bulletin* 67 of the Experiment Station, by Arthur N. Talbot. The conclusions and recommendations contained in this bulletin form the basis of the following discussion of footing design.

127. Plain Concrete Footings. The area of the base of footing may be found by dividing the total load on the footing, including its own weight, by the allowable soil pressure. The top area must be large enough to provide for a proper distribution of the load from the wall or column to the footing. Where a great difference between the areas of the top and base exists, the upper surface of the footing is usually either sloped or stepped.

The depth of the footing must be sufficient to keep the tension in the concrete within the allowable value. In a stepped footing the relation of the depth of step to the projection of each step should be such that the tensile strength of the concrete is not exceeded at any point. In determining the depth required, the amount of moment should be computed by treating the projection as a cantilever with uniform upward soil pressure. The critical section is at the face of the column. The width of footing to be used in the computation of fiber stress is variable. In the tests at the University of Illinois the full width of the footing was used in calculating the modulus of rupture, instead of taking into account the variation in stress across the section. The values of moduli of rupture thus found were smaller (averaging about one-third less) than those of the control beams of the same concrete. The method of calculating the bending moment was the same as that used in the reinforced footings. *Bulletin* 67 concludes: "as is usually the case when plain concrete is used in flexure, the unreinforced footings show considerable variation in results. The variations are such as not to permit a method of determining the effective width of resisting section to be established, or to obtain a formula for resisting moment."

128. Reinforced Concrete Footings. In the majority of cases reinforced concrete is preferable to plain concrete for footing construction due to the saving in excavation, in material, and in weight of the foundation itself. This is the result of the smaller depth required to provide for the existing flexural stresses.

129. Analysis of Wall Footings.[3] The principles of beam action are, in general, applicable to wall footings. Figure 54 shows a wall footing and a typical set of external forces acting upon it. Although it is evident that the maximum bending moment occurs at a section which passes through the middle of the wall, an analysis of the resisting moments justifies the assumption that the critical section for moment will occur at the face of the wall. The results of the tests bear out this assumption, and measurements of deformation in the reinforcement indicate that the calculated tensile stress in the bars at this section is probably somewhat higher than the maximum tensile stress developed. The maximum bending moment will then be given by the equation $M = \frac{1}{8}w(l - a)$.[2]

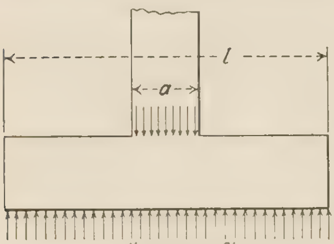


Fig. 54.

The calculation for bond stress in the tested footings, based on the total external vertical shear at the section at the face of the wall, and calculated as for ordinary beams, evidently gives stresses higher than actually occur. An analysis of the conditions existing at this section tends to confirm this statement. As bond resistance, however, is so important an element of strength in a short cantilever beam, this method of calculation and the use of the working value of bond stress ordinarily assumed in design seem only reasonably conservative, and may be recommended for general practice. Anchorage of bars by bending upward and back in a long curve, or by looping in a horizontal plane, was found to add materially to bond resistance. The

[3] The analysis of wall footings is substantially from *Bulletin* 67, Engineering Experiment Station, University of Illinois.

allowable bond stresses as specified by the Joint Committee are given in Art. 133.

The tests indicate that the vertical shearing stresses developed at the face of the wall, calculated by the usual method, are higher than the vertical shearing stresses which are found to exist in simple beams with concentrated loading when diagonal tension failures are developed. It was found that these start at a point some distance away from the section at the face of the wall. This observation, and certain analytical considerations such as the probable greater proportion of shear taken in the compressive area at sections near the face of the wall, show that in calculating the vertical shearing stress which shall be used as a basis for judging the resistance to diagonal tension, a section some distance from the face of the wall should be used. The tests and the discussion indicate that a section d distant from the face of the wall (d being the distance from the center of reinforcing bars to the top of footing) may properly be used as the critical section for calculating the vertical shearing stress for this purpose, and that at this section the ordinarily accepted working stress may be used for calculating resistance to diagonal tension failure. Web reinforcement, while adding to diagonal tension resistance, is not especially effective, and since it is not very convenient to place, it is usually better to design the footing so that the vertical shearing stress is within the limit of the working stress permitted in beams without web reinforcement.

130. Design of a Typical Wall Footing. A 16-in. wall supports a total load of 23,100 lb. per lin. ft., and rests on soil whose safe bearing power is 2 tons per sq. ft. Design a footing for this wall that will satisfy the requirements of the Joint Committee. A 2000-lb. concrete is to be used.

Assuming the weight of footing as 900 lb. per lin. ft., the total width of footing required $= \frac{24,000}{4,000} = 6.0$ ft. The maximum moment is

$$M = 3850 \times \frac{(6 - 1.33)^2}{8} \times 12 = 126,000 \text{ in.-lb.}$$

With allowable unit stresses of 16,000 and 800, Table IV gives $K = 146.7$ and $j = .857$

$$d = \sqrt{\frac{126{,}000}{146.7 \times 12}} = 8.5 \text{ in.}$$

An effective depth of 9 in. is used, which, with 3-in. insulation, gives a total thickness of 12 in. The weight per linear foot is then 900 lb. as assumed.

$$A_s = \frac{126{,}000}{16{,}000 \times .857 \times 9} \doteq 1.02 \text{ sq. in. per ft.}$$

Assuming deformed bars anchored at both ends by means of hooks with a diameter of 6 in., and increasing the allowable unit bond stress of $.05f'_c$ by 50 per cent,[4]

$$\Sigma_0 = \frac{2\ 33 \times 3850}{150 \times .857 \times 9} = 7.7 \text{ in. per ft.}$$

These requirements are satisfied by using ½-in. square bars, 3 in. center to center. Investigating for diagonal tension, the total external shear on a section 9 in. from the face of the wall is

$$\frac{23{,}100}{6} \times \frac{18}{12} = 5775 \text{ lb.}$$

and the unit shear, which is a measure of diagonal tension, is

$$v = \frac{5775}{12 \times .857 \times 9} = 62 \text{ lb. per sq. in.}$$

This is but 2 lb. in excess of the allowable ($.03 \times 2000 = 60$) and the design may be considered satisfactory.

131. Single Column Footings. *Flexure Analysis, Footings Reinforced in Two Directions.* Ordinarily in single column footings, the load may be considered as applied uniformly over the bearing area of the column, and the upward pressure as uniformly distributed over the base of the footing. The footing is then analogous to a cantilever slab supported at the top over a central area and loaded with a uniform upward load. As the projecting portion of the footing deflects upward, its surface assumes the shape of a bowl.

In calculating the strength of column footings of ordinary dimensions, a study of the results of tests justifies the assumption that the critical section for the bending moment for one direction

[4] This increase of 50 per cent is not a part of the Joint Committee's recommendations. See Arts. 80 and 133.

occurs on a vertical section passing through the face of the column. A common method used in design is to consider that the total upward pressure on a trapezoid, the parallel sides of which are the face of the column and the adjacent edge of the footing, acts at its center of gravity. The bending moment, the product of this amount of load and the distance of the assumed center of pressure from the critical section, is then considered to be resisted by a beam *ABCD* (Fig. 55) of a width somewhat greater than the width of the column. This infers that only the steel within the assumed width of beam is effective in resisting the bending moment determined as above. Steel is placed at a more or less arbitrary spacing outside of the assumed effective width to carry the load on that portion to the beam at right angles to *ABCD*.

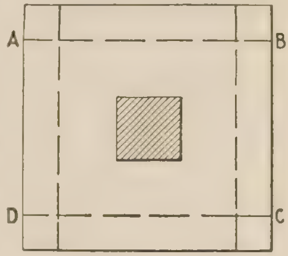


Fig. 55.

This method of computing the bending moment takes no cognizance of any variation in the distribution of the upward load along the various portions of the footing projection. It is evident that the proportion of the upward pressure which is taken by each beam from any elementary area is dependent upon the relative deflection of the beams in the two directions. The total load acting on any element on the rectangle between the face of the pier and the edge of the footing may rightly be considered to act on the beam perpendicular to that face. The load on any element in the corner squares is distributed between the two beams in proportion to the deflection of the beams at that point. A rational analysis of the problem involving some approximations, and a study of the flexure curves obtained on a number of column footings tested, have led to the conclusion that "all of the upward load on the rectangle lying between the face of the pier and the edge of the footing is considered to act at a point half-way out from the pier, and half of the upward load on the two corner squares is considered to act at a center of pressure located at a point 0.6 of the width of the projection from the given section." This conclusion applies only to square footings supporting square columns. The principles involved, how-

ever, may, with necessary modifications, be extended to other forms.[5]

The width of footing to be used in flexure computations, that is, the width of beam *ABCD* (Fig. 55), depends upon the size of the column, the thickness of the footing, the dimension of the projecting portion, and the amount and distribution of the reinforcement. Talbot states that "a study of the observations and results of tests of the footings made in the laboratory indicates that the bars for some distance on either side of the pier

[5] The Joint Committee specifies that the critical section for bending shall be taken at the face of the column or pedestal. Where steel or cast iron column bases are used, the moment in the footing shall be computed at the edge of the base and at the center. In calculating this moment the column or pedestal load shall be uniformly distributed over its base. For a square footing supporting a concentric square column the bending moment at the critical section is that produced by the upward pressure on the trapezoid bounded by one face of the column, the corresponding outside edge of the footing, and the portions of the two diagonals. The center of application of the reaction on the two corner triangles of this trapezoid shall be taken at a distance from the face of the column equal to 0.6 of the projection of the footing. The center of application of the reaction on the rectangular portion of the trapezoid shall be taken at its center of gravity. This gives a bending moment expressed by the formula:

$$M = \frac{w}{2}(a + 1.2c)c^2$$

M = bending moment at critical section of footing in foot-pounds.
a = width of face of column or pedestal in feet.
c = projection of footing from face of column in feet.
w = upward reaction per unit of area of base of footing in pounds per square foot.

Square footings supporting a round or octagonal column shall be treated in the same manner as for a square column, using for the distance a the side of a square having an area equal to the area enclosed within the perimeter of the column.

Rectangular or irregularly shaped footings shall be calculated by dividing them into rectangles or trapezoids tributary to the sides of the column, using the distance to the actual center of gravity of the area as the moment arm of the upward force.

The reinforcement necessary to resist the bending moment in each direction in the footing shall be determined as for a reinforced concrete beam; the effective depth of the footing shall be the depth from the top to the plane of the reinforcement.

have nearly the same stress as those under the pier." As a working basis applicable when the spacing of the bars is uniform or nearly so, the conclusion was reached that the resisting moment of the footing in each of the two directions may be based upon the amount of steel in a width of beam equal to the width of pier plus twice the depth of footing to the reinforcement, plus one-half the remainder of the width of footing. The use of this amount of steel will determine the maximum steel stress. If this width is greater than the width of the footing, then the width of the beam may be taken as the full width of the footing.[6]

The extreme fiber stress in compression in the concrete shall be kept within the limits specified for ordinary beams. No failures by compression have been observed in the tests and none would be expected with the low percentages of steel used.

132. Single Column Footings. *Flexure Analysis, Footings with Four-way Reinforcement.* In footings with reinforcement in the direction of the diagonals in addition to that parallel to the sides, the usual method of computation is to assume that the bending moment on each trapezoid, computed as for a two-way footing, is resisted by the bars in two bands, one band normal to the parallel faces of the trapezoid and a half band along each of the other sides. The steel is placed so that approximately all of it passes under the column. The only other requisite is that no portion of the area of base of the footing be unreinforced (see Fig. 57).

133. Bond Stresses. Bond resistance is one of the most important features of strength of column footings, and probably much more important than has been appreciated by the

[6] The Joint Committee specifies that the required area of reinforcement shall be spaced uniformly across the footing, unless the width of the footing is greater than the side of the column or pedestal plus twice the effective depth of the footing, in which case the width over which the reinforcement is spread may be increased to include one-half the remaining width of the footing. In order that no considerable area of the footing shall remain unreinforced, additional bars shall be placed outside of the width specified, but such bars shall not be considered as effective in resisting the calculated bending moment. For the extra bars a spacing double that used for the reinforcement within the effective belt may be used.

average designer. The calculations of bond stress in footings of ordinary dimensions where large reinforcing bars are used show that the bond stress may be the governing element of strength.

The method proposed for calculating maximum bond stress in column footings having two-way reinforcement evenly spaced, or approximately so, is to use the ordinary formula for bond stress, and to consider the circumference of all the bars which were used in the calculation of tensile stress, and to take for the external shear that amount of upward pressure or load which was used in computing the bending moment at the given section. The use of small bars, closely spaced, is often essential to provide the necessary bond resistance, especially in the two-way footings. The use of short bars placed with their ends staggered increases the tendency to fail by bond, and cannot be considered as acceptable practice in footings of ordinary proportions.[7]

134. Diagonal Tension. Tests indicate that diagonal tension develops in a critical way at a distance from the edge of the column equal to the depth of the footing. In the tests at the University of Illinois, the vertical shearing stress calculated at the vertical sections formed upon the square which lies at a distance from the face of the pier equal to the depth of the footing was used as a means of measuring resistance to diagonal tension failure. This calculation gives values for the shearing stress for the footings which failed by diagonal tension, which agree fairly closely with the values obtained in tests of simple beams. The formula used in this calculation is $v = \frac{V}{bjd}$, where V is the total vertical shear at this section and is equal to the upward pressure on the area of the footing outside of the section considered, b is the total distance around the four sides of the section, and

[7] The Joint Committee specifies that the bond stress on a section of footing shall be computed as for ordinary beams. Only the bars counted as effective in bending shall be considered in computing the number of bars crossing a section. The bond stress computed in this manner on sections at the face of the column or outside of the column shall not exceed $.04f'_c$ for plain bars, nor $.05f'_c$ for deformed bars; for footings where reinforcement is required in more than one direction these values of the permissible bond stress shall be reduced 25 per cent. The bond stresses for bars adequately anchored at both ends may be increased in accordance with the recommendations of Art. 80.

jd is the distance from the center of the reinforcing bars to the centroid of the compressive area.[8]

135. Punching Shear. The tendency of the column to punch its way through the footing is overcome by the shearing resistance of the concrete. This shearing resistance is measured by the shearing strength of the concrete on an area equal to the effective depth multiplied by the perimeter of the column. The amount of shear to be resisted is equal to the net upward soil pressure on the portion of the footing outside of a vertical plane surrounding the column. The value of the allowable unit stress generally recommended for punching shear is $.06f'_c$. The depth of footing is usually governed by this requirement.

136. Stepped and Sloped Footings. According to the Joint Committee, footings in which the depth has been determined by the requirements for shear may be sloped between the critical section and the edge of the footing, provided the shear on no section outside the critical section exceeds the value specified, and provided further that the thickness of the footing above the reinforcement at the edge shall not be less than 6 in. for footings on soil nor less than 12 in. for footings on piles. The top of the footing may be stepped instead of sloped, provided the steps are so placed that the footing will have at all sections a depth at least as great as that required for a sloping top. Stepped footings shall be cast monolithic. The extreme fiber stress in such footings shall be based on the exact shape of the section for a width not greater than that assumed effective for reinforcement. The requirements for the top area of the footing are given in Art. 137.

Talbot calls attention to the fact that: "In stepped footings, the abrupt change in the value of the arm of the resisting moment

[8] The Joint Committee specifies that the shearing stress shall be computed as for rectangular beams. When so computed, the stress on the critical section defined below, or on sections outside of the critical section, shall not exceed $.02f'_c$ for footings with straight reinforcing bars, nor $.03f'_c$ for footings in which the reinforcing bars are adequately anchored at both ends. The critical section for diagonal tension in footings bearing directly on the soil shall be taken on a vertical section through the perimeter of the lower base of the frustum of a cone or pyramid which has a base angle of 45 degrees and has for its top the base of the column or pedestal and for its lower base the plane of the center of longitudinal reinforcement.

at the point where the depth of footing changes may be expected to produce a correspondingly abrupt increase of stress in the reinforcing bars. Where a step is large in comparison to the projection, the bond stress must become abnormally large. It is evident that the distribution of bond stress is quite different from that in footings of uniform thickness. The sloped footing also gives a distribution of stress which is different from that in a footing of uniform thickness. For footings of uniform thickness, however, the bond stress is a maximum at the section at the face of the pier; in a sloped footing the bond stress at the section at the face of the pier would be less accordingly than in a footing of uniform thickness, and a moderate slope may be found to distribute the bond stress more uniformly throughout the length of the bar. This is not of advantage if the full embedment of the bar is effective in resisting any pull due to bond."

137. Bearing of Column on Footing. The upper surface of sloped or stepped footings should project at least from 4 to 6 in. beyond each face of the column supported by the footing. The compressive stress in the longitudinal reinforcement at the base of the column is transferred to the pedestal or footing either by dowels or distributing bases. When dowels are used, there should be at least one dowel for each column bar, and the total sectional area of the dowels should not be less than the sectional area of the longitudinal reinforcement in the column. The dowels should extend into the column and into the pedestal or footing not less than 50 diameters of the dowel bars for plain bars, or 40 diameters for deformed bars.[9]

[9] When metal distributing bases are used, the Joint Committee requires that "the area and thickness of these bases shall be sufficient to transmit safely the load from the longitudinal reinforcement in compression and bending. The permissible compressive unit stress on top of the pedestal or footing directly under the column shall be not greater than that determined by the formula

$$r_a = .25f'_c\sqrt[3]{\frac{A}{A'}},$$

where r_a = permissible working stress over the loaded area;

A = total area at the top of the pedestal or footing;

A' = loaded area at the column base.

"In sloped or stepped footings, A may be taken as the area of the top horizontal surface of the footing or as the area of the lower base of the

138. Design of a Two-way Block Footing. A column 24 in. square supports a total load of 400,000 lb. Design a single slab concrete footing reinforced in two directions to support this column on a soil, the safe bearing capacity of which is 5000 lb. per sq. ft. Follow the specifications of the Joint Committee. Assume a 2000-lb. concrete.

Assuming the weight of footing as 40,000 lb., the bearing area required equals $\frac{440{,}000}{5000}$, or 88 sq. ft. A base 9 ft.-6 in. square, furnishing 90.25 sq. ft., is selected. The unit soil pressure due to the load on the column equals $\frac{400{,}000}{90.25}$, or 4440 lb. For punching shear at the edge of the column,

$$d = \frac{(90.25 - 4) \times 4440}{4 \times 24 \times 120} = 33.0 \text{ in.}$$

With 3 in. insulation, the total thickness = 36 in., and the weight of footing = 40,500 lb. (see Fig. 56).

The total net upward pressure outside of a square concentric with the column, and each of whose sides is 33 in. distant from the corresponding face of the column equals $4440[(9.5)^2 - \left(\frac{90.0}{12}\right)^2] = 150{,}000$ lb.

$$v = \frac{150{,}000}{4 \times 90.0 \times \frac{7}{8} \times 33} = 15 \text{ lb. per sq. in.}$$

This is satisfactory.

The effective width of footing is

$$24 + 2 \times 33 + \frac{2 \times 12}{2} = 102 \text{ in.}$$

The moment at the edge of the column is

$$M = 4440\left[2 \times \frac{\overline{3.75}^2}{2} + .6(3.75)^3\right] \times 12 = 2{,}445{,}000 \text{ in.-lb.}$$

---

largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base the loaded area A', and having side slopes of 1 vertical to 2 horizontal.

"The allowable compressive unit stress on the gross area of a concentrically loaded pedestal or on the minimum area of a pedestal footing shall not exceed $.25f'_c$, unless reinforcement is provided and the member designed as a reinforced concrete column."

Assuming $j = .9$

$$A_s = \frac{2,445,000}{16,000 \times .9 \times 33} = 5.15 \text{ sq. in.}$$

Twenty-six ½-in. rounds furnish 5.13 sq. in.

$$p = \frac{5.13}{102 \times 33} = .0015$$

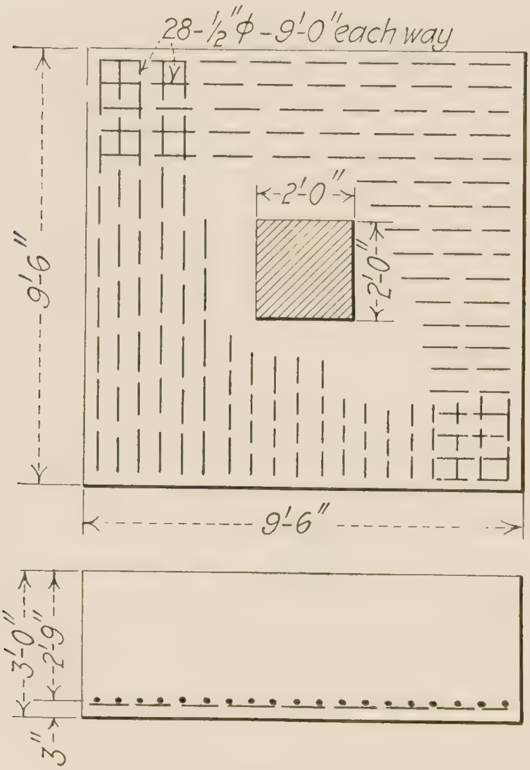


Fig. 56.

Table IV shows that the concrete fiber stress is considerably below the allowable. Table V gives $j = .938$ and the revised steel area is

$$A_s = \frac{2,445,000}{16,000 \times .938 \times 33} = 4.95 \text{ sq. in.}$$

This result still requires twenty-six ½-in. rounds.

One additional bar is placed on each side of the footing outside of the effective width to assist in distributing the load from

that portion. The bond stress on the effective reinforcement equals

$$u = \frac{\frac{1}{4} \times 4440(90.25 - 4)}{26 \times 1.57 \times .938 \times 33} = 76 \text{ lb. per sq. in.}$$

This is allowable with deformed bars. Larger bars, with a consequent increase in bond stress could have been used if special anchorage were provided in accordance with Art. 80.

139. Design of a Four-way Sloped Footing. Design a sloped footing with four-way reinforcement according to the Joint Committee, to support a 23-in. square column, the total load on which is 400,000 lb. The safe bearing capacity of the soil is 4000 lb. per sq. ft. Use a 2000-lb. concrete.

Assuming the weight of footing as 37,000 lb., the bearing area required equals $\frac{437,000}{4000}$, or 109.25 sq. ft. A base 10 ft.-6 in. square furnishes a bearing area of 110.25 sq. ft. and is selected.

The net upward pressure from the soil is $\frac{400,000}{110.25}$ or 3630 lb. per sq. ft. The total amount of punching shear at the edge of the column is

$$3630[110.25 - (23/12)^2] = 387,000 \text{ lb.}$$

$$d = \frac{387,000}{4 \times 23 \times 120} = 35.1 \text{ in.}$$

Using an effective depth of 36 in. and allowing 4-in. insulation from the center of gravity of the steel, the total height of footing is 40 in. The top of the footing is made 32 in. square, and the total thickness at the edge 12 in. The actual weight of footing is 37,000 lb. as assumed (see Fig. 57).

An investigation for diagonal tension along a vertical plane 36 in. from the face of the column shows that, with an effective depth at that plane equal to

$$36 - \frac{31.5 \times 28}{47} = 17.2 \text{ in.}$$

and an area outside of the plane equal to

$$110.25 - \left[\frac{(2 \times 36) + 23}{12}\right]^2 = 47.7 \text{ sq. ft.}$$

$$v = \frac{3630 \times 47.7}{4 \times 95 \times \frac{7}{8} \times 17.2} = 30.2 \text{ lb. per sq. in.}$$

This can be carried safely by the concrete.

The bending moment at the edge of the column is

$$M = 3630\left[\frac{23}{12} \times \left(\frac{51.5}{12}\right)^2 \times \frac{1}{2} + .6 \times \left(\frac{51.5}{12}\right)^3\right] \times 12 = 2{,}830{,}000 \text{ in.-lb.}$$

Since this is resisted by two bands as explained in Art. 132, the steel required in each band, assuming $j = .9$, is

$$A_s = \frac{1}{2} \times \frac{2{,}830{,}000}{16{,}000 \times .9 \times 36} = 2.74 \text{ sq. in.}$$

This is furnished by nine $\frac{5}{8}$-in. round rods. Further revision to correct the value of j does not affect the number of bars required.

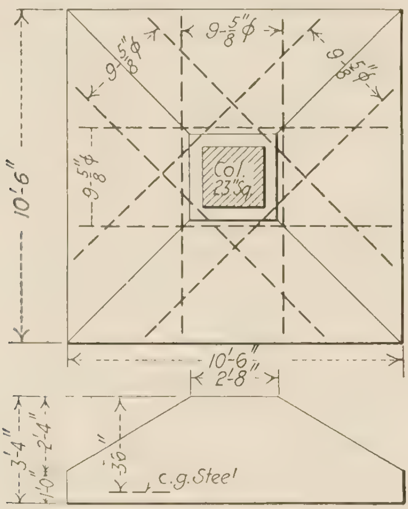


FIG. 57.

The bond stress at the face of the column is

$$u = \frac{\frac{1}{8} \times 387{,}000}{9 \times 1.964 \times .9 \times 36} = 84 \text{ lb. per sq. in.}$$

This is but slightly in excess of the allowable value for deformed bars without hooks, *i.e.*, $.75 \times 100 = 75$ lb. per sq. in. By hooking each bar at each end, adequate bond resistance is furnished.

See Art. 172 for the design of a square sloped footing supporting a round column.

140. Multiple Column Footings. In cases where the available ground area is limited, it may be impossible to construct simple spread footings to support the exterior columns. Such a condition obtains when the face of the exterior column coincides with, or is near, the edge of the building lot. The center of gravity of the column load could not be made to coincide with the center of gravity of the concrete base if a single footing were used. This would result in unequal distribution of the soil pressure with the possibility of uneven settlement and bending in the columns. To avoid this, the wall column footing may be connected to the nearest interior column footing by means of a concrete strap, or a single footing may be designed to support both columns. The latter type of construction is known as the combined footing, and the former as the cantilever footing. Multiple column footings may support more than two columns in case such procedure is deemed advisable or necessary.

141. Combined Column Footings. The simplest type of a combined footing is a slab of uniform thickness whose center of gravity coincides with the center of gravity of the loads that the footing sustains. If both columns are placed near the opposite edges of the footing, the slab must have a trapezoidal shape, assuming the column loads are unequal, in order to fulfil the requirement of uniform pressure distribution. If a projection of sufficient length is possible beyond the heavier load, a rectangular shape may be used, the length of the projection being sufficient to cause the centers of gravity of the downward loads and the upward pressure to coincide. In some types of combined footings, an inverted T-section is used for the slab.

The following design illustrates the manner of determining the shape of the trapezoidal slab combined footing.

Two columns 12 ft.-0 in. center to center, one having a cross-section of 18×18 in. and the other a cross-section of 24×24 in., sustain loads of 200,000 and 300,000 lb., respectively. The allowable pressure on the soil is 5000 lb. per sq. ft. Design a combined footing to support these columns, the unit stresses not to exceed the following: $f_c = 650$, $f_s = 16{,}000$, $u = 100$, $v = 120$. In proportioning stirrups, use the first method outlined in Art. 74. $n = 15$.

Assuming the weight of footing as 400 lb. per sq. ft., the bearing area required for the two loads is

$$\frac{500{,}000}{5000 - 400} = 108.7 \text{ sq. ft.}$$

Allowing the footing to project 6 in. beyond the edge of the larger column, the total length of footing is 14.25 ft. In order to secure uniform pressure on the soil, the center of gravity of the footing must be at a distance of

$$\frac{200{,}000}{500{,}000} \times 12 + 1.5 = 6.30 \text{ ft. from } b_1 \text{ (Fig. 58)}$$

Using the equations involving the area and center of gravity of a trapezoid,

$$b_1 + b_2 = \frac{2 \times 108.7}{14.25} = 15.26$$

$$b_1 + 2b_2 = \frac{3 \times 15.26 \times 6.30}{14.25} = 20.24$$

from which equations $b_1 = 10.28$ ft. and $b_2 = 4.98$ ft.

The maximum moment occurs at the point of zero shear. Let the distance of this point from b_1 be called y, the load on the larger column P, and the net upward soil pressure (the difference between the allowable soil pressure and the weight of the footing) w. Equating the upward pressure and the downward load,

$$P = w\left[b_1 y - \frac{(b_1 - b_2)y^2}{2l}\right]$$

from which the distance $y = 7.33$ ft. The width of the footing at this point, b_4, is 7.55 ft., and the distance of the center of gravity of the trapezoid bounded by b_1 and b_4, from b_4, is 3.85 ft. The maximum moment is found by concentrating the upward pressure on the trapezoid b_4b_1 at its center of gravity as located above.

$$M = 300{,}000(7.33 - 1.5) - 3.85 \times \frac{7.55 \times 10.28}{2} \times 7.33 \times 4600$$
$$= 590{,}000 \text{ ft.-lb.}$$

or 7,080,000 in.-lb.

The depth of footing required to provide for this moment equals

$$d = \sqrt{\frac{7{,}080{,}000}{107.7 \times (7.55 \times 12)}} = 27.0 \text{ in.}$$

An approximate solution is sometimes used in which the moment at the section through the center of gravity of the entire footing is used, thus avoiding the computations for finding the point of zero shear. Such a solution gives a bending moment which differs very little from the true bending moment.

The depth required for punching shear at the larger column,

$$d = \frac{300{,}000 - 5000 \times 4}{4 \times 24 \times 120} = 24.3 \text{ in.}$$

and at the smaller column, similarly, $d = 21.8$ in., both of which are less than that required for moment.

The width of the transverse distributing beam under the larger column is taken as 36 in., and that under the smaller column as 18 in. The moment at the edge of the column due to the upward pressure of the soil is

$$\frac{1}{2} \times \frac{300{,}000}{10.28} \times \left(\frac{10.28 - 2}{2}\right)^2 \times 12 = 3{,}000{,}000 \text{ in.-lb.}$$

for the longer beam, and

$$\frac{1}{2} \times \frac{200{,}000}{4.98} \times \left(\frac{4.98 - 1.5}{2}\right)^2 \times 12 = 732{,}000 \text{ in.-lb.}$$

for the shorter beam.

The maximum shear at the edge of the column for the former is

$$\frac{300{,}000}{10.28} \times \frac{10.28 - 2}{2} = 121{,}000 \text{ lb.}$$

and for the latter

$$\frac{200{,}000}{4.98} \times \frac{4.98 - 1.5}{2} = 70{,}000 \text{ lb.}$$

The depths required are 27.9 in. and 19.5 in., respectively. An effective depth of 28 in. is adopted for the entire footing; with 4 in. of insulation, the total depth is 32 in. and the weight per square foot, 400 lb. as assumed. In case the depth required for either of the distributing beams were considerably in excess of that required for the main slab, it would be economical to use the greater thickness only for the distributing beam itself.

In the main slab, the distance from the point of zero shear to the point where the unit shear reaches the allowable value for plain concrete is determined by solving the equation for the unit shear at any point z ft. away from the plane of zero shear, sub-

stituting for the unit shear v the definite allowable value for plain concrete v', as follows:

Let b_5 equal the width of footing at the required section, and w the net upward soil pressure in lb. per sq. ft.

$$v' = \frac{V}{b_5 jd}$$

$$b_5 = b_4 + \frac{b_1 - b_2}{l} \times z \text{ and } V = \frac{b_4 + b_5}{2} \times z \times w$$

Substituting the values of b_5 and V in the equation for v',

$$z^2\left[\frac{w}{2l}(b_1 - b_2)\right] + z\left[wb_4 - \frac{v'jd(b_1 - b_2)}{l}\right] - v'b_4jd = 0$$

In using this equation, inches must be used with pounds per square inch, and feet with pounds per square foot. In the present footing z is found to be 2.70 ft., and the width of footing at this point, b_5, is 8.56 ft. The distance q, over which web reinforcement is required, is 2.13 ft.

The maximum shear in the main slab along the plane b_6, the width of which is 9.35 ft., is

$$\frac{7.55 + 9.35}{2} \times 4.83 \times 4600 = 187{,}700 \text{ lb.}$$

and the unit shear $= \dfrac{187{,}700}{9.35 \times 12 \times .875 \times 28} = 68.3$ lb. per sq. in.

The average unit shear over the distance q is 54.15 lb. per sq. in., the average width of footing 8.95 ft., and the total area of web reinforcement required is

$$\frac{\frac{2}{3} \times 54.15 \times 8.95 \times 2.13 \times 144}{16{,}000} = 6.20 \text{ sq. in.}$$

This requires thirty-two ½-in. round single stirrups. On account of the short distance over which these must be spaced, three rows of eleven each are used. In order to stress equally all of the web reinforcement, the triangle representing the shear to be resisted by the stirrups is divided into three parts of equal area, and one row of stirrups placed at the center of gravity of each part. The length of base of this triangle is $2.13 \times 12 = 25.56$ in., and the altitude $9.35 \times 12 \times (\frac{2}{3} \times 68.3) = 5100$ lb. per lin. in. The division may be made graphically by dividing the base into three equal lengths, constructing a semicircle on

the base as a diameter, erecting perpendiculars from the third points of the base to intersect the circle, and striking arcs of circles from these intersections, using the zero ordinate point of the triangle base as a center. Vertical planes through the points of intersection of the arcs just described divide the triangle into three equal parts.

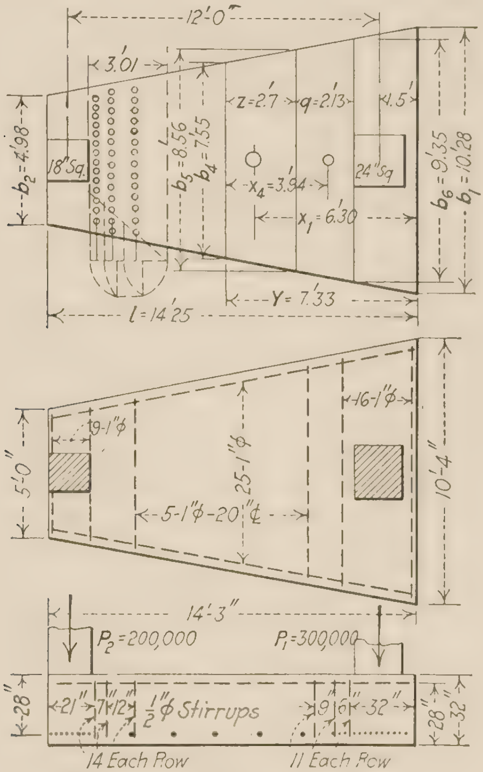


Fig. 58.

The proper location of the three rows of stirrups, determined above, is shown in Fig. 58. In a similar manner it is found that forty-two ½-in. round single stirrups spaced over a distance of 3.01 ft. are required at the other end of the footing, the spacing being as shown in Fig. 58.

The same condition with regard to diagonal tension failures may be assumed for the distributing beams as is assumed in other

footings. The unit shear on the longer beam 28 in. from the edge of the column supported by it is found as for a simple beam and equals 59.7 lb. per sq. in. Stirrups are required from this point to the point where the unit shear is equal to 40 lb. per sq. in., which is at a distance of 33 in. from the edge of the column. One double-looped ½-in. round stirrup placed 30 in. from the edge of the column is sufficient. No stirrups are required in the shorter distributing beam on account of the small shear existing at the critical section.

The area of longitudinal steel required in the main slab is found as follows:

$$A_s = \frac{7{,}080{,}000}{16{,}000 \times .875 \times 28} = 18.1 \text{ sq. in.}$$

$$\Sigma_o = \frac{187{,}700}{100 \times .875 \times 28} = 76.6 \text{ in.}$$

Twenty-five 1-in. round deformed bars furnish sufficient area and surface. They are placed 4 in. from the top of the slab as shown in Fig. 58.

Similarly for the transverse beams, sixteen and nine 1-in. round deformed bars are necessary for the long and short beams, respectively. These are placed 4 in. from the bottom of the beams as shown in Fig. 58. Five 1-in. round bars about 20 in. center to center are placed between the two distributing beams in order to add to the rigidity of the footing.

142. Cantilever Footings. In the cantilever type of construction the wall column footing is connected to the nearest interior column footing by means of a beam or strap. The eccentric load from the exterior column is resisted by some downward pressure from the interior column, the effect of which is transmitted through the strap. The wall column load is supported directly by the strap, and distributed to the soil by means of bars at right angles to the strap, in the exterior footing.

The principles involved in the design of a cantilever footing are shown in Art. 174.

143. Concrete Foundations on Piles. Where a soil of a compressible nature is encountered, and where the amount of excavation which would be required to reach a firm stratum would be excessive, economy might dictate the use of a foundation

supported by piles, the latter being long enough either to reach the firm substratum or to offer sufficient skin friction to overcome the loads to which they are subjected.

This type of foundation consists essentially of a concrete slab, plain or reinforced, supported directly by the piles. The heads of the piles are allowed to project a short distance above the ground so that the concrete may encase these portions of the piles and form with them a solid unit. A minimum embedment of 6 in. is considered satisfactory in most cases. If desirable, the material around the piles may be excavated, the depth of excavation depending upon soil conditions, and the space thus made filled in with gravel or other solid material, on which the concrete is laid as stated above. Such procedure utilizes the increased bearing power of the earth surrounding the piles.

When piles are supported entirely by the friction between their sides and the earth, the load is transmitted to a deep ground level in a conoid of pressure through the earth above it. Such piles should be driven so far apart, or to such a depth, that the increased area of bearing developed by the conoid of pressure, which has the required altitude to contain the frictional resistance, reaches a level whose material will afford the required support before it intersects the corresponding conoid of an adjacent pile. A. M. Wellington recommends that bearing piles should be spaced at least not closer than 3 ft. center to center, and that they are worse than wasted if driven less than 2½ ft. on centers. E. P. Goodrich recommends as an absolute minimum spacing 2.7 ft., and suggests 3 ft. be used wherever possible. In good practice, timber piles are never spaced closer than 2½ ft. center to center, and preferably not closer than 3 ft.

The allowable bearing on any pile will depend, among other things, upon the soil conditions, the size and spacing of the piles, and the depth to which they are driven.[10]

[10] Goodrich recommends that "the best practice is to assume a given load per pile, to design all footings accordingly, and to require the superintendent of construction to provide and drive piles which will sustain this assumed load. In that case the designer's care will be to provide just the proper number under each footing, and to space them so that each pile will develop

The main difference in the design of a concrete foundation supported on piles, and the design of a footing resting directly on the soil is in the manner in which the load on the footing is resisted by the foundation bed. In the former case, a series of concentrated upward loads must be considered, while in the latter case, uniform distribution under the entire concrete area is assumed.[11]

144. Design of Footing Supported on Piles. A concrete footing resting on a pile foundation is used to support a column 18 in. square, the total load on which is 225,000 lb. The safe bearing power of each pile is 11 tons. Design the footing in accordance with the specifications of the Joint Committee using an ultimate compressive strength of concrete of 2000 lb. per sq. in.

Assuming the weight of footing as 38,000 lb., the number of piles required $= \frac{263{,}000}{22{,}000} = 12.$

In order to keep a minimum spacing of piles of 2 ft.-6 in., and to distribute the load equally between the 12 piles as nearly as practicable, the arrangement shown in Fig. 59 is adopted. For punching shear

$$d = \frac{225{,}000}{4 \times 18 \times 120} = 26 \text{ in.}$$

its full proportion of the given load. To this end, groups should be made as nearly circular as possible, especially when they consist of any considerable number of piles. The corner piles of square groups of 16 piles might just as well be omitted."

[11] The Joint Committee includes the following recommendations for concrete footings supported on piles: The thickness of concrete above the reinforcement at the edge of a sloped footing shall not be less than 12 in.

A mat of reinforcing bars consisting of not less than .20 sq. in. per ft. of width in each direction shall be placed 3 in. above the tops of the piles.

The critical section for diagonal tension shall be taken on a vertical section at the inner edge of the first row of piles entirely outside a section midway between the face of the column or pedestal and the vertical section through the perimeter of the lower base of the frustum of a cone or pyramid which has a base angle of 45 degrees, and has for its top the base of the column or pedestal and for its lower base the plane of the center of the longitudinal reinforcement, but in no case outside of this vertical section. The critical section for piles not grouped in rows shall be taken midway between the face of the column and the perimeter of the base of the frustum described above.

The total thickness of concrete, allowing a 6-in. embedment of the piles and 4 in. from the top of the piles to the center of the reinforcing steel, equals 36 in. The actual weight of footing is then 38,000 lb., which checks the assumed value.

The net load per pile is $\frac{225,000}{12} = 18,750$ lb. The total bending moment existing around the entire perimeter of the column is $M = (18,750 \times 3 \times 8 + 18,750 \times .5 \times 4) \times 12 =$

5,860,000 in.-lb.

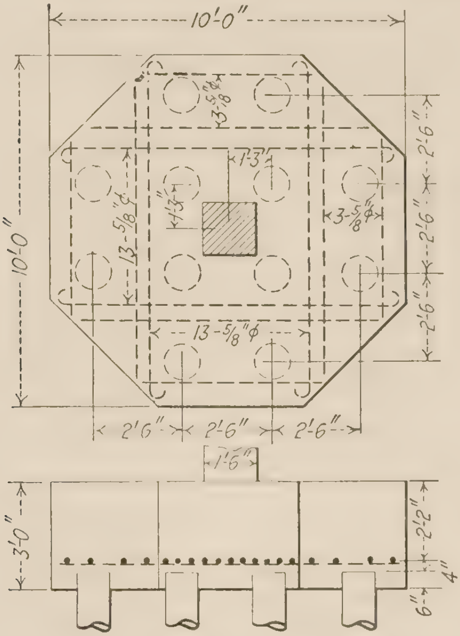


FIG. 59.

Assuming the reinforcement in two bands, the area of steel required in each band is

$$A_s = \frac{\frac{1}{4} \times 5,860,000}{16,000 \times .9 \times 26} = 3.92 \text{ sq. in.}$$

Thirteen $\frac{5}{8}$-in. round deformed bars hooked at both ends are used in each band and placed in a width of 54 in. centered about the column. The remaining portion of the footing outside this effective width is reinforced by three $\frac{5}{8}$-in. round deformed bars as shown in Fig. 59.

The unit bond stress on the effective steel is

$$u = \frac{\frac{1}{4} \times 225{,}000}{13 \times 1.964 \times .9 \times 26} = 94 \text{ lb. per sq. in.}$$

which may be considered safe for anchored bars (see Art. 80).

A more exact determination of the lever arm jd is not justified because such a supposedly theoretically correct value would depend upon the effective width of the band which in itself is an assumption. Due to the low percentage of steel the concrete fiber stress is well below the allowable.

The critical section for diagonal tension, according to the specifications, occurs at a distance of 13 in. from the face of the column. The unit shear at a vertical plane through the critical section

$$v = \frac{\frac{8}{12} \times 225{,}000}{4 \times 44 \times .9 \times 26} = 36 \text{ lb. per sq. in.}$$

145. Miscellaneous Foundations. In designing foundations to rest on soil, the safe bearing power of which is very small, it sometimes becomes necessary to extend the footings to cover practically all of the area of the building, one connected to the other. Such foundations may consist of a solid flat slab of concrete, a series of beams with slabs at the top or bottom, or an extension of the multiple column principle to the entire area.

CHAPTER VIII

REINFORCED CONCRETE BUILDINGS

146. Reinforced concrete has gradually become one of the leading building materials of the present day, chiefly because of its durability, its fire-resisting qualities, its adaptability to various types of design, and its pleasing architectural appearance. When used with any other type of construction, as for example, the floors in a steel frame structure, or by itself in a building all of whose constituent structural parts are of reinforced concrete, its suitability is well recognized.

In determining the type of structure to be used for any particular building, usually the two most important considerations are the time required before the building may be occupied, and the relative economy of the selected type as compared with the other available structures. While the actual erection of a steel frame building may be completed in considerably less time than a reinforced concrete building, in most cases the length of time necessary for the fabrication of the steel will result in the lapse of a longer period of time from the letting of the contract to the completion of the structure than in the case of all-reinforced-concrete construction.

Certain contracting firms maintain in stock, completely fabricated, all of the steel necessary for buildings of various types and sizes. In such cases the length of time required for the finished building from the letting of the contract will obviously be shorter than for the reinforced concrete type. The limitations as to size and adaptability of such a class of buildings, however, naturally restrict their use to a small percentage of present-day structures.

Steel frame structures in which no attempt is made to encase the steel may be lower in first cost than those of reinforced concrete. If, however, an attempt is made to have the steel structure as fireproof as the reinforced concrete structure, the

ratio of relative first costs may be reversed. This is especially true of certain types of buildings in which long spans and heavy loads exist. A real comparison between buildings of different materials should be made only after a consideration of the first cost and subsequent annual expenditures.

147. Floor and Roof Loads. The first step in any design is the selection of the ultimate load that may be placed on the structure. This depends upon the character of the building and also upon the requirements of the building code that applies to the locality in which the erection will take place. The roof load will depend to a certain extent upon the geographical location of the building, being affected by the amount of snowfall. Good practice is well illustrated by the following table of floor and roof loads taken from several building code requirements.

148. Building Code Requirements for Live Load. All floors shall be constructed to bear a safe live load per superficial square foot of not less than the following amounts:

Apartments	50	Offices	75
Assembly Halls	100	Schools	75
Dwellings	50	Stables, garages	100
Hospitals	60		
Hotels	60	Stairways	75
Manufacturing	150	Roofs, slope under 20°	40
Mercantile—stores, warehouses, etc.	200	Wind pressures	30

The values in the above table are general and in all cases the governing building code should be consulted before any selection of live load is made. Where necessary, the effect of impact should be considered, especially in the case of floors which support heavy vibrating or oscillating machinery. In all cases the dead weight of the floor must be included in the total design load.

149. Floor Systems. The different systems of reinforced concrete floors may be divided into five general classes:

1. Beam and girder floors.
2. Flat slab floors.
3. Hollow tile or steel tile floors.
4. Floors of unit construction.
5. Floors in which a reinforced concrete slab is supported directly on steel beams.

The beam and girder floor consists of a series of parallel beams supported at their extremities by girders which in turn frame into columns placed at more or less regular intervals over the entire floor area. This framework is covered by a reinforced concrete slab, the load from which is transmitted first to the beams and thence to the girders and columns. The beams are usually spaced so that they come at the mid-points, at the third points, or at the quarter points of the girders, and in some extreme cases only at the columns. The arrangement of beams and spacing of columns should be determined by economical and practical considerations. These will be affected by the use to which the building is to be put, the size and shape of the ground area, and the load which must be carried. A comparison of a number of trial designs and estimates should be made if the size of the building warrants, and the most satisfactory arrangement selected. The design of a typical interior beam and girder floor bay is given in Arts. 159 to 162.

The flat slab floor consists of a reinforced concrete slab supported directly on concrete columns without the aid of beams or girders. This type is considered in detail in Arts. 163 to 168.

The tile floor is a modification of the solid slab floor and is widely used for long spans and light loads. A part of the concrete is replaced by rows of hollow clay tile or steel tile pieces, so placed that they form small parallel T-beams of the concrete. Such construction reduces the volume of concrete and, consequently, the weight of the floor, without materially reducing the resisting moment. The above arrangement infers that beams, either steel or concrete, are placed in one direction between the columns to take the load from the T-beams. The clay tiles remain embedded in the concrete and form a part of the finished slab, while the steel tiles are usually removable and are merely used as forms. Such an arrangement constitutes what is commonly known as a one-way tile floor.

Another form of tile floor that is sometimes used consists of hollow clay tiles or steel tile pieces placed in the concrete in such positions that a series of small T-beams is formed in two directions, one parallel to each side of the panel. This is known as a two-way tile floor. Either steel or concrete beams, placed along

all four edges of the panel, may be used to support the T-beams. The two types of tile floors are illustrated in Fig. 60. Steel tile pieces are available in various sizes; their exact dimensions may be obtained from the manufacturers' catalogues.

A floor of unit construction is made of precast concrete structural units such as beams, girders, and columns, erected in a manner similar to steel members. Suitable pockets are molded in members which are to support other units, and these supported members held in place by means of projecting bars, by a small

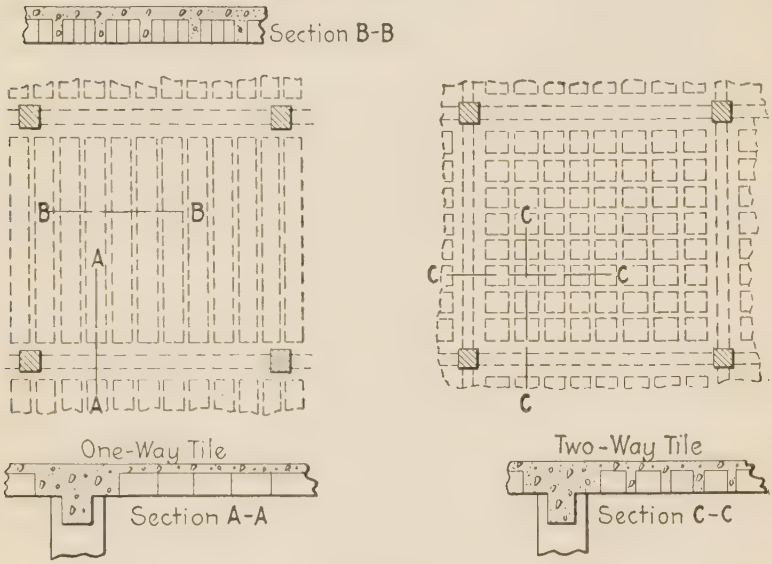


Fig. 60.

amount of fresh concrete, or by other suitable details. Such construction does not have the strength of a monolithic floor due to the lack of continuity in the members. Under certain conditions a unit-built floor may be cheaper than one poured in place. The economy arises from the fact that the same forms may be used for several units, the amount of falsework required is reduced to a minimum, the work of molding the units may be carried on under cover in all kinds of weather, and the number of men required in the actual construction is comparatively small.

In floors consisting of a reinforced concrete slab supported directly on steel beams, the slab may rest on top of the beams or

be supported by the lower flanges. If the concrete encases the steel completely, the fire-resisting qualities of the floor are greatly increased. The usual maximum spacing of beams in such construction is about 6 ft. The use of one-way floors resting on steel beams is very common for longer spans up to about 20 ft. Illustrations of modifications of this type of construction are given in Fig. 61.

150. Floor Surfaces. A mortar or granolithic finish is probably the most common type of wearing surface for concrete floors. The usual proportions for such a surface are one part Portland cement, one part sand, and one part crushed stone which passes

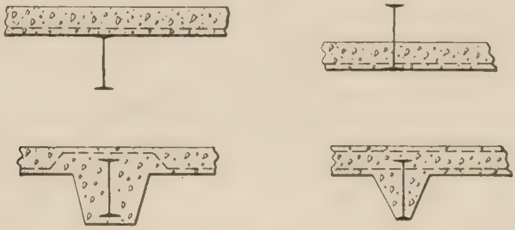


FIG. 61.

a 1/4-in. sieve. A thickness of 1 to 2 in. should be used, depending on the time of pouring, 1 in. being sufficient if the main slab has not been allowed to set thoroughly before the placing of the mortar.

A wooden floor may be provided for if desired, by embedding nailing strips or "sleepers," usually 2 × 4's laid flat, in a layer of concrete on top of the main slab. A spacing of 16 in. for the sleepers has been found satisfactory for the ordinary floor.

Small vitrified clay flat tiles embedded in a 2-in. layer of Portland cement mortar also provide a satisfactory floor surface. Linoleum placed over a smooth cement base has been used in many cases.

The durability of any wearing surface depends in a large measure upon the method of placing the surface. If not properly constructed, the granolithic finish might spall, the wood floor "dry-rot," the tiles curl, and the linoleum crack. In order to insure the maximum degree of serviceability from a given type of floor surface, a special study of methods which have proved

successful for laying that particular type of floor should be made.[1]

151. Concrete Roofs. The design of roofs is similar to the design of floors. In addition to the structural requirements, however, the roofs must be impervious to the passage of water, provide for adequate drainage, and furnish protection against condensation.

In order to provide adequate drainage, the roof slab may be pitched slightly, or a filling of some light material such as cinder concrete, covered with a suitable roofing material, may be used, the thickness of the filling being varied so as to give the required slope to the roof surface. The amount of slope required for drainage will depend upon the smoothness of the exposed surface. A value of $\frac{1}{8}$ in. per ft. might be used with a surface of hard tile. Felt and gravel roofs should have a pitch of at least $\frac{3}{8}$ in. per ft. Some form of flashing is required along the parapets to prevent the drainage from seeping into the building at the edges of the roof slab.

Condensation may best be guarded against by proper ventilation and insulation. The form of insulation to be used will depend upon the particular class of building under consideration. The main types of insulation are:[2]

Roofing felts and quilts.

Cinder fill (with cement finish upon which the roofing is laid).

Cinder concrete fill covered with roofing.

Hollow tile (with mortar top coat upon which roofing is laid).

Combination hollow tile and cinder fill.

[1] W. P. Anderson, in *Factory*, 1916, sums up the relative merits of wood finish floors and cement floors for industrial buildings as follows: "Wood finish floors are high in initial cost and call for a higher insurance rate. They give satisfactory wearing service and are easier to repair than cement floors. Cement finish floors are cheaper. They wear well when properly laid. They seem to offer no really authenticated objection in the matter of producing fatigue in employees standing on them. They are easy to clean. They do not prevent the ready installation of machinery and are not difficult to repair."

[2] See Article on "Prevention of Condensation on Concrete Roof Surfaces," by Albert M. Wolf, *Concrete-Cement Age*, May, 1914.

Double concrete roof (light concrete slab above the main roof slab).

Suspended ceilings.

Imperviousness may best be provided for by the application of some form of separate roof covering, such as a combination of felt and gravel in alternate layers cemented together and to the slab by means of coal-tar pitch or asphalt, vitrified tile embedded in asphalt, or any of the commercial types of built-up roofings. Tin, corrugated iron, or copper roofings are sometimes placed on reinforced concrete buildings but are usually more expensive and less permanent than other types of coverings. If it is not desirable to use any separate roof covering, the main slab may be made reasonably water-proof by the methods mentioned in Art. 20. Such procedure is not recommended except for structures where absolute imperviousness is not essential, because of the difficulty of preventing entirely the formation of shrinkage cracks and the attending seepage.

152. Curtain Walls. As a general rule, the exterior walls of a reinforced concrete building are supported at each floor by the skeleton framework of the building, their only function being to enclose the building completely. Such walls are called curtain walls. They may be of concrete, brick, tile, or other similar material. The thickness will vary according to the material, the type of construction and the building requirements governing the particular locality where the construction takes place. A minimum thickness of 12 in. for brick, tile, and stone masonry curtain walls is usually considered satisfactory; for some classes of structures a thickness of 8 in. is sufficient.

The pressure of the wind is practically the only load that need be considered in determining the theoretical thickness of a reinforced concrete curtain wall. Designed as a slab supported on four sides for a wind pressure of 30 lb. per sq. ft., the walls in buildings of ordinary proportions need only be from 3 to 4 in. thick. This is rather thin to permit of practical and economical construction and to assure complete protection against seepage and condensation. Most building codes require a minimum thickness of 8 in. for curtain walls of reinforced concrete. The amount of steel necessary is usually governed by the necessity

of guarding against the formation of cracks caused by expansion and contraction due to temperature changes. Small bars running horizontally and vertically are placed near each face of the wall center to center not more than 12 or 18 in. in both directions. On account of the probability of greater temperature variation on the exposed face, more steel should be placed near that face than on the inside unless the lateral pressure requirements govern these amounts. These bars should extend into the columns and wall beams if the walls are poured at the same time. If they are poured after the columns and beams, anchorage should be provided for by means of dowels projecting from the latter units. It is good practice in such cases to mortise the wall into the columns and wall beams by leaving grooves in these latter members when they are poured. The grooves may be formed by nailing wooden strips on the inside of the forms.

Good practice in reinforced concrete curtain wall construction is illustrated by the New York City and Cleveland Building Codes, as follows:

New York City Code. Enclosure walls of reinforced concrete shall be securely anchored at all floors. The thickness shall not be less than $1/25$ of the unsupported height, but in no case less than 8 in. Steel reinforcement, running both horizontally and vertically, shall be placed near both faces of the wall; the total weight of such reinforcement shall be not less than ½ lb. per sq. ft. of wall.

Cleveland Code. Buildings having a complete skeleton construction of steel or of reinforced concrete or a combination of both, may have exterior walls of reinforced concrete 8 in. thick, provided, however, that such walls shall support only their own weight, and that such walls shall have steel reinforcement of not less than three-tenths of 1 per cent in each direction, vertically and horizontally, the rods spaced not more than 12 in. on centers and wired to each other at each intersection. The steel rods shall be combined with the concrete and placed where the combination will develop the greatest strength; and the rods shall be staggered or placed and secured so as to resist a pressure of 30 lb. per sq. ft. either from the exterior or from the interior on each and every square foot of each wall panel.

Where a small amount of window area is inserted in a curtain wall, the reinforcement may remain as for a solid wall, but additional bars should be placed near all edges of the openings. Where light is essential, as in a factory building, practically the entire wall panel may be enclosed by windows, the construction then consisting of a wall beam at each floor and a spandrel between the wall beam and the window. Sometimes this spandrel is made of reinforced concrete and constructed as a part of the wall beam. In other instances the spandrel is considered independent of the wall beam, and is reinforced only for temperature stresses. The advantage in using independent spandrels lies in the fact that they may be placed after the structural framework is completed and greater care can be taken in the finishing than would be possible if they were part of the load-bearing skeleton of the structure. Brick or other suitable material may be used for this portion of the wall.

153. Bearing Walls. A bearing wall may be defined as one which carries any load in addition to its own weight. Such walls may be constructed of stone masonry, brick, hollow building blocks, or concrete. Occasional projections or pilasters add to the general appearance and strength of the wall. In small reinforced concrete commercial buildings and residences the bearing wall type of construction may be used with economy and expediency. In larger commercial and manufacturing buildings where the element of time is an important factor, the delay necessary for the erection of the bearing wall and the attending increased cost of construction often dictate the use of some other arrangement.

The thickness of bearing walls may be decreased toward the upper stories of the building. The minimum thickness often specified is 12 in. Walls of reinforced concrete in most cases must be of the same thickness as brick walls. This requirement prevents the extensive use of reinforced concrete in bearing walls because of the relatively large cost of the concrete. Among other requirements, the New York City Code specifies that the thickness of masonry walls of public and business buildings shall not be less than the following:

(*a*) When over 75 ft. in height, 16 in. for the uppermost 25 ft., 20 in. for the next lower 35 ft., 24 in. for the next lower 40 ft., and increasing 4 in. for each additional lower section of 40 ft.

(*b*) When over 60 ft. and not above 75 ft. in height, 16 in. for the uppermost 50 ft. and 20 in. below that.

(*c*) When not over 40 ft. in height, 12 in. throughout.

Bearing walls may be either of single or double thickness, the advantage of the latter type being that the air space between the walls renders the interior of the building less liable to temperature variations, and makes the wall itself more nearly moisture-proof. On account of the greater gross thickness of the double wall, such construction reduces the available floor space. This feature is often sufficient in itself to warrant the selection of the solid wall unless the factors of condensation and temperature are of great importance.

154. Basement Walls. In determining the thickness of basement walls, the lateral pressure of the earth, if any, must be considered in addition to the other structural features. If part of a bearing wall, the lower portion may be designed either as a slab supported by the basement and first floors, or as a retaining wall, depending upon the material used. If columns and wall beams are available for support, each basement wall panel of reinforced concrete may be designed to resist the earth pressure as a simple slab reinforced in either one or two directions.

The New York City Building Code requires that brick or concrete foundation walls shall be at least 4 in. thicker than the walls next above them, but not less than 12 in. thick in any case. For each additional 10 ft. or part thereof, below the depth of 12 ft. below the curb level, the thickness shall be increased 4 in.

Care should be taken to brace a basement wall thoroughly from the inside if the earth is backfilled before the wall has obtained sufficient strength to resist the lateral pressure without such assistance.

155. Parapet Walls. In the case of buildings with flat roof slabs on which drainage slopes are built, parapet walls are necessary architecturally to give a more finished appearance to the top of the structure, and practically to provide a backing for the drainage slopes. They are usually of brick or concrete or a com-

bination of both. Concrete is, in most cases, preferable from an economical standpoint. In order to give a better appearance it may have a veneer of brick or terra cotta.

The chief point to be considered in the construction of parapet walls is the necessity of providing the right amount of reinforcement to prevent cracks caused by excessive temperature changes (on account of exposed position) and expansion and contraction at corners.

156. Veneer for Exterior Walls. In order to give an attractive appearance to the building, it sometimes becomes necessary to cover up the entire exterior wall surface with a veneer of brick, terra cotta tile, marble, or other finishing material. A method of securing a covering of face brick to concrete consists in placing corrugated copper or galvanized iron ties, usually about 3/4 in. wide and 6 in. long, at frequent intervals in the wall or column forms so that about 4 in. of the tie strip will project into the concrete when poured and the remaining 2 in. will lie flat against the form and tacked lightly to it. When the form is removed, this latter portion is bent outward and is bonded into the brick veneer by means of the joint mortar. A brick veneer should be supported by a concrete ledge at every floor. Terra cotta and stone facings are generally supported by ledges in the concrete or by angle irons, and are provided with anchors commensurate with the size of the veneer units.

157. Partition Walls. Interior walls used for the purpose of subdividing the floor area may be made of concrete, metal lath and plaster, terra cotta tile, plaster block, or brick. Reasonably adequate fire protection is afforded by a solid concrete wall 3 or 4 in. thick. If properly reinforced and anchored at the top and bottom, such a wall becomes desirable in nearly every respect. The reinforcement should be similar to that in curtain walls but need not be as great in quantity. Suitable anchorage may be obtained by permitting the vertical rods to project into the floor and ceiling. If it is convenient, as is usually the case, to pour the wall after the structural framework of the building is completed, a groove should be left in the floor and one in the ceiling to receive the partition. Two objectionable features of the solid concrete partition wall are its weight and cost of installation. In buildings

where many lives would be endangered by a rapid spread of a fire once started, these objectionable features become insignificant.

The New York City Building Code specifies a minimum thickness of 3 in. for gravel concrete partition walls if properly reinforced with steel, and 4 in. if unreinforced, the mix to be not leaner than 1:3:6. For cinder concrete walls the limits are 4 in. for the reinforced wall and 5 in. for the unreinforced wall, the proportion of materials remaining the same.

The most common form of metal lath and plaster partition consists of some form of vertical steel studding suitably anchored to the floor and ceiling, with metal lath fastened to both sides. Each side is plastered with a mixture of lime and cement mortar, thus forming a hollow wall from 3 to 6 in. thick, which has, if proper bond is secured between the plaster and lath, a fair amount of fire resistance. This form may be modified by filling the space between the plaster sides with cinder concrete, or by omitting the metal lath on one side of the vertical studding, and plastering both sides of the remaining sheet of lath. This results in a solid wall, usually about 2 in. thick, the reliability of which is rather uncertain. All openings should be framed with steel sections to which the wood frames or other trim may be fastened.

Terra cotta tile partitions are usually made of blocks from 4 to 6 in. thick, although the blocks may be obtained in thicknesses varying from 2 to 12 in. This type of partition is light in weight, and satisfactory under ordinary conditions. The blocks may be plastered on one or both sides, the thickness of wall being increased by about ¾ in. for each plastered side.

Partitions made of plaster blocks usually vary from 4 to 8 in. in thickness. The blocks are made of gypsum or plaster of paris, with an admixture of cinders, asbestos fiber, wood chips or vegetable fiber, and laid in gypsum plaster or cement mortar tempered with lime. They are light and easy to handle and place, but offer decidedly poor resistance to the action of fire and water.

The main use of brick inner walls is for the enclosing of stairs and elevator shafts, and in fire walls, the express purpose of which is to divide the floor area into sections to prevent the spreading of fire from one part of the structure to another. Reinforced concrete partitions are also used for the same purpose. These,

as well as all other permanent partitions, should be independently supported at each floor on the fire-proof construction of the floor. The use of partitions of pressed metal and glass or of wood and glass should be restricted to the subdivision of rooms or spaces enclosed by fire-proof partitions.

158. Concrete Stairs. The simplest form of reinforced concrete stairway consists of an inclined slab supported at the ends upon beams, with steps formed upon its upper surface. Such a stair slab is usually designed as a simple slab of a span equal to the horizontal distance between supports. This method of design requires steel to be placed only in the direction of the length of the slab. Transverse steel, usually one rod to each tread, is used only to assist in distribution of the load and to provide temperature reinforcement. The usual stairway load in commercial and manufacturing buildings varies from 75 to 100 lb. per sq. ft. of horizontal surface.

It sometimes becomes necessary to include a platform slab at one or both ends of the inclined slab. Many successful designs made as outlined above for the simple inclined slab indicate that the effect of the angle that occurs in a slab of this type can safely be disregarded. In cases where both ends of the stair slab are fixed and dowels inserted to provide for negative moment, or where one end is continuous across an intermediate beam, and negative moment steel provided there, the moment coefficient for a partially continuous beam can safely be used in the design.

It is advisable to keep the unsupported span of a stair slab reasonably short. If no break occurs in the flight between floors, intermediate beams, supported either by the structural framework of the building, or by additional short posts from the floor below, may be employed. If the stair between floors is divided into two or more flights, beams as described above may be used to support the intermediate landing, and these in turn supported as above for the long straight flight, or the intermediate slab may be suspended from a beam at floor level by means of rod hangers. Where conditions permit, the intermediate slab may be supported directly by the exterior walls of the building.

When it becomes necessary to use a stair slab of comparatively great length with no possibility of intermediate support, the use of

inclined side beams may be advisable. The slab type of stair with the intermediate supports will usually prove cheaper than the type in which side beams are employed, and should, therefore, ordinarily be used where possible. In a flat slab building it will be necessary to insert special beams at the floor level around the opening of the stairway. In a beam and girder building, the regular floor beams may be used to support the stair slab at that level, or special beams may be inserted for the purpose.

The vertical height of a stair step is called the rise, and the horizontal distance between the vertical faces of two consecutive steps is called the run. In order to give a satisfactory and comfortable ratio of rise to run, various rules have been adopted. One requires that for steps without nosings, the sum of the rise and run shall be $17\frac{1}{2}$ in. A rise of less than $6\frac{1}{2}$ in. or more than $7\frac{3}{4}$ in. is not desirable in the usual case. The New York City Building Code requires that run and rise shall be so proportioned that the product of the run, exclusive of nosing, and the rise in inches, shall be not less than 70 nor more than 75, but risers shall not exceed $7\frac{3}{4}$ in. in height, and treads, exclusive of nosing, shall be not less than $9\frac{1}{2}$ in. wide. A minimum width of tread of 12 in. should be used when no nosings are employed. The width of the slab will vary with the occupancy of the building, and the total number of stairways. A minimum unobstructed width of $3\frac{1}{2}$ ft. will prove satisfactory for the ordinary building, but the width should be increased as the probable number of occupants becomes relatively large.

In cases where the stairway is constructed after the main structural framework of the building, recesses should be left in the beams to support the stair slab, and dowels provided to furnish the necessary anchorage. The steps may be poured monolithic with the slab, or they may be molded after the main slab is in place. In the latter instance, provision must be made for securing the step to the slab. The nosing, where used, may be constructed by offsetting the upper portion of the vertical form of the step. A satisfactory wearing surface for the upper face of the step may be obtained by finishing with a 1-in. layer of cement mortar. Metal treads embedded in the concrete are

often used for a wearing surface. A complete design of a concrete stairway, with details of construction, is given in Art. 177.

Beam and Girder Floors

159. Design of a Beam and Girder Floor. In order to illustrate the application of the principles of reinforced concrete to the design of a reinforced concrete floor of the beam and girder type, let it be required to design a typical interior floor bay to sustain a live load of 200 lb. per sq. ft. The columns supporting the floor are to be spaced 21 ft.-0 in. center to center in one direction and 23 ft.-0 in. center to center in the other direction. The beams span the 23 ft.-0 in. direction, and are placed one at each column and one at each third point of the supporting girders, thus making the distance center to center of beams 7 ft.-0 in. A 1-in. granolithic finish is to be included in the dead load on the slab, but is not to be considered as part of the effective depth of the slab. The allowable unit stresses are to be as specified by the Joint Committee for a 2000-lb. concrete. The general arrangement of beams is shown in Fig. 65.

160. *Design of Slab.* Assume weight of slab and finish 60 lb. per sq. ft. The total load is then 260 lb. per sq. ft.

For a 12-in. width of slab

$$M = \tfrac{1}{12} \times 260 \times 7^2 \times 12 = 12{,}800 \text{ in.-lb.}$$

From Table IV, $k = .429$, $j = .857$, and $K = 146.7$

$$d = \sqrt{\frac{12{,}800}{146.7 \times 12}} = 2.7 \text{ in.}$$

Selecting $d = 3$ in., and allowing 1 in. of insulation below the center of the steel, the total weight of the slab and finish is 62 lb. per sq. ft., which agrees closely with the assumed value. A thickness of slab less than 4 in. is not advisable in ordinary building construction. Some building codes specify a minimum thickness of 5 in. for floor slabs.

The area of steel required for a 12-in. width of slab is

$$A_s = \frac{12{,}800}{16{,}000 \times .857 \times 3.0} = 0.31 \text{ sq. in.}$$

This is furnished by $\frac{3}{8}$-in. round bars 4 in. center to center.

In order to provide for negative moment over the beams, the arrangement of slab steel shown in Fig. 62 is used. This gives equal amounts of positive and negative moment steel, furnishes a small amount of steel at the bottom over the support, and permits of bending every bar to the same shape.

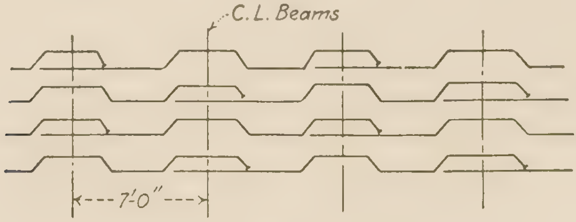


Fig. 62.

An alternate method would be to bend every other bar up over the support, and continue the remaining bars straight through the support as shown in Fig. 62*a*. This method furnishes only one-half as much steel for negative moment as for positive moment, and is, therefore, not to be regarded as theoretically perfect. In all cases the steel should be bent up at approximately the quarter point of the slab span.

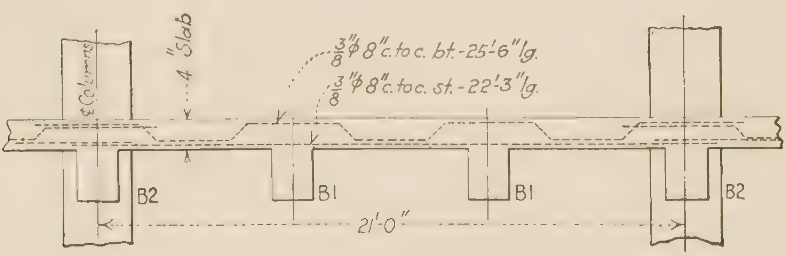


Fig. 62*a*.

In order to prevent shrinkage and temperature cracks, $\frac{3}{8}$-in. round bars will be placed parallel to the beams about 18 in. apart, four to a panel. These bars also assist in distributing the load on the slab over a comparatively large width, and assist in binding the entire structure together.

161. *Design of Cross-beams.* Since the slab and beams are poured at the same time and thoroughly bonded together, the

latter may be designed as T-beams. The span of the cross-beams is 23 ft.-0 in.

Total load from slab per lin. ft. = $7 \times 260 = 1820$ lb.

Assuming the weight of the stem of the beam as 230 lb. per ft., the total load per linear foot on the beam is

$$w = 1820 + 230 = 2050 \text{ lb.}$$

$$M = \tfrac{1}{12} \times 2050 \times \overline{23}^2 \times 12 = 1{,}085{,}000 \text{ in.-lb.}$$

$$V = 2050 \times \tfrac{23}{2} = 23{,}600 \text{ lb.}$$

$$b'd = \frac{23{,}600}{\tfrac{7}{8} \times 120} = 224 \text{ sq. in.}$$

If $b' = 8$ in., $d = 28.0$ in.

If $b' = 10$ in., $d = 22.4$ in.

If $b' = 12$ in., $d = 18.7$ in.

In order to keep the ratio of $\frac{b'}{d}$ within the limits of $\frac{1}{2}$ to $\frac{1}{3}$, and to furnish sufficient width to provide for the required steel, an effective section of 10×22.5 in. is selected. Allowing for two rows of steel, 2 in. center to center vertically, the center of the lower row being 2 in. above the lower surface of the beam, the depth of beam below the slab = $22.5 + 3 - 4 = 21.5$ in.

The weight of the stem is then 225 lb., which agrees very closely with the assumed weight.

Assuming $jd = d - \tfrac{1}{2}t$,

$$A_s = \frac{1{,}085{,}000}{16{,}000(22.5 - 2.0)} = 3.30 \text{ sq. in.}$$

This is furnished by eight $\frac{3}{4}$-in. round bars, the area of which is 3.53 sq. in. As explained in Art. 96, some excess of positive moment steel is advisable in order to provide for the negative moment over the support, the required tension steel at the latter point being greater on account of the difference in values of j between the center and support.

The beam may be reviewed to determine the value of the concrete stress. Diagram 6 is used.

The effective width of flange is in this case $\frac{1}{4}$ of the span, or $\frac{1}{4} \times 23 \times 12 = 69$ in.

$$p = \frac{3.53}{66 \times 22.5} = .0023 \qquad \frac{t}{d} = \frac{4.0}{22.5} = .178$$

From Diagram 6, $k = .240$ and $j = .929$

$$f_s = \frac{1{,}085{,}000}{3.53 \times .929 \times 22.5} = 14{,}700 \text{ lb. per sq. in.}$$

$$f_c = \frac{14{,}700 \times .240}{15(1 - .240)} = 310 \text{ lb. per sq. in.}$$

The determination of steel area and the investigation of concrete stress could also be done with the aid of Diagram 3. Since

$$\frac{M}{bd^2} = \frac{1{,}085{,}000}{69 \times (22.5)^2} = 31.1 \text{ and } \frac{t}{d} = .178$$

from Diagram 3, $f_c = 330$ lb. per sq. in. and $j = .935$

The difference in values of f_c is due to the fact that, in the first case, the steel stress is considerably below 16,000 lb. per sq. in., while in the latter case the steel stress assumed is 16,000 lb. per sq. in. A decrease in steel stress effects a corresponding decrease in concrete stress.

$$A_s = \frac{1{,}085{,}000}{16{,}000 \times .935 \times 22.5} = 3.23 \text{ sq. in.}$$

Eight $\frac{3}{4}$-in. round bars are required as in the first method.

The foregoing solution provides for the stresses existing at the center of the beam only. Investigation must be made of the stresses at the support, and provision made for resisting them.

Four rods from each beam are bent up and carried over the support to the third point of the adjoining span. The four remaining bars of each beam are carried straight through the support into the adjoining span far enough to develop their strength in bond. This arrangement furnishes a total of eight bars in both the top and bottom over the support. To allow for the girder rods, the center of the top row of bars in the beam at the support is placed $2\frac{1}{2}$ in. from the upper surface of the slab. This arrangement also permits of proper intermeshing of the slab and girder rods. (See discussion in Art. 97.) The effective depth of the tension steel at the support is then 22.0 in., as shown in Fig. 63.

Investigating the unit stresses over the support,

$$\frac{d'}{d} = \frac{3}{22} = .14 \text{ and } p = p' = \frac{3.53}{10 \times 22} = .0160$$

From Diagram 8, $k = .400$ and $j = .870$

$$f_s = \frac{1{,}085{,}000}{3.53 \times .870 \times 22} = 16{,}050 \text{ lb. per sq. in.}$$

$$f_c = \frac{16{,}050 \times .400}{15(1 - .400)} = 720 \text{ lb. per sq. in.}$$

$$f'_s = 16{,}050 \times \frac{.400 - .136}{1.0 - .400} = 7100 \text{ lb. per sq. in.}$$

The maximum unit bond stress on the tension bars is

$$u = \frac{23{,}600}{8 \times 2.356 \times .870 \times 22} = 66 \text{ lb. per sq. in.}$$

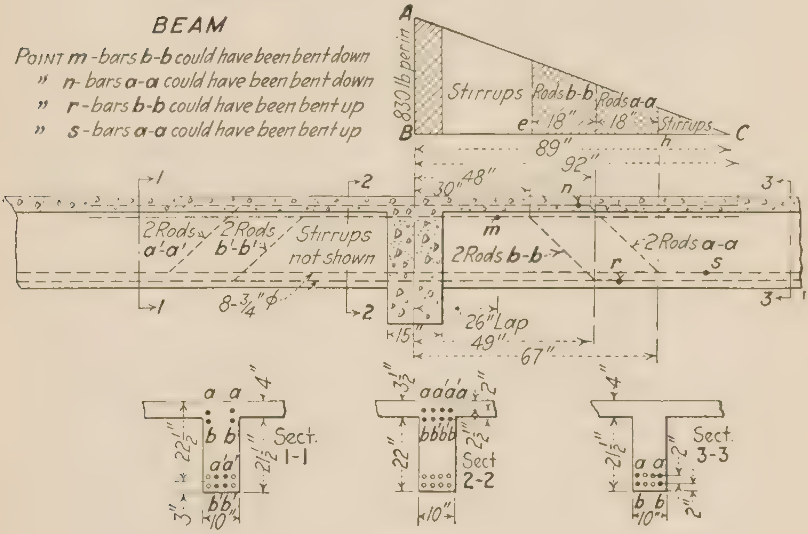


FIG. 63.

The unit shear at the support is

$$v = \frac{23{,}600}{10 \times .870 \times 22} = 123 \text{ lb. per sq. in.}$$

which is but slightly in excess of the allowable.

The four rods are bent up in pairs. From Diagram 1 the first pair may be bent $.3 \times 23 \times 12 = 83$ in. from the center of the girder, and the second pair $.21 \times 23 \times 12 = 58$ in. from the same point.

Assume that the negative moment becomes zero at the one-third point of the span. To provide fully for this moment at all

points, the first pair of rods to be bent up must reach the top of the beam not nearer the center of the support than

$$\frac{1}{2} \times \frac{23 \times 12}{3} = 46 \text{ in.}$$

and the second pair

$$\frac{1}{4} \times \frac{23 \times 12}{3} = 23 \text{ in.}$$

Provision must now be made for the diagonal tension stresses. The amount of shear as a measure of diagonal tension to be provided for by the web reinforcement is represented by the triangle ABC (Fig. 63) in which

$$AB = \frac{23{,}600 - (40 \times 10 \times .870 \times 22)}{.870 \times 22} = 830 \text{ lb. per lin. in.}$$

The numerator in the above equation (15,900 lb.) represents the amount of external shear in excess of the strength of the concrete.

$$BC = x_1 = \frac{23}{2} - \frac{40 \times 10 \times .929 \times 22.5}{2050} = 7.4 \text{ ft. or } 89 \text{ in.}$$

The bars are bent as close to the support as possible because of the greater shearing stress there. Since the inclined bars may be assumed as adding to diagonal tension resistance for a distance of $\frac{45}{45 + 10} \times 22.5 = 18$ in. from the point of bending, and since the amount of inclined stress to be resisted by each pair of rods over this distance is less than the tensile strength of the two rods, stirrups are needed only over the portions Be and hC.

The required spacing of $\frac{3}{8}$-in. round U-stirrups at the support

$$s = \frac{2 \times .1104 \times 16{,}000 \times .870 \times 22}{15{,}900} = 4.3 \text{ in.}$$

at point e

$$s = \frac{2 \times .1104 \times 16{,}000 \times .870 \times 22}{15{,}900 - {}^{31}\!/\!_{12} \times 2050} = 6.4 \text{ in.}$$

at point h

$$s = \frac{2 \times .1104 \times 16{,}000 \times .929 \times 22.5}{15{,}900 - {}^{67}\!/\!_{12} \times 2050} = 16.8 \text{ in.}$$

The maximum allowable spacing = .45 × 22.5 = 10 in. The first stirrup is placed about 2 in. from the edge of the girder, and the remaining stirrups towards the center of the span are placed six at 4 in., five at 10 in., and four at 12 in. Stirrups are placed arbitrarily over the portion *eh* and over the central portion of the beam to assist in binding the web and flange together.

The straight bars must be continued beyond the center of support a distance of

$$\frac{720 \times 15}{4 \times 80} \times \frac{3}{4} = 26 \text{ in.}$$

The bent bars are continued to the point of zero negative moment, assumed $\frac{1}{3} \times 23 \times 12 = 92$ in. from the girder center. The steel details are shown in Figs. 63 and 65.

An alternate method of bending the steel in similar beams is shown in Fig. 81.

162. *Design of Girder.* The girder has a span of 21 ft.-0 in., with concentrated loads of 2 × 23,600 = 47,200 lb. at each of the one-third points. The maximum moment due to the concentrated loads is

$$M_1 = \tfrac{2}{9} \times 47{,}200 \times 21 \times 12 = 2{,}650{,}000 \text{ in.-lb.}$$

Assume the weight of the stem of the girder as 500 lb. per lin. ft. The maximum moment due to the uniform load is

$$M_2 = \tfrac{1}{12} \times 500 \times \overline{21}^2 \times 12 = 220{,}000 \text{ in.-lb.}$$

and the total maximum moment is 2,870,000 in.-lb.

The total maximum shear is $47{,}200 + 500 \times {}^{21}\!/_{2} = 52{,}450$ lb.

$$b'd = \frac{52{,}450}{\tfrac{7}{8} \times 120} = 500 \text{ sq. in.}$$

Taking into consideration space for bars, economical depth, headroom, etc., the width of the stem is made 15 in. which requires an effective depth of 33.3 in.

With the arrangement of steel proposed in the design of the cross-beams, the center of the upper row of girder steel at the support is 1½ in. from the top of the slab and the vertical distance center to center of rows, 2 in. At the center of the girder an insulation of 2½ in. below the center of the lower row

of steel is provided, and the vertical distance center to center of rows is 2 in. The effective depth at the support (which is governed by the shear requirement) is 1 in. greater than that at the center. With an effective depth at the center equal to 32.5 in. the value of d at the support is 33.5 in. which provides for the shear as computed above, and the total height of beam, assuming two rows of steel, is $32.5 + 3.5 = 36.0$ in. The depth below the slab is 32.0 in., and the weight of stem 500 lb. per ft. as assumed.

At the center of the girder the effective width of flange is $\frac{1}{4}$ of the span, or 63 in.

$$\frac{M}{bd^2} = \frac{2{,}870{,}000}{63 \times 32.5^2} = 43.1 \text{ and } \frac{t}{d} = \frac{4.0}{32.5} = .123$$

From Diagram 3, $f_c = 450$ lb. per sq. in. and $j = .940$

$$A_s = \frac{2{,}870{,}000}{16{,}000 \times .940 \times 32.5} = 5.87 \text{ sq. in.}$$

Eight 1-in. round bars, furnishing 6.28 sq. in., are selected.

In order to provide for the negative moment, four bars from each girder are bent up and carried over the support to the point of inflection in the adjoining girder. In addition, two more bars from each side are bent up and hooked into the column merely to assist in resisting diagonal tension stresses. The remaining bars are carried straight through the support a sufficient distance to develop their strength in bond. This arrangement furnishes eight 1-in. round bars in tension and four 1-in. round bars in compression at the support. The center of the upper row of bars at the support is brought to within $1\frac{1}{2}$ in. of the top of the slab as planned above. The effective depth at this point is therefore 33.5 in., and the value of d' is 2.5 in. since only one row of steel remains at the bottom at the support (see Fig. 64).

Investigating the unit stresses over the support,

$$\frac{d'}{d} = \frac{2.5}{33.5} = .075 \text{ and } p' = .5p = \frac{3.14}{15 \times 33.5} = .0062$$

From Diagram 8, $k = .400$ and $j = .883$

$$f_s = \frac{2{,}870{,}000}{6.28 \times .883 \times 33.5} = 15{,}500 \text{ lb. per sq. in.}$$

$$f_c = 15{,}500 \times \frac{.400}{15(1 - .400)} = 690 \text{ lb. per sq. in.}$$

$$f'_s = 15{,}500 \times \frac{.400 - .075}{1 - .400} = 8410 \text{ lb. per sq. in.}$$

The maximum unit bond stress on the tension bars at the point of maximum shear is

$$u = \frac{52{,}450}{25.1 \times .883 \times 33.5} = 71 \text{ lb. per sq. in.}$$

The inclined bars are so placed as to take as much of the diagonal tension as possible and thus keep the number of stirrups required a minimum. The amount of diagonal tension to be provided for by the web reinforcement is represented by the trapezoid $ABCD$ (Fig. 64), the length of which equals $\frac{1}{3} \times 21 \times 12$ or 84 in., and the parallel sides of which are

$$AB = \frac{52{,}450 - (40 \times 15 \times .883 \times 33.5)}{.883 \times 33.5} = 1170 \text{ lb. per lin. in.}$$

and

$$CD = \frac{34{,}750 - (7 \times 500)}{.940 \times 32.5} = 1020 \text{ lb. per lin. in.}$$

Since the unit shear to the right of the first concentrated load is

$$v = \frac{52.450 - (7 \times 500) - 47{,}200}{15 \times .940 \times 32.5} = 4 \text{ lb. per sq. in.,}$$

no web reinforcement is required between the two loads.

Assume that the point of zero positive moment occurs at a distance of $\frac{2}{3}$ of $\frac{l}{3}$ measured toward the support from the point of application of the concentrated load. This is justified by a study of bending moments in continuous beams with concentrated loads at the third points of the span. Consider the positive moment diagram to be a straight line between the maximum and zero values. Then the first pair of bars may be bent up at a distance of

$$\frac{1}{4} \times \frac{2}{3} \times \frac{21 \times 12}{3} = 14 \text{ in. to left of point } C.$$

The next pair may be bent at a point 28 in., and the third pair 42 in. from C.

The same assumption for the negative moment variation may be made as for positive moment, the point of zero moment, however, being $\frac{2}{3}$ of $\frac{l}{3} = 56$ in., measured toward the concentrated load from the support.

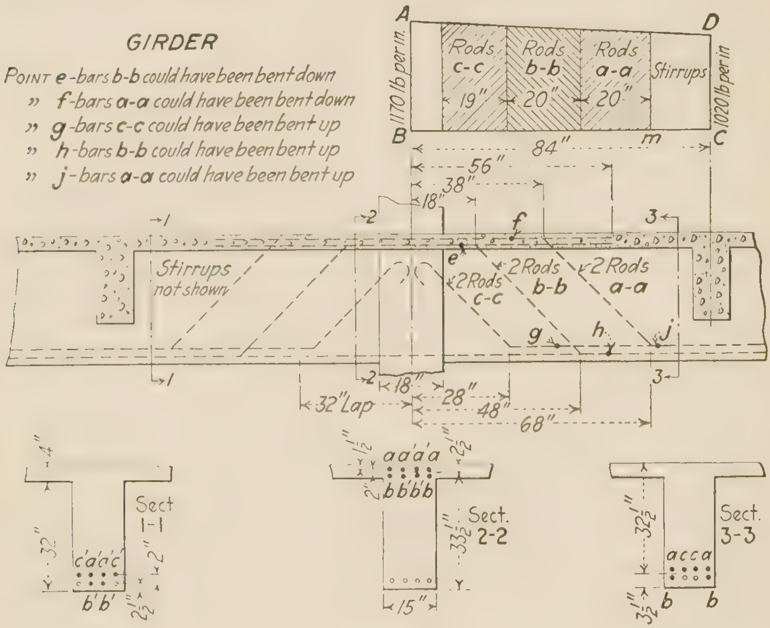


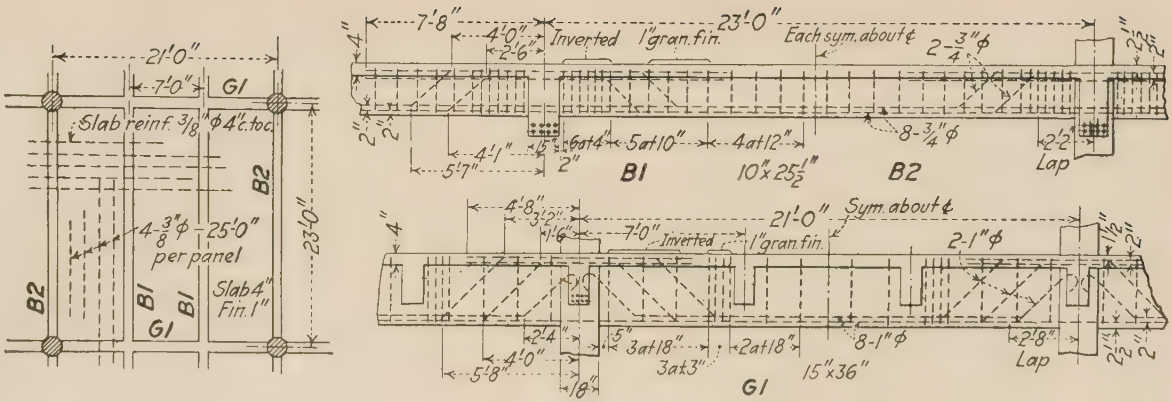
FIG. 64.

Assume the eight bars over the support to be stressed equally. Two of them may be bent down at a point

$$\frac{1}{4} \times \frac{2}{3} \times \frac{21 \times 12}{3} = 14 \text{ in.}$$

and two more at a point 28 in. from the center of support. The theoretical distances are increased by 2 in. in order to allow for any variations in the moments as assumed above.

In order to stress the bent-up bars equally, they are bent up at about equal spacings, since the shear diagram, which is a measure of diagonal tension, is approximately a rectangle. Taking into account the allowable points of bending, and assuming the width of column, inside of which no web reinforcement is required, as 18 in., the bars are bent, at 45 degrees, as shown



BEAM STEEL BENDING SCHEDULE

SIZE	LENGTH	MARK	NO PER BEAM	NO PER PANEL	DETAILS
$\frac{3}{4}$" φ	33'-6"	B1 B2	4 4	12	21'-1" 1'-7" 10'-2"
$\frac{3}{4}$" φ	33'-6"	B1 B2	4 4	12	19'-7" 1'-7" 11'-8"
1" φ	29'-4"	G1	4	4	19'-8" 2'-6" 6'-2"
1" φ	27'-4"	G1	4	4	3" rad 13'-0" 2'-6" 7'-10"

STIRRUP SCHEDULE

SIZE	LENGTH	MARK	NO. PER BEAM	NO. PER PANEL	DETAILS
$\frac{3}{8}$" φ	5'-0"	B1 B2	18 18	54	1'-10" o. to o. 7" inside 4"±
$\frac{3}{8}$" φ	5'-0" (inverted)	B1 B2	14 14	42	1'-10" o. to o. 7" inside 4"±
$\frac{3}{8}$" φ	7'-3"	G1	12	12	2'-9" o. to o. 12" inside 4"±
$\frac{3}{8}$" φ	7'-3" (inverted)	G1	6.	6	2'-9" o. to o. 12" inside 4"±

FIG. 65.

in Fig. 64. It should be noted that the allowable spacing of $\frac{45}{45 + 10} \times d = 26$ in. has not been exceeded, and that the pairs of bars are not overstressed by the inclined tension (see Art. 76).

Stirrups are required between the concentrated load and the point of bending of the first pair of bars, the required spacing being governed by the external shear at the point m. Use ⅜-in. round U-stirrups.

$$s = \frac{2 \times .1104 \times 16{,}000 \times .940 \times 32.5}{\left(52.450 - \frac{68}{12} \times 500\right) - (40 \times 15 \times .940 \times 32.5)} = 3.4 \text{ in.}$$

It would be advisable to use ⅜-in. round U-stirrups, spaced at about 18 in., over the remaining portions of the girder to assist in securing unity of action of the two parts of the Tee, as shown in Fig. 65.

The bent bars are continued 56 in. beyond the center of the column, and the straight bars

$$\frac{690 \times 15}{4 \times 80} \times 1 = 32 \text{ in. beyond the same point.}$$

Complete details for the typical bay designed above are shown in Fig. 65.

Flat Slab Buildings

163. Description of General Type. A flat slab floor, as its name implies, is one consisting of a reinforced concrete floor slab built monolithic with the columns and supported directly by the columns without the aid of beams and girders. The slab may be of uniform thickness throughout the entire floor area, or a part of it, symmetrical about the column, may be made somewhat thicker than the rest of the slab, the thickened portion of the slab thus formed constituting what is known as a dropped panel, or drop (see Fig. 67).

The columns themselves in practically all cases flare out toward the top, forming a capital of a shape somewhat similar to an inverted truncated cone. This capital gives a wider support for the floor slab, which results in a decrease in the bending moment

which the slab is called upon to resist, and a decrease in the punching shear to be taken by the concrete, and tends to a more rigid structure. Modifications of the preceding general arrangement are made whenever called for by any peculiarities of the problem in hand.

164. Advantages of Flat Slab Buildings. Structurally, the flat slab type of building has many advantages over the ordinary beam and girder type. The most important of these may be enumerated as follows:

1. For ordinary spans with heavy loads, under average conditions, the flat slab type is more economical than the beam and girder floor.

2. In a multi-storied building, the same number of stories of a given clear height may be obtained with a smaller total building height. This is well illustrated by a comparison of the designs in Arts. 159 and 168.

3. The slab formwork is much simplified.

4. The flat slab type, owing to the lack of many sharp corners, is better able to resist continued exposure to fire than the beam and girder type. It has been found by actual experience that the worst damage caused to reinforced concrete by severe fires has occurred at places where there may be spalling, that is, at exposed edges and sharp corners.

5. Automatic sprinkler protection may be made more complete under a flat slab floor since the nozzles may be placed well up near the under side of the slab without obstruction to the path of the spray.

6. More light may be admitted into the building if desired, by placing the wall beams above the floor level, and thus allowing the windows to be extended to the under side of the slab. The absence of deep beams and girders also removes the obtruction to the passage of light within the building.

7. Due to the large number of smaller rods extending in several directions over the entire area of the floor, the danger of sudden failure or collapse is less than in the beam and girder type of floor. The relatively large breadth of structure also makes the effect of local variations in the concrete less than would be the case for narrow members like beams.

8. The opportunity for inspecting the position of the reinforcement is excellent, and the conditions attending deposition and placing of the concrete are favorable to securing uniformity and soundness in the concrete.

165. Analysis of Stresses in Flat Slab Floors. The theoretical analysis of stresses in a flat slab floor is rather complicated and based on numerous assumptions. At best, such an analysis can give only approximate results. The analyses which have been made have proved to be ultraconservative as compared to the results of tests. It should be borne in mind, however, that results of tests are apt to be exceptionally high, due to the fact

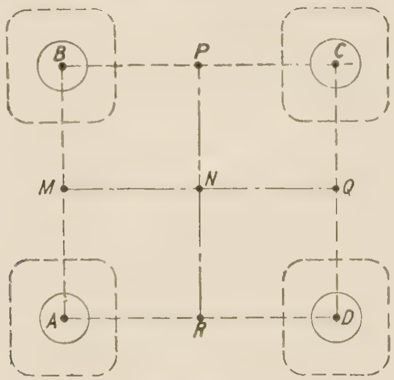


FIG. 66.

that excessive care may have been taken in preparing the panels—a condition not approximating actual practice. By comparing the stresses determined by a sound theoretical analysis with those obtained by actual tests, a method of design may be devised which will give rational and safe results.

This process has resulted in the development of a number of empirical methods of design as typified by the Flat Slab Regulations of the Special Committee of the American Society of Civil Engineers, of the American Concrete Institute, and in the building codes of various cities.

Figure 66 represents a portion of a flat slab floor including four column supports, equally spaced, the load on the floor being uniformly distributed. The full circles represent the column heads underneath the slab. It is evident that, considering any radial

line from the column center, the curvature of the slab along this line will be convex upward for a certain distance, then concave upward, then convex upward again. This implies that at some point along each radial line there is a point of inflection where the radial bending moment changes from positive to negative. The locus of all these points may be represented by the dotted approximate circles centered about the column capitals.

As the slab is loaded, deflection occurs. The point N at the mid-point of the panel, being the farthest away from the support, will deflect more than a point M, P, Q, or R midway between any two adjacent columns. The points M, P, Q, R will therefore be higher than point N but lower than the supports. This results in a negative moment along the line MQ at M, and a positive moment at N. The condition is similar along line PR. As described by Turneaure and Maurer, "There will exist ridges along the lines AB, BC, CD, and DA, with low points or saddles at the center points. The moments transverse to these ridges are negative at all points."

After the total amount of moment existing in the slab has been computed the proportion of the total resistance that exists as positive moment and that as negative moment must be determined. These proportions will vary somewhat with the design of the slab. The Joint Committee, in their recommendation, consider $2/3$ of the total as negative, and $1/3$ as positive moment.

With reference to variations in stress along the sections, it is evident from conditions of flexure that the resisting moment is not distributed uniformly along either the sections of positive moment, or those of negative moment. This may be illustrated by a consideration of the negative moment existing normal to a vertical plane AB. A strip 1 ft. wide along the line BC may be expected to be more rigid than a similar strip along the line MQ. The amount of negative moment taken by the strip BC at B is greater than that taken by the strip MQ at M. The same analysis holds true of the positive moments perpendicular to a vertical plane through PR.

As the law of distribution is not known definitely, it is necessary to make an empirical apportionment along the sections. For purposes of computation each panel of the slab is usually divided

into sets of strips such as strip A and strip B in Fig. 67. Strips A extend from column to column and have a width equal to $\frac{l}{2}$ centered about the column line; strips B occupy the space between strips A and likewise have a width of $\frac{l}{2}$. The strips are for purposes of design only and are not necessarily the boundary lines of any steel used.

The positive moment portion of strip A is usually called the Outer Section, and the negative moment portion the Column-

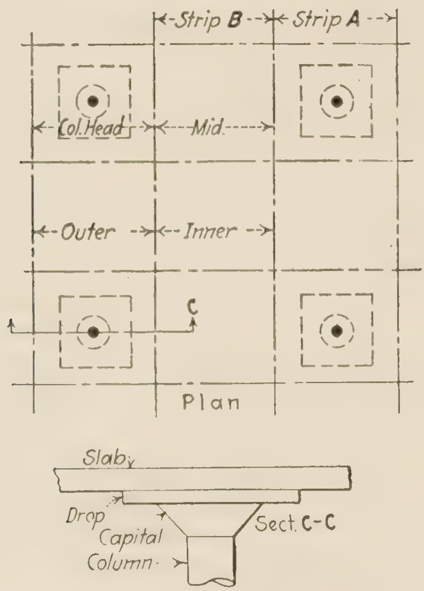


FIG. 67.

head Section. The positive moment portion of strip B is called the Inner Section, and the negative moment portion the Mid-section. The total negative moment existing in the slab is apportioned between the column head and mid-sections, and the total positive moment between the outer and inner sections. As mentioned above, the column-head section furnishes more of the negative resisting moment than the mid-section, and the outer section provides for more of the positive moment than the inner section.

The exact amounts of moment, both positive and negative, to be taken by each strip are given in the various building codes and technical committee recommendations. A typical and very satisfactory municipal building code governing flat slab construction is that of New York City; this code is given in Appendix C.[3]

At right angles to the strips mentioned above, the panel is divided into similar strips A and B, having the same widths and relations to the column lines, and with the same designation of moment sections as the above strips.

166. Methods of Reinforcing Flat Slab Floors. There are, in the main, four different methods or systems of reinforcing the slab in this type of floor: (1) *Two-way system*, (2) *Four-way system*, (3) *Three-way system*, (4) *Circumferential system.*

In the two-way system small bars are placed parallel to the lines of columns over the entire area of the floor at small intervals. The maximum spacing allowable varies in the different codes and specifications, but is seldom greater than one and one-half times the thickness of the slab, or greater than 12 in.

The four-way system consists in two main bands of steel running parallel to the lines of columns, each band centered about the column lines, and two diagonal bands of sufficient width to fill up the floor area left uncovered by the direct bands. In some cases short bars are placed near the top of the slab at right angles to the direct bands over the middle portion of the band to resist the negative moment over that portion; these latter bars constitute what are known as the across-direct bands.

The three-way system involves a special arrangement of columns, such that the lines connecting their center lines form a series of equilateral triangles. The reinforcement then follows the sides of these triangles, each band being centered about one of the panel sides. The three-way system is peculiarly adapted

[3] The New York City Flat Slab Regulations fix $\frac{1}{17}Wl$ as the total moment in either rectangular direction, in which W is the total load on the panel and l the average span of the panel. For slabs with drops, one-third of this is considered positive and two-thirds negative. For slabs without drops, four-tenths is considered positive and six-tenths negative. These moments are then apportioned between the various sections, both positive and negative, according to the relative stiffnesses of the sections, as indicated in Appendix C.

to such structures as car barns, garages, etc., on account of the large radius of curvature permitted by the arrangement of columns.

In the circumferential system, radial steel emanating from the column head, and circumferential steel in the form of concentric rings symmetrical about the column head are used. Concentric rings are also placed about the mid-point of the slab, and about the mid-points of the four edges of each panel. The three-way and circumferential systems are not in such general use as the two- and four-way systems. Of the last two, the four-way system is the more theoretically correct, while the two-way system is simpler in design and construction.

167. Factors to Be Considered in the Design of Flat Slab Buildings. Flat slab floors are ordinarily designed to carry only a uniform load over the entire surface, the assumption being that no breaks in the continuity occur. Where heavy concentrated loads are to be sustained in addition to the uniform load, beams should be introduced in such positions as will enable them to carry the weight of the concentrations. Where openings in the slab occur, they should be framed by beams which will have the effect of restoring continuity to the slab. These beams should be designed to carry a portion of the floor load in addition to any concentrated loads that may rest upon them.

The columns should be designed to provide for bending stresses such as might be caused by unequally loaded panels. This is especially important in the exterior columns where both the dead and live loads cause continual bending, and where the direct loads are relatively small. The interior roof columns are not likely to be subjected to such eccentric loading, and the ratio of the possible bending stress to the direct load stress decreases as the number of floors to be supported increases. Hence, bending in the interior columns is not so important as in the exterior columns. It should be investigated, however, especially in the upper stories. The amount of bending moment to be assumed is usually stated in the various regulations governing flat slab design. The spacing of columns is governed by practically the same factors as in the case of the beam and girder type.

To provide proper drainage, a slight pitch may be given to the roof slab without any change in the theoretical computations. Sudden changes in slope, or steps, on the other hand, require special attention.

It should be remembered at all times that careful compliance with the building code pertaining to the place of construction is not only necessary to the acceptance of the design, but is also conducive to safety. As in all other types of construction, failures of flat slab buildings have occurred. The causes of such failures may in most cases be traced to one or more of the following:

1. Strong commercial competition leading to the tendency to use thinner sections than good design dictates, especially in the absence of good building codes.

2. Faulty construction, such as poor mixing of concrete, inaccurate placing of the steel, too early removal of forms, or placing of concrete in freezing weather.

3. Overloading of the floors beyond the load allowed in the design.

4. Faulty design, due to a lack of knowledge on the part of the designer.

The first essential of a safe and economical design is the removal of all agencies such as are stated above, that might lead to failure, or on the other hand, to needless waste.

168. Design of Flat Slab Building. The method of design of flat slab floors as well as other details involved in a reinforced concrete building is illustrated below in the complete design of a flat slab building, 66 × 105 ft. in plan, consisting of two stories and a basement. The height of the upper stories is 12 ft.-0 in. and that of the basement 10 ft.-0 in. The floor plan is shown in Fig. 78. The live load to be supported by the floors is 200 lb. per sq. ft., and by the roof 40 lb. per sq. ft. A 1-in. granolithic finish is to be considered in the dead weight of the floors, but this finish is not to be assumed as part of the effective slab thickness. An additional dead load of 40 lb. per sq. ft. is to be allowed for in the design of the roof to provide for a cinder concrete surface to prevent condensation. Adequate drainage will be provided by inclining the exterior slabs in the

short direction of the building and by varying the thickness of the surfacing over the middle panel. The New York City Flat Slab Regulations (see Appendix C) will be used in the design of the floor slabs. The specifications to be followed in the design of the other parts of the building are given in the following articles.

Design of Floor and Roof Slabs

Interior Floor Panel

SLAB—*Rules No. 12B and 8.* Average span—21.5 ft.

Assume $t = 9$ in., weight $= 112$ lb. per sq. ft.

Total load on slab including 1-in. finish $= 324$ lb. per sq. ft.

$t = .02l\sqrt{w} + 1 = .02 \times 21.5\sqrt{324} + 1 = 8.8$ in.

$$\tfrac{1}{32} \times 21.5 \times 12 = 8.1 \text{ in.}$$

The assumed thickness of 9 in. is satisfactory.

CAPITAL—*Rule No. 6*

$.225 \times 21.5 \times 12 = 58$ in. Assume diameter of capital $= 5$ ft.-0 in.

DROP—*Rule No. 7*

$.33 \times 21.5 \times 12 = 85$ in. Assume drop 7 ft.-6 in. square.

$.33 \times 9 = 2.87$ in. Assume drop 3 in. thick.

SHEARING STRESSES—*Rule No. 4*

Unit shearing stress on bjd section around drop not to exceed 60 lb.

$V = 324[(21 \times 22) - (7.5)^2] = 131{,}500$ lb.

Assuming the distance to the center of gravity of steel as $1\tfrac{1}{2}$ in.

$$v = \frac{131{,}500}{4 \times 7.5 \times 12 \times \tfrac{7}{8} \times 7.5} = 56 \text{ lb. per sq. in.}$$

Unit shearing stress on bd section around column capital not to exceed 120 lb.

$$V = 324\left[(21 \times 22) - \frac{\pi \times 5^2}{4}\right] = 143{,}000 \text{ lb.}$$

$$v = \frac{143{,}000}{\pi \times 60 \times 10.5} = 72 \text{ lb. per sq. in.}$$

The assumed dimensions of capital and drop need no revision.

BENDING MOMENTS—*Rule No. 12A*

$W = 324 \times 21 \times 22 + \frac{3}{12} \times 150 \times \overline{7.5}^2 = 152{,}000$ lb.

$l = 21.5 \times 12 = 258$ in.

$Wl = 152{,}000 \times 258 = 39{,}200{,}000$ in.-lb.

Column-head section moment $= -\frac{1}{32} \times 39{,}200{,}000 = -1{,}222{,}000$ in.-lb.

Mid-section moment $= -\frac{1}{133} \times 39{,}200{,}000 = -295{,}000$ in.-lb.

Outer-section moment $= +\frac{1}{80} \times 39{,}200{,}000 = +491{,}000$ in.-lb.

Inner-section moment $= +\frac{1}{133} \times 39{,}200{,}000 = +295{,}000$ in.-lb.

REINFORCEMENT—*Rule No. 9*

Inner section: Assuming ½-in. bars and ¾-in. clear insulation, the effective depth of the upper row of bars is $9 - 1\frac{1}{2} = 7\frac{1}{2}$ in.

$A_s = \dfrac{295{,}000}{16{,}000 \times .874 \times 8} = 2.63$ sq. in. Fourteen ½-in. rounds.

Mid-section: Assuming ½-in. bars and ¾-in. clear insulation, the effective depth is $9 - 1 = 8$ in.

$A_s = \dfrac{295{,}000}{16{,}000 \times .874 \times 7.5} = 2.81$ sq. in. Fourteen ½-in. rounds.

The mid-section steel is furnished by bending up one-half of the inner-section bars from the adjacent panels.

Outer section: Assuming ¾-in. bars and ¾-in. clear insulation, the effective depth is $9 - 1\frac{1}{8} = 7\frac{7}{8}$ in.

$A_s = \dfrac{491{,}000}{16{,}000 \times 874 \times 7.87} = 4.45$ sq. in. Ten ¾-in. rounds.

Column-head section: Assuming ¾-in. bars and ¾-in. clear insulation, the effective depth of the lower row of bars is $12 - 1\frac{7}{8} = 10\frac{1}{8}$ in.

$A_s = \dfrac{1{,}222{,}000}{16{,}000 \times .862 \times 10.12} = 8.76$ sq. in. Twenty ¾-in. rounds.

Ten of these are furnished by bending up one-half of the outer-section bars from each adjacent panel. The other ten are straight bars placed in the top over the column head, of a length equal to $.6l$

FIBER STRESS IN CONCRETE—*Rules No. 4b and 7*

At the column-head section:

$$p = \frac{20 \times .4418}{90 \times 10.12} = .0097$$

From Table V, $k = .412$ and $j = .863$.

$$f_c = \frac{2 \times 1{,}222{,}000}{.412 \times .863 \times 90 \times 10.12^2} = 746 \text{ lb. per sq. in.}$$

The thickness of drop, therefore, needs no revision, since 750 lb. per sq. in. is allowed at this section. At all other sections, the moment is so much less and the effective width so much greater than at the column-head section that further investigation for the compressive stresses in the concrete may be omitted. The actual stresses are well below the allowable value of 650 lb. per sq. in.

Exterior Floor Panel—Rule No. 12C

Inner section:

$A_s = 2.81 \times 1.2 = 3.37$ sq. in. Eighteen ½-in. rounds.

Outer section:

$A_s = 4.45 \times 1.2 = 5.34$ sq. in. Twelve ¾-in. rounds.

Mid-section at first interior row of columns:

$A_s = 2.63 \times 1.2 = 3.15$ sq. in. Sixteen ½-in. rounds.

Mid-section at wall:

$A_s = 3.15 \times .5 = 1.58$ sq. in. Nine ½-in. rounds.

Column-head section at the first interior row of columns:

$A_s = 8.76 \times 1.2 = 10.5$ sq. in. Twenty-four ¾-in. rounds.

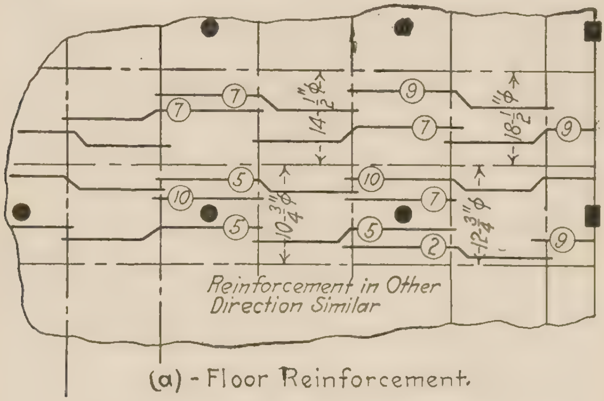
Column-head section at wall:

$A_s = 10.5 \times .8 = 8.40$ sq. in. Nineteen ¾-in. rounds.

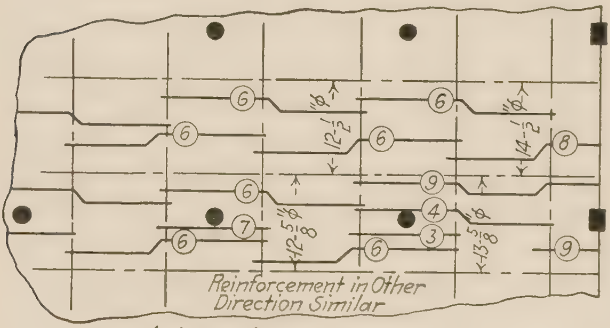
The arrangement of bars that is used to satisfy the above requirements and Rule 9 of the code is shown in Fig. 68*a*.

Complete steel details are shown in Fig. 83.

The method of detailing shown in Fig. 83 bends practically every bar at one end only. An alternate method consists in bending one-half of the positive moment steel at both ends, leaving the remaining one-half straight. The additional negative moment steel is then furnished by short straight bars as necessary. Figure 83*a* gives the complete steel details for the above floor under this method.



(a) - Floor Reinforcement.



(b) Roof Reinforcement

FIG. 68.

A third method consists in bending all of the positive moment steel at both ends, lapping over the negative moment portion as required. The bending should be done in at least two places so as to prevent the formation of a continuous crack along the line of bending. In most cases under this system the use of additional straight bars in the negative moment portions becomes unnecessary.

The bent bars extend to the point of inflection, $.3l$ beyond the column line. The bends are made approximately at the quarter point of the span.

Interior Roof Panel

SLAB—*Rules No. 12B and 8*

Average span, 21.5 ft. Assume $t = 6\frac{1}{2}$ in.; weight = 81 lb. per sq. ft.

Total load, including surfacing = 161 lb. per sq. ft.

$$t = .02 \times 21.5\sqrt{161} + 1 = 6.47 \text{ in.}$$

$$\tfrac{1}{40} \times 21.5 \times 12 = 6.46 \text{ in.}$$

CAPITAL—From floor design, diameter of capital = 5 ft.-0 in.

DROP—The drop will be 7 ft.-6 in. square, and 3 in. thick to correspond with that under the lower floor slabs.

SHEARING STRESSES—*Rule No. 4*

Unit shearing stress on bjd section around drop not to exceed 60 lb.

$V = 161[21 \times 22 - (7.5)^2] = 65{,}500$ lb.

Assuming the distance to center of gravity of steel to be $1\frac{3}{8}$ in.,

$$v = \frac{65{,}500}{(4 \times 7.5 \times 12) \times 5\frac{1}{8} \times \frac{7}{8}} = 41 \text{ lb. per sq. in.}$$

Unit shearing stress on bd section around column capital not to exceed 120 lb.

$$V = 161\left[(21 \times 22) - \frac{\pi \times 5^2}{4}\right] = 71{,}200 \text{ lb.}$$

$$v = \frac{71{,}200}{\pi \times 60 \times 8.12} = 46 \text{ lb. per sq. in.}$$

BENDING MOMENTS—*Rule No. 12A*

$W = 161 \times 21 \times 22 + \frac{3}{12} \times 150 \times \overline{7.5}^2 = 76{,}500$ lb.

$l = 258$ in.

Column-head section moment	= − 617,000 in.-lb.
Mid-section moment	= − 148,500 in.-lb.
Outer-section moment	= + 247,000 in.-lb
Inner-section moment	= + 148,500 in.-lb.

REINFORCEMENT—*Rule No.* 9

As in the computations for the steel area in the floor slab,

Inner section: $A_s = 2.13$ sq. in. Twelve ½-in. rounds.

The extra bar is added to simplify the steel detailing in the present case.

Mid-section: $A_s = 1.93$ sq. in. Twelve ½-in. rounds.

These bars are furnished by bending up one-half of the inner section bars from the adjacent panels.

Outer section: $A_s = 3.25$ sq. in. Twelve ⅝-in. rounds.

Column-head section: $A_s = 5.72$ sq. in. Nineteen ⅝-in. rounds.

Twelve of these are furnished from the adjacent outer sections; the remaining seven are extra straight bars.

FIBER STRESS IN CONCRETE—*Rules No.* 4*b and* 7

At the column-head section:

$$p = \frac{5.84}{90 \times 7.81} = .0083 \qquad k = .388 \text{ and } j = .871$$

$$f_c = \frac{2 \times 617{,}000}{.388 \times .871 \times 90 \times (7.81)^2} = 665 \text{ lb. per sq. in.}$$

Exterior Roof Panel—Rule No. 12*C*

Outer section:

$A_s = 3.25 \times 1.2 = 3.9$ sq. in. Thirteen ⅝-in. rounds.

Inner section:

$A_s = 2.13 \times 1.2 = 2.56$ sq. in. Fourteen ½-in. rounds.

Mid-section at first interior row of columns:

$A_s = 1.93 \times 1.2 = 2.31$ sq. in. Twelve ½-in. rounds.

Mid-section at wall:

$A_s = 2.31 \times .5 = 1.16$ sq. in. Eight ½-in. rounds.

These bars will be considered satisfactory even though the spacing (about 15 in.) is somewhat large. This number is available from the inner section.

Column-head section at first interior row of columns:

$A_s = 5.72 \times 1.2 = 6.86$ sq. in. Twenty-two ⅝-in. rounds.

Column-head section at wall:

$A_s = 6.86 \times .8 = 5.48$ sq. in. Eighteen ⅝-in. rounds.

The arrangement of bars is shown in Fig. 68*b*.

Complete steel details are shown in Fig. 84.

169. *Design of Interior Columns.* The interior columns are to be of 2500-lb. concrete, round, with spirals, and designed in accordance with the recommendations of the Joint Committee (see Chap. V). In selecting the size of columns required for the given direct load, allowance must be made for the stresses caused by the bending due to unequally loaded panels, as stated in the Flat Slab Regulations (Rule 12*D*). The minimum diameter of interior column permitted by the regulations (Rule 5) is governed in this case by $\frac{1}{15}$ of the average span of slab supported by the column; $\frac{1}{15} \times 21.5 \times 12 = 17.2$ in.

The fundamental principles in Chaps. IV and V should be reviewed and their application to the following design noted. In flat slab construction, the unsupported length of a column is equal to the clear distance from the floor to the bottom of the capital. In spiral columns, only that area enclosed within the spirals is considered effective in resisting stress; the diameter d of this effective core area is taken as the distance center to center of spiral wire. Two inches of insulation outside of this core area are allowed in the following design. The longitudinal steel is placed directly inside of the spirals, making the diameter of the circle on which the steel is placed equal to the effective diameter d, less one diameter of spiral and one diameter of the longitudinal bar. This diameter will vary from $\frac{3}{4}$ to $1\frac{3}{4}$ in. less than the effective diameter d, and will, for the average column, be approximately 1 in. less than the effective diameter. The moment of inertia of the steel may therefore, without appreciable error, be taken from Table XII. If a more exact solution is desired, the moment of inertia of the steel may be found from the equation $I_s = \frac{A_s d'^2}{8}$, in which d' is the diameter of the circle on which the steel is placed.

The assumed sizes of columns and areas of steel are given in the following table. The spirals are proportioned so as to furnish a volume equal to one-fourth the volume of the longitudinal steel, as required by the Joint Committee; the maximum pitch of spirals is given by the Committee as one-sixth of the core diameter, but not to exceed 3 in. in any case. Table XIV may be used in selecting the required size and spacing of spirals.

Floor	Load from	Amount of load	Effective Diameter	Vertical steel	Per cent	Spirals
Second	Roof Column Capital	76,500 3,000 2,000	14 in.	5 – ⅝ in. rounds	1.0	¼ in. @ 2¼ in.
	Total	81,500				
First	Floor Column Capital	152,000 5,200 1,600	20 in.	10 – 1 in. rounds	2.5	5/16 in. @ 2¼ in.
	Total	240,300				
Basement	Floor Column Capital	152,000 5,800 1,400	24 in.	10 – 1 in. rounds	1.7	5/16 in. @ 3 in.
	Total	399,500				

The 1-in. vertical bars from the lower columns are extended 2 ft.-0 in. above the floor level to lap with those of the column next above.

170. *Investigation for Bending Stresses.* According to Rule 12*D*, any two superimposed columns must withstand a bending moment caused by unequally loaded panels of $\frac{1}{40} W_1 l$, in which W_1 is the total live load on one panel and l the average span of the slab supported by the lower column. The two columns are to resist this bending moment in direct proportion to their values of $\frac{I}{h}$, in which I is the total moment of inertia of the column, and h its unsupported height in inches, measured from the top of slab to base of capital. The maximum total stress exists only when the panels at the floor line under consideration are unequally loaded, a full live load occurring on alternate panels. A full live load is assumed on all the floors above the one considered.

TOP OF ROOF COLUMN[4]

$$M = \frac{1}{40} \times 40 \times 21 \times 22 \times 21.5 \times 12 = 119{,}000 \text{ in.-lb.}$$

[4] On account of the improbability of an eccentric live load on the roof, this investigation for the top of the roof column may usually be omitted.

This must be resisted entirely by the roof column. See following tables for values of moments of inertia, etc.

$$f_c = \frac{119{,}200 \times 7}{2243} = 370 \text{ lb. per sq. in.}$$

The simultaneous direct load stress,

$$f_c = \frac{78{,}500 - \dfrac{(40 \times 21 \times 22)}{2}}{153.9 + 11 \times 1.53} = 405 \text{ lb. per sq. in.}$$

The total combined stress = 775 lb. per sq. in.

BOTTOM OF ROOF COLUMN

$M = \frac{1}{40} \times 200 \times 21 \times 22 \times 21.5 \times 12 = 596{,}000$ in.-lb.

$\dfrac{20.1}{20.1 + 105.0} \times 596{,}000 = 96{,}000$ in.-lb. to be resisted by the roof column.

The bending stress

$$f_c = \frac{96{,}000 \times 7}{2243} = 300 \text{ lb. per sq. in.}$$

The direct load stress

$$f_c = \frac{81{,}500}{153.9 + 11 \times 1.53} = 478 \text{ lb. per sq. in.}$$

The total combined stress = 778 lb. per sq. in.

The investigation for the remaining columns follows as above; the results are tabulated in the following tables.

The allowable unit stresses are 780, 960, and 870, for the second floor, first floor, and basement tiers, respectively (see Art. 108). At the bottom of the basement column, since no bending is considered, the allowable stress is 725 lb. per sq. in. The actual stress is but slightly in excess of the allowable, and the design may be considered satisfactory.

Column	I_c in.^{4} (Table XI)	$(n - 1)I_s$ in.^{4} (Table XII)	$I = I_c + (n - 1)I_s$ in.^{4}	h in.	$\frac{I}{h}$
Second......	1,886	357	2,243	112	20.1
First........	7,854	3,900	11,754	112	105.0
Basement....	16,286	5,598	21,884	90	243.2

Column	Point	$A_t = A + (n - 1)A_s$ sq. in.	Effective direct load, lb.	f_c direct	Moment in. lb.	f_c bending	f_c total
Second	Top	170.8	69,260	405	119,200	370	775
	Bottom		81,500	478	96,000	300	778
First	Top	400.6	188,800	473	500,000	425	898
	Bottom		240,300	600	179,000	153	753
Basement	Top	538.8	347,500	645	417,000	228	873
	Bottom		399,500	742	0	0	742

171. *Design of Exterior Columns.* The exterior columns are to be made of 2000-lb. concrete, rectangular in shape, and the longitudinal steel tied together by means of ties spaced 8 in. center to center, the diameter of tie being $\frac{1}{4}$ in. for the upper two tiers and $\frac{5}{16}$ in. for the lower tier, in accordance with the rule suggested in Art. 104. The dimension parallel to the wall is kept constant, that perpendicular to the wall being varied as required. In addition to the load from the floors the upper floor exterior columns must support the weight of the walls enclosing the floor next above. The basement wall is to be of solid concrete 14 in. thick, poured monolithic with the basement columns and first floor slab, and, will, therefore, act as a bearing wall to support the spandrel load from the first floor, thus relieving the basement tier columns of this weight.

It should be noted that with this type of construction, the basement columns are in reality pilasters in the bearing wall, and as such need not be designed to support the entire load from the first floor slab. The amount of bending moment to be taken by these columns is also considerably less than that stated in the flat slab regulations. The following design of the basement columns, which disregards these modifications entirely, is, therefore, very conservative, but is justified by the rigidity of structure obtained at an insignificantly larger cost.

In estimating the weight of the enclosure walls, the wall beams and brick spandrel underneath the windows are assumed 12 in. thick, the spandrel 2 ft.-6 in. deep, and the wall beam 2 ft.-0 in.

deep. The brick parapet wall at the roof is assumed 12 in. thick and 3 ft.-6 in. deep. The weight of windows, including sash, is taken as 8 lb. per sq. ft. The general arrangement of a typical wall panel is shown in Fig. 69.

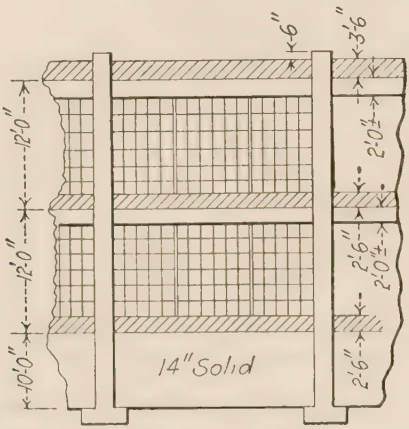


FIG. 69.

The following tables contain a summary of the design. In investigating the stresses induced by eccentric loading, Rule 12*E* of the flat slab regulations requires that the total live load and dead load on one complete panel be substituted for the live load only in the formula $\frac{1}{40}Wl$. The general investigation

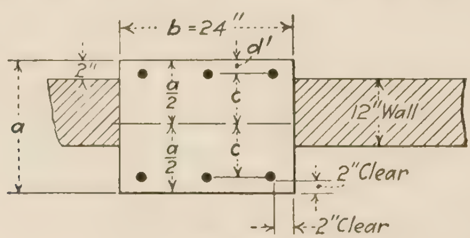


FIG. 70.

is similar to that for the interior columns, the difference in shape necessarily causing certain changes in the determination of the moments of inertia. Since the bending stresses in the exterior columns often exceed the direct load stresses, thus causing tension on one face, the diagrams of Chap. VI permit of easier solu-

tion than the method used in the design of the interior columns. The following tables illustrate their use. Figure 70 illustrates the terms used in the following table for moments of inertia.

Floor	Load from	Amount of load	Size of column	Vertical steel	Per cent	$A_t = A + (n-1)A_s$ sq. in.
Second	Roof Wall Bracket Column[5]	38,200 14,200 500 7,300	24 × 18	6 ¾-in. rounds	0.6	469.2
	Total	60,200				
First	Floor Wall Bracket Column	76,000 12,000 400 6,000	24 × 20	6 ⅞-in. rounds	0.8	530.5
	Total	154,600				
Basement	Floor Bracket Column	76,000 300 6,000	24 × 24	8 1-in. rounds	1.1	664.0
	Total	236,900				

Column	$I_c = \frac{1}{12}ba^3$ in.^{4}	$(n-1)I_s = 14(A_sc^2)$ in.^{4}	$I = I_c + (n-1)I_s$ in.^{4}	h in.	$\frac{I}{h}$
Second	11,700	1,630	13,330	112	119
First	16,000	2,900	18,900	112	169
Basement	27,650	5,950	33,600	90	374

In computing the stress at the top of the basement column, the value of d' was assumed 2½ in., *i.e.*, from the face of the column parallel to the wall to the center of the three bars parallel to that face (see Fig. 72). While not theoretically correct, it simplifies the computations without introducing an appreciable error, giving but a slightly lower stress, 575 lb. per sq. in., than the true stress, 589 lb. per sq. in., calculated by equation (38), Art. 99.

[5] This value includes the weight of the column above the roof (see Fig. 69).

Column	Point	N lb.	$\frac{d'}{a}$	M in.-lb.	$\frac{e}{a}$	K	k	$\frac{M}{ba^2f_c}$	f_c
Second	Top	55,100	.132	493,000	.496		.45	.106	597
	Bottom	60,200		405,000	.373		.57	.111	468
First	Top	148,600	.122	575,000	.193		.96	.103	580
	Bottom	154,600		305,000	.099	1.41			453
Basement	Top	230,900	.104	675,000	.121	1.40			575
	Bottom	236,900		0					357

The allowable unit stress for all exterior columns where bending is included is .3 × 2000, or 600 lb. per sq. in. The complete column schedule is shown in Fig. 85.

172. *Design of Interior Column Footings.* The interior column footings are to be square, sloped, and reinforced in two directions. The report of the Joint Committee will be followed in the design of all footings. The allowable soil pressure is 3 tons per sq. ft. A 2000-lb. concrete is to be used.

Diameter of round column = 28 in. Total load = 399,500 lb.

Side of equivalent square column = 25 in.

Assume weight of footing as 25,000 lb.

Bearing area required $= \frac{424,500}{6000} = 70.8$ sq. ft. Use base 8 ft.-6 in. square.

Net upward pressure from soil $= \frac{399,500}{72.3} = 5540$ lb. per sq. ft.

Punching shear at perimeter of 28-in. round column is

$$5540(72.3 - 4.3) = 377,000 \text{ lb.}$$

$$d = \frac{377,000}{\pi \times 28 \times 120} = 35.6 \text{ in.} \quad \text{Use 36 in.}$$

Allowing 4 in. of insulation below the center of gravity of the steel, the total height of footing is 40 in. The top of footing is made 3 ft.-0 in. square, and the total thickness at edge, 12

in. The weight of footing is then 25,000 lb. as assumed (see Fig. 72).

The effective depth at a vertical plane 36 in. from the face of the round column is

$$d' = 36 - \tfrac{32}{33} \times 28 = 8.8 \text{ in.}$$

The area outside of this plane $= 72.3 - \left(\frac{100}{12}\right)^2 = 2.7$ sq. ft.

$$v = \frac{2.7 \times 5540}{4 \times 100 \times .875 \times 8.8} = 5 \text{ lb. per sq. in.}$$

From the equation in Art. 131, the bending moment at the edge of the equivalent square column is

$$M = \frac{5540}{2}\left[\frac{25}{12} + 1.2 \times \frac{38.5}{12}\right] \times \left(\frac{38.5}{12}\right)^2 \times 12 = 2{,}030{,}000 \text{ in.-lb.}$$

For each band, $A_s = \dfrac{2{,}030{,}000}{16{,}000 \times .9 \times 36} = 3.92$ sq. in.

For bond, assuming deformed bars hooked at both ends, and allowing an increase of 50 per cent over the allowable bond stress given by the Joint Committee (see Arts. 80 and 133), each band requires

$$\Sigma_o = \frac{\frac{1}{4} \times 377{,}000}{112.5 \times .9 \times 36} = 25.8 \text{ in.}$$

These requirements are satisfied by using twenty ½-in. round deformed bars, hooked at each end, per band. The effective width of footing for each band of steel is

$$28 + 2 \times 36 + \tfrac{1}{2} \times 2 = 101 \text{ in.}$$

The spacing of bars is approximately 5¼ in., a satisfactory value. No additional bars are required outside of the effective width of band.

Ten 1-in. round dowels, 4 ft.-0 in. long, are placed in the footing to lap with the longitudinal bars in the basement column. These dowels are placed so as to project 2 ft.-0 in. above the top of the footing.

173. *Design of Exterior Column Footings.* The exterior column footings on all sides except the front will consist of a solid block, rectangular in plan, and reinforced in two directions.

Column 24 × 24 in. Total load 236,900 lb. Assume weight of footing = 12,000 lb.

$$\text{Bearing area required} = \frac{284{,}900}{6{,}000} = 41.5 \text{ sq. ft.}$$

A square footing would be the most economical type to use in supporting a square column, a typical design for which is given in Art. 138. In order to illustrate the design of a rectangular footing, the size of footing selected for the present case is 6 ft.-0 in. × 7 ft.-0 in.

$$\text{The net upward soil pressure} = \frac{236{,}900}{42.0} = 5650 \text{ lb. per sq. ft.}$$

For punching shear at the perimeter of the column

$$d = \frac{(42.0 - 4)5650}{96 \times 120} = 18.6 \text{ in.} \quad \text{Use 19 in.}$$

Allowing 4 in. of insulation below the center of the steel, the total height is 23 in., and the weight of footing is 12,000 lb. as assumed.

On a vertical plane, 19 in. from each face of the column

$$V = \left[42.0 - \frac{62 \times 62}{144}\right]5650 = 86{,}600 \text{ lb.}$$

and

$$v = \frac{86{,}600}{248 \times .9 \times 19} = 20 \text{ lb. per sq. in.}$$

The maximum moment occurs along the face of column parallel to the 6 ft.-0 in. side of footing.

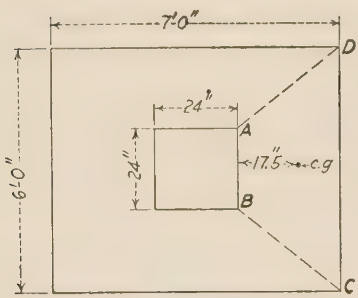


Fig. 71.

For rectangular footings, the Joint Committee requires the upward pressure on the trapezoids contributory to the column face to be concentrated at the center of gravity of the trapezoid in computing bending moment.

The total upward pressure on the trapezoid *ABCD* (Fig. 71),

$$= \frac{5650}{144} \times \frac{24 + 72}{2} \times 30 = 56{,}500 \text{ lb.}$$

The distance from the face AB to the center of gravity of the trapezoid is

$$\frac{24 \times 30 + \frac{2}{3} \times 30 \times (72 - 24)}{72 + 24} = 17.5 \text{ in.}$$

$$M = 56{,}500 \times 17.5 = 990{,}000 \text{ in.-lb.}$$

$$A_s = \frac{990{,}000}{16{,}000 \times .9 \times 19} = 3.61 \text{ sq. in.}$$

$$\Sigma_o = \frac{56{,}500}{112.5 \times .9 \times 19} = 29.3 \text{ in.}$$

To satisfy the above requirements, nineteen ½-in. round deformed bars are used. No additional bars are required outside of the effective beam width because of the small portion of footing remaining on each side. Further revision of the value of j is not attempted because of the approximations which would be involved in determining the percentage of steel. The effective width of footing for this band of steel is

$$24 + 2 \times 19 + 10/2 = 67 \text{ in.}$$

The spacing of the bars is approximately 3½ in., a satisfactory value.

The number of bars required in the beam parallel to the 6-ft.-0 in. side of the footing is governed by the bond stress requirement. The total amount of shear to be resisted by this band is 51,000 lb., and requires

$$\Sigma_o = \frac{51{,}000}{112.5 \times .9 \times 19} = 26.6 \text{ in.}$$

which is furnished by seventeen ½-in. round, deformed bars. The effective width of this band is 73 in. No additional bars are required outside of the effective area (see Fig. 72).

Eight 1-in. round dowels, 4 ft.-0 in. long, are placed in the footing to lap with the longitudinal bars in the basement columns. These dowels are placed so as to project 2 ft.-6 in. above the top of footing.

174. *Design of Cantilever Footings.* To illustrate the fundamental principles involved in the design of a cantilever footing, the front of the building is assumed placed directly on the street line, so that no encroachment beyond the columns is possible. The exterior column footings along that side are therefore, eccentric with respect to the loads they support, and must be tied to the nearest interior column footing by means of a reinforced concrete beam or strap to prevent overturning (see Fig. 74).

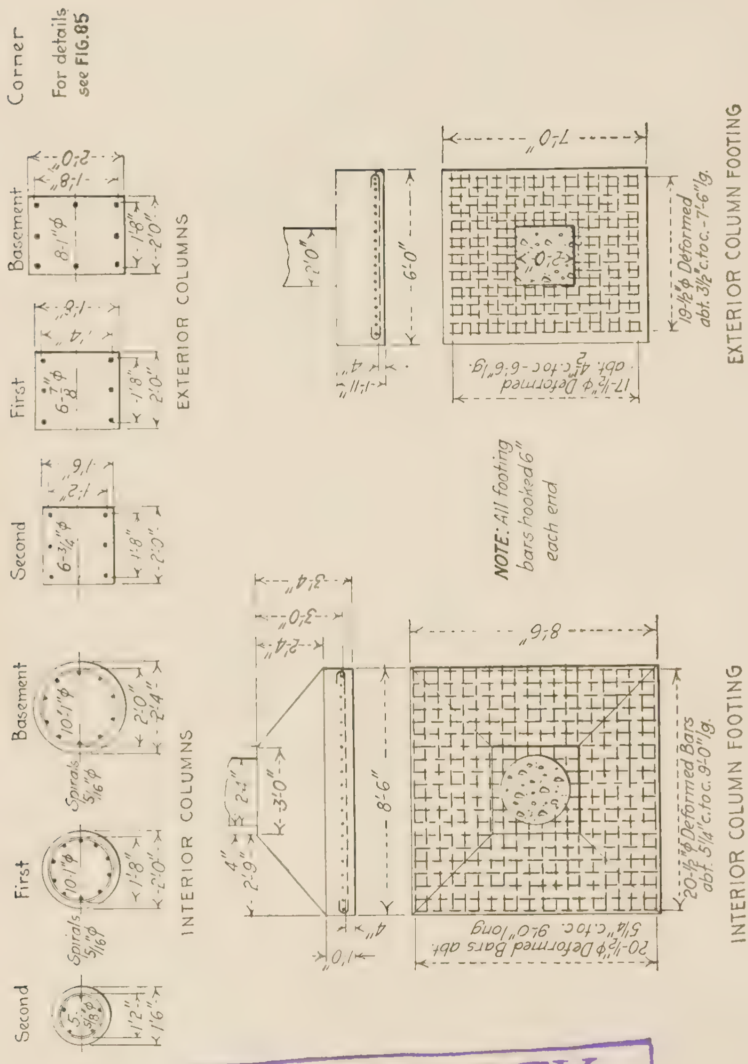

Corner
For details see FIG. 85
Basement
First
Second
EXTERIOR COLUMNS
Spirals
INTERIOR COLUMNS
NOTE: All footing bars hooked 6" each end
17-1/2" φ Deformed abt. 4 1/2" c.to c.-6'-6" lg.
19-1/2" φ Deformed abt. 3 1/2" c.to c.-7'-6" lg.
EXTERIOR COLUMN FOOTING
20-1/2" φ Deformed Bars abt. 5 1/4" c.to c. 9'-0" long
INTERIOR COLUMN FOOTING

Fig. 72.

The design assumes that all of the loads are resisted directly by the strap; the portion under the exterior column is widened and reinforced at right angles to the strap so as to distribute the pressure from the column and strap over an area sufficient to keep the unit soil pressure below the allowable. The total depth of strap and exterior footing is made the same as that of the interior footing to which they connect. The width of exterior footing parallel to the wall is made the same as the corresponding dimension of the interior footing.

The area of wall column footing required for the load on the column $= \dfrac{236,900}{6000} = 39.5$ sq. ft.

Allowing approximately 25 per cent to provide for the weight of the strap and footing a base 8 ft.-6 in. $\times$ 6 ft.-0 in. is selected.

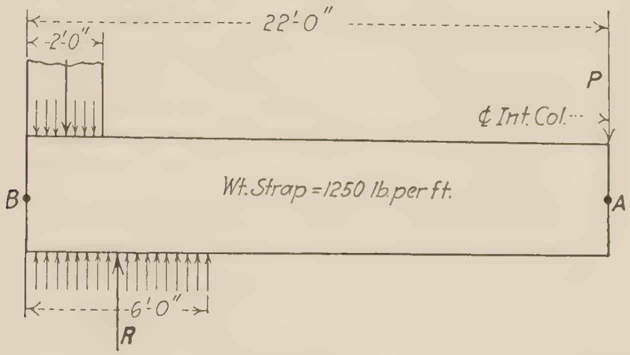


FIG. 73.

Taking the strap out as a free body, the external loads upon it are as shown in Fig. 73. The eccentricity of the exterior column load is resisted by a downward pressure P from the interior column, thus counterbalancing the tendency for the exterior footing to overturn. The load on the exterior column is considered uniformly distributed over the column base, and the upward soil reaction R is assumed uniformly distributed over the 6-ft. length of the exterior footing.

The total height of interior footing is 40 in. The effective depth of the strap is taken as 37 in., thus allowing 3 in. of insula-

tion. Assuming the weight of strap as 1250 lb. per lin. ft., the upward soil pressure R is determined by taking moments about A.

$$-236{,}900 \times 21.0 - 1250 \times 22 \times 11 + R \times 19 = 0,$$

from which $R = 277{,}800$ lb.

The downward pressure P is determined by taking moments about B.

$$236{,}900 \times 1.0 + 1250 \times 22 \times 11 + P \times 22 - 277{,}800 \times 3 = 0,$$

from which $P = 13{,}400$ lb.

The maximum moment occurs at the point where the shear is zero. Zero shear occurs at some point near the interior edge of the exterior footing. Equating to zero the shear at a point x ft. from B,

$$-236{,}900 + \frac{277{,}800x}{6} - 1250x = 0, \text{ from which } x = 5.3 \text{ ft.}$$

The moment at this point is

$$M = -236{,}900 \times 4.3 - 1250 \times \frac{\overline{5.3}^2}{2} + \frac{277{,}800}{6} \times \frac{\overline{5.3}^2}{2}$$

$$= -387{,}500 \text{ ft.-lb. or } -4{,}650{,}000 \text{ in.-lb.}$$

$$bd^2 \text{ required} = \frac{-4{,}650{,}000}{146.7} = 31{,}700 \text{ in.}^3$$

Since $d = 37$ in., $b = 23.2$ in. for moment.

The maximum shear occurs at the inner face of the exterior column.

$$V = -236{,}900 - 1250 \times 2.0 + \frac{277{,}800}{6} \times 2.0 = -146{,}800 \text{ lb.}$$

$$b \text{ required for shear} = \frac{146{,}800}{37 \times \frac{7}{8} \times 150} = 30 \text{ in.}$$

It is reasonable to allow a unit shearing stress of at least 150 lb. per sq. in. since the strengthening action of the projecting portions of the exterior footing has been disregarded in computing the shearing area.

A width of 30 in. is selected for the strap. The actual weight is then 1250 lb. per lin. ft., as assumed. The actual weight of the exterior footing, exclusive of the strap, is 18,000 lb., and

the soil pressure under that footing $\frac{277{,}800 + 18{,}000}{8.5 \times 6} = 5810$ lb. per sq. ft., which is satisfactory.

$$A_s \text{ required in strap} = \frac{4{,}650{,}000}{16{,}000 \times .875 \times 37} = 9.0 \text{ sq. in.}$$

$$\Sigma_o \text{ required in strap} = \frac{146{,}800}{150 \times .875 \times 37} = 30.2 \text{ in.}$$

Nine 1-in. square deformed bars hooked at both ends, placed in one row, the center of which is 3 in. below the top of strap, are selected.

$p = \frac{9.0}{30 \times 37} = .0081$, and $j = .872$, which checks the assumed value sufficiently close to avoid the necessity of revision.

Four of the bars may be cut off at a point 13.0 ft. from the exterior face of the wall column, since at this point the moment is four-ninths the maximum. They are continued 1 ft.-0 in. beyond this theoretical point of cut-off.

Stirrups are required from the inner edge of the exterior column, the point of maximum shear, to the point where the unit shear is 60 lb., that is, x ft. from B, x being determined from the following relation:

$$v = \frac{236{,}900 - \frac{280{,}000}{6}x + 1250x}{30 \times .875 \times 37} = 60$$

from which

$$x = 4.0 \text{ ft.}$$

The required spacing of ½-in. round triple-looped stirrups at the inner edge of the exterior column is

$$s = \frac{6 \times .1963 \times 16{,}000 \times .875 \times 37}{146{,}800 - (60 \times 30 \times .875 \times 37)} = 7.0 \text{ in.}$$

Five stirrups are used, at a constant spacing of 7 in., the first one placed 2 in. from the inner edge of the exterior column.

The unit shearing stress at the inner edge of the exterior footing is

$$v = \frac{13{,}400 + 1250 \times 16}{30 \times .875 \times 37} = 34 \text{ lb. per sq. in.}$$

No web reinforcement other than that mentioned above is required.

The unit load on each of the cantilever portions of the exterior footing, exclusive of its own weight, is

$$\frac{277,800}{8.5 \times 6} = 5450 \text{ lb. per sq. ft.}$$

The maximum moment along the edge of the strap per foot of width is

$$M = 5450 \times 3 \times 1.5 = 24,500 \text{ ft.-lb., or } 294,000 \text{ in.-lb.}$$

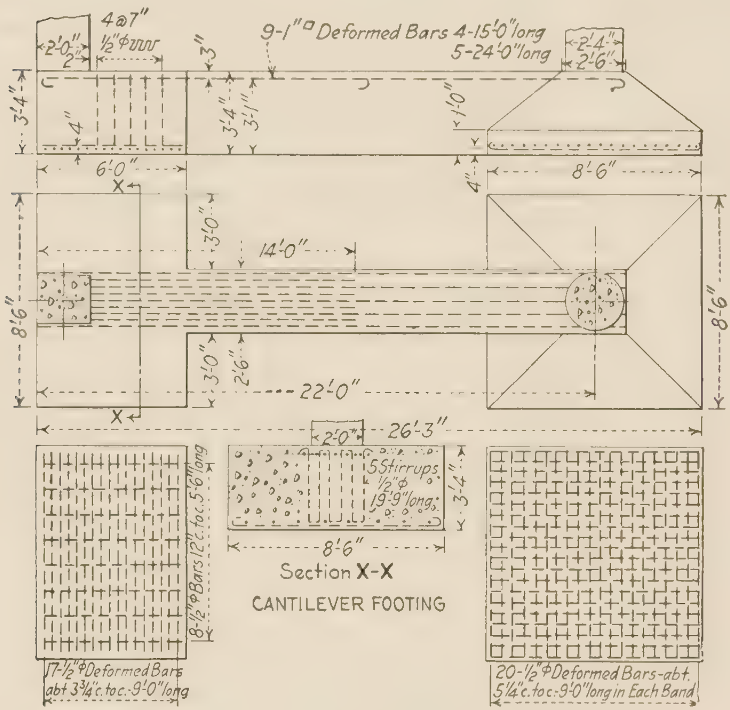


FIG. 74.

The area of steel in the exterior footing, perpendicular to the strap, per foot of width, is

$$A_s = \frac{294,000}{16,000 \times .9 \times 37} = .554 \text{ sq. in.}$$

The total area for the entire footing is then $.554 \times 6 = 3.32$ sq. in., which is furnished by seventeen ½-in. round deformed

bars, hooked at both ends as shown in Fig. 74. The bond stress on these bars is

$$u = \frac{5450 \times 6 \times 3}{17 \times 1.571 \times .9 \times 37} = 110 \text{ lb. per sq. in.}$$

which is satisfactory.

175. *Design of Wall Beams.* With the proposed floor plan as shown in Fig. 78, the unsupported spans of the end wall beams are 18 ft.-6 in. and 17 ft.-6 in., while those of the interior wall beams are 20 ft.-0 in. and 19 ft.-0 in. for the short and long sides of the building, respectively. Since it is desirable to keep the depth of the wall beams constant throughout the entire building on account of architectural appearance, it is first necessary to determine the cross-section required for the maximum shear and moment. The width of beams is taken as 12 in. in all cases. The allowable unit stresses are taken as follows:
$f_s = 16{,}000$, $f_c = 650$, $u = 100$, $v = 120$, $v = 40$, and $n = 15$.

The maximum shear occurs on the second floor interior beams along the short side of the building. According to the New York City Flat Slab Regulations, the wall beams must, in addition to supporting the spandrel load, be capable of providing for 20 per cent of the total live and dead loads on one panel. The load per lineal foot is, therefore, made up as follows:

Brick sill $2\frac{1}{2} \times 1 \times 140$	=	350 lb.
Windows 7.5×8	=	60
Stem of beam, assumed	=	140
Floor load, $.2 \times 324 \times 21$	=	1360
Total	=	1910 lb.

The maximum shear $= 1910 \times 10 = 19{,}100$ lb.

$$\text{The effective depth required} = \frac{19{,}100}{120 \times .875 \times 12} = 15.2 \text{ in.}$$

The maximum moment occurs in the end wall beams on the short side of the building. These beams are continuous over one support only and are designed for a moment of $\frac{1}{10}wl^2$.

$$M = \tfrac{1}{10} \times 1910 \times \overline{18.5}^2 \times 12 = 785{,}000 \text{ in.-lb.}$$

Since the wall beams in reality are T-beams with the Tee on one side only, the effective width of flange, according to the Joint Committee's recommendation, is

$$b = \tfrac{1}{10} \times 18.5 \times 12 = 22 \text{ in.}$$

The approximate depth for moment is determined from equation (26) of Art. 84.

$$M_c = \tfrac{1}{2} f_c bt(d - \tfrac{1}{2}t)$$

$$785{,}000 = \tfrac{1}{2} \times 650 \times 22 \times 9 \times (d - 4.5)$$

from which

$$d = 16.7 \text{ in.}$$

An effective depth of 18 in. is used to furnish a desired exterior appearance to the building. Allowing 3 in. insulation below the center of the steel, the total height of beam is 21 in. With this value of d

$$\frac{M}{bd^2} = \frac{785{,}000}{22 \times 18 \times 18} = 110.5$$

$$\frac{t}{d} = \frac{9}{18} = .5$$

Diagram 3 shows that the neutral axis is in the flange and hence the beam must be designed as a rectangular beam 22 in. wide. The revised d required for moment under these conditions is

$$d = \sqrt{\frac{785{,}000}{107.7 \times 22}} = 18.0 \text{ in.}$$

176. *Design of 2L1 and 2L2.* Since it has been found above that a cross-section of 12 × 21 in. is satisfactory for all wall beams, it is now merely necessary to determine the area of steel required for each beam and fully to provide for shearing stresses.

Assuming $j = .875$, for the end beam, $2L1$,

$$A_s = \frac{785{,}000}{16{,}000 \times .875 \times 18} = 3.1 \text{ sq. in.}$$

This may be provided by four 1-in. round bars. With this amount of steel,

$$p = \frac{3.14}{22 \times 18} = .0079.$$

From Table V, $j = .873$, approximately as assumed.

Before investigating this beam over the support (first interior support) it is necessary to determine the amount of steel in the interior beams. The moment existing in the interior beams $2L2$, is

$$M = \tfrac{1}{12} \times 1910 \times \overline{20}^2 \times 12 = 764{,}000 \text{ in.-lb.}$$

and the area of steel required is

$$A_s = \frac{764{,}000}{16{,}000 \times .873 \times 18} = 3.04 \text{ sq. in.}$$

Since this agrees closely with the area furnished in the end beam, the same selection of rods is made.

Two rods from each beam are bent up to provide for negative moment at the support; the tensile steel furnished at each support is then 3.14 sq. in. and the compressive steel area 3.14 sq. in.

$$\frac{d'}{d} = \frac{3}{18} = .17$$

$$p = \frac{3.14}{12 \times 18} = .0145 = p'$$

$$\frac{M}{bd^2} = \frac{785{,}000}{12 \times 18 \times 18} = 202$$

By interpolation from Diagram 9, f_c = 720 lb. per sq. in. and from Diagram 10, f_s = 16,200, both of which are satisfactory. Since there are only three panels in this side of the building, no other supports need be investigated.

The maximum unit bond stress is

$$u = \frac{19{,}100}{4 \times 3.14 \times \tfrac{7}{8} \times 18} = 100 \text{ lb. per sq. in.}$$

Therefore deformed bars are necessary.

Since only two bars are bent up in each beam, and these at one place, their strength is disregarded in providing for diagonal tension; stirrups are placed at suitable intervals to furnish all of the web strength necessary. In $2L2$

$$x_1 = \frac{20}{2} - \frac{40 \times 12 \times .875 \times 18}{1910} = 6.0 \text{ ft. from edge of column.}$$

The required spacing of $\frac{3}{8}$-in. round U-stirrups at the edge of the column, assuming the distribution of shear as given in the first case of Art. 74, is

$$s = \frac{3}{2} \times \frac{2 \times .1104 \times 16{,}000 \times .875 \times 18}{19{,}100} = 4.4 \text{ in.}$$

Three feet from edge of column

$$s = 4.4 \times \frac{19{,}100}{13{,}370} = 6.2 \text{ in.}$$

The maximum allowable spacing $= .45 \times 18 = 8$ in. Placing the first stirrup 2 in. from the edge of the column, the remaining spacings from each end, as selected, are as follows: 5 at 4 in., 5 at 6 in., and 2 at 8 in.

In $2L1$, $x_1 = \frac{18.5}{2} - \frac{40 \times 12 \times .875 \times 18}{1910}$
5.25 ft. from edge of column.

At the support

$$s = \frac{3}{2} \times \frac{2 \times .1104 \times 16{,}000 \times .875 \times 18}{1910 \times 9.25} = 4.7 \text{ in.}$$

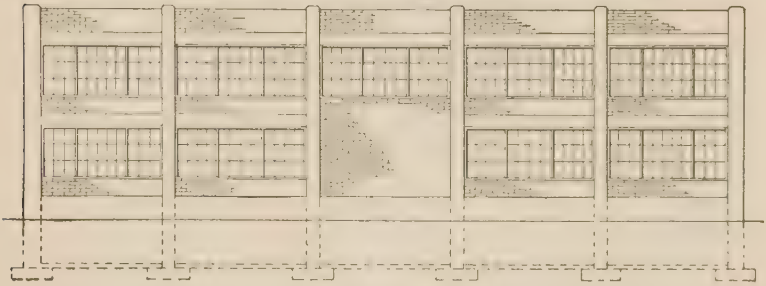
Placing the first stirrup 2 in. from the edge of column, the remaining spacings from each end, as selected are as follows: 4 at $4\frac{1}{2}$ in., 4 at 7 in. and 1 at 8 in.

The point at which the two bars, 50 per cent of the steel, may be bent up is found from Diagram 1, in $2L1$, $.18 \times 18.5 = 3.33$ ft. from the edge of the column. They are bent 3.0 ft. from the column. In $2L2$, the two bars may be bent up $.21 \times 20 = 4.2$ ft. from the edge of the column. They are bent 4.0 ft. from the column. Negative moment is also provided for by this arrangement.

The bent bars are continued along the top into the adjacent beam to the point of inflection, assumed at $\frac{1}{4}$ span in all cases, measured from the outer edge of the columns. The straight bars are continued 2 ft.-6 in. beyond the center of the column.

In a manner similar to the above, the areas of steel required in the other wall beams, floor and roof, and the required arrangement of longitudinal and web reinforcement are determined.

The results are indicated in Fig. 81. The roof wall beams are made the same depth as those at the second floor. In investigating the beams along the long side of the building it should be borne in mind that the first interior supports are designed for $\frac{1}{10}$ moment coefficient, while the other interior supports are designed for $\frac{1}{12}$ coefficient. The end supports at the corners of the building are arbitrarily reinforced for negative moment by bending up one-half of the longitudinal steel in the adjacent beam. The necessary steel details are shown in Figs. 86 and 87.



REAR ELEVATION

Fig. 75.

177. *Design of Stairway Slab.* The stairway, which extends from the basement to the second floor level, is located as shown in Fig. 78. The opening made by the stair well and future elevator shaft is framed by a series of beams as shown. In order to keep the thickness of the stair slab down to a minimum, beam B-9 is placed at the wall edge of the floor level landing slab. The stair slab is designed as a simple slab, the horizontal span of which is equal to the horizontal distance from the middle of beam B-9 to the middle of the wall support of the intermediate landing slab. The allowable unit stresses for use in the design of the stair slab and all beams framing the opening are as follows: $f_s = 16{,}000$, $f_c = 650$, $u = 100$, $v = 40$ or 120.

The dead load on the stair slab is made up of the weight of the intermediate landing slab, the weight of the inclined slab, and the weight of the treads. For purposes of computation it is sufficiently accurate to assume this total dead load as uniformly distributed over the horizontal span. The live load is assumed as 100 lb. per sq. ft. of horizontal surface.

The widths of stair slabs and landing slabs as shown in Fig. 80 will prove satisfactory in the ordinary building of this size. The width of each tread exclusive of the nosing is made $10\frac{1}{4}$ in., and the rise of each tread about $7\frac{3}{16}$ in. This selection satisfies the rules for convenient climbing (Art. 158), and also gives a uniform rise of treads between landing slabs.

Allowing 6 in. bearing on the brick exterior wall, the horizontal span of the stair slab = 12 ft.-1 in. The total length of slab between the supports is

$$\sqrt{6^2 + 7.66^2} + 3.83 = 13.5 \text{ ft.}$$

Assuming the weight of slab as 80 lb. per sq. ft., the total load on a 1-ft. strip of slab is as follows:

Treads	$9 \times \dfrac{7.19 \times 10.25}{2 \times 144} \times 150$	= 350 lb.
Slab	13.5×80	= 1080
Live load	12×100	= 1200
		2630 lb.

The maximum bending moment is

$$M = \tfrac{1}{10} \times 2630 \times 12.1 \times 12 = 38{,}200 \text{ in.-lb.}$$

$$d = \sqrt{\frac{38{,}200}{107.7 \times 12}} = 5.45 \text{ in.}$$

Using 5.5 in. and allowing 1 in. insulation, the weight of slab is 81 lb. per sq. ft., and no revision is required.

$$A_s = \frac{38{,}200}{16{,}000 \times .874 \times 5.5} = .500 \text{ sq. in. per ft. width of slab.}$$

This is furnished by $\frac{1}{2}$-in. round bars, $4\frac{1}{2}$ in. center to center One $\frac{1}{2}$-in. round bar, 3 ft.-10 in. long, is placed under each tread as shown on Fig. 80.[6] The floor level landing slab is made

[6] The detailing shown in Fig. 80 assumes that the stairway slab will be poured with the structural framework of the building. The coefficient of $\frac{1}{10}$ used in determining the moment is justified by the continuity provided across *B*-9. A more widely used method of construction places the stair slabs after the main structure has been completed. In such cases, recesses must be left in the beams in order to furnish support for the future stair slabs, and dowels must be placed in the beams so as to project into the stair slab when it is poured. The stair slab steel must extend only to the face of the supporting beams instead of continuing beyond this face as in Fig. 80. A moment coefficient of $\frac{1}{8}$ should be used for such a stair slab.

$6\frac{1}{2}$ in. thick, and every other bar of the stair slab reinforcement continued across this slab for reinforcement. The remaining stair slab bars extend across *B*-9 far enough to develop the necessary bond. Two $\frac{1}{2}$-in. round bars 10 ft.-0 in. long are placed at right angles to the main landing slab reinforcement in order to assist in distributing the load on that slab and to provide for temperature stresses. Negative moment stresses in the stair slab at *B*-9 are provided for by means of short bent bars placed in the top of the slab at this point, as shown in Fig. 80.

178. *Design of Beams Framing Stair Well.*

Beam B-9. This beam supports a uniform load along its entire length, consisting of one-half of the stairway slab load and one-half of the floor level landing slab load in addition to its own weight. The span of the beam is 9 ft.-6 in., the distance center to center of beams *B*-6 and *B*-7. The beam is designed as a T-beam with the Tee on one side only; this is necessary because of the break between the up and down stair slabs at the landing.

Beam B-6. This beam is a simply supported rectangular beam, the load on which consists of the weight of a 4-in. hollow tile partition and the concentrated load from *B*-9 in addition to its own weight.

Beam B-8. This beam carries the concentrated load from *B*-6, one-half of the load from the landing slab, and the weight of a 4-in. hollow tile partition in addition to its own weight. *B*-8 has been placed at the edge of the outer section band of steel. This fact, together with the improbability of having the full live load on the portion of the floor near *B*-8, justifies the neglect of considering any floor load as supported by *B*-8. Due to the open shaft on one side of the beam at the point of maximum moment, it must be designed as a T-beam with the Tee on one side only. In order to illustrate the method of design where such irregular loads are involved, the design of *B*-8 is included below.

The loads on the beam (see Figs. 76 and 78) are as follows:

Uniform load over entire length of beam,

Partition 20×11.5	= 230 lb. per lin. ft.
Weight of beam	= 200 lb. per lin. ft.
Total	= 430 lb. per lin. ft.

Additional uniform load from stairway landing slab,

$$\frac{1}{2} \times 3.5 \times 194 = 340 \text{ lb. per lin. ft.}$$

Concentrated load from B-6 = 9260 lb.

$$R_L = \frac{9260 \times 9.5 + 340 \times \frac{\overline{9.5}^2}{2} + 430 \times \frac{\overline{21}^2}{2}}{21} = 9450 \text{ lb.}$$

$$R_R = 12{,}100 \text{ lb.}$$

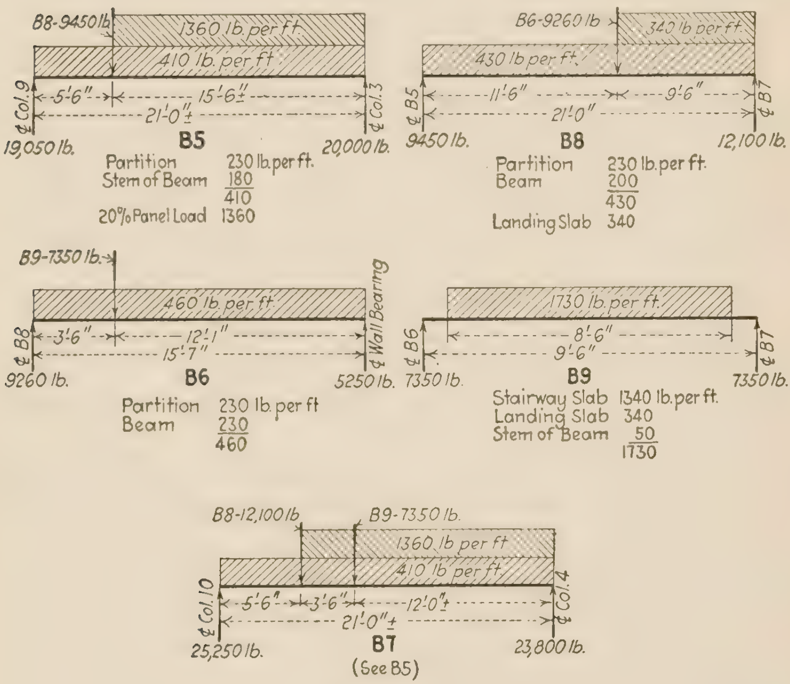


FIG. 76.

The point of zero shear, which locates the point of maximum moment, occurs under the concentrated load. The maximum moment is

$$M = 9450 \times 11.5 - 430 \times \frac{\overline{11.5}^2}{2} =$$

$$80{,}400 \text{ ft.-lb., or } 965{,}000 \text{ in.-lb.}$$

The Joint Committee limits the effective width of flange to $\frac{1}{10}$ span for beams with a Tee on one side only. If the flange is

relatively thick in comparison with the depth of beam, this value may be increased without endangering the safety of the beam. For such beams a value of b equal to one-half that recommended for beams with a Tee on both sides may be used *i.e.*, $\frac{1}{2} \times \frac{1}{4}$ span, or $\frac{1}{8}$ span. This value of the effective flange width is used in the present case.

$$b = \tfrac{1}{8} \times 21 \times 12 = 31.5 \text{ in.}$$

Assuming, on account of the thickness of the slab forming the flange, that the neutral axis is in the flange,

$$d = \sqrt{\frac{965{,}000}{31.5 \times 107.7}} = 16.8 \text{ in.}$$ Use 17 in. for the effective depth.

$$b' = \frac{12{,}100}{120 \times .875 \times 17} = 6.8 \text{ in.}$$ Use 10 in. for width of stem.

Allowing 3 in. insulation to provide for two rows of steel, the total height of beam is 20 in. and the weight per foot 200 lb. as assumed.

$$\frac{M}{bd^2} = \frac{965{,}000}{31.5 \times 17^2} = 106.0 \qquad \frac{t}{d} = \frac{9}{17} = .53.$$

Diagram 3 shows that the neutral axis is in the flange as assumed, and a rectangular beam section, of a width equal to 31.5 in., actually exists. The above solution needs no revision.

$$A_s = \frac{965{,}000}{16{,}000 \times .875 \times 17} = 4.05 \text{ sq. in.}$$

With four 1-in. square bars $p = .0075$, $j = .876$, and the revised area of steel required is 4.05 sq. in.

Since this beam is poured monolithic with the beams supporting it, there will exist some negative moment at the supports. The amount of this bending moment is dependent upon too many factors to permit of an accurate determination. In order to provide for the stresses of negative moment, one-half of the steel is bent up and hooked over the support. Adequate bond resistance is furnished by the two deformed bars remaining. The points at which this steel may be bent up may best be found by determining the points where the bending moment is one-half of the maximum by means of an equation involving the distance

x from the support. For the left end of the beam as shown in Fig. 78, this equation is as follows:

$$9450x - 430\frac{x^2}{2} = 40{,}200, \text{ from which } x = 4.75 \text{ ft.}$$

The two bars on this side are bent up 4 ft.-6 in. from the center of B-5.

A similar equation for the right end of the beam locates the point where the bending moment is 40,200 ft.-lb., 3.75 ft. from the center of B-7. The two bars are bent up 3 ft.-6 in. from that point.

Since only two bars are bent up, and these at a single point, they will not be depended upon for diagonal tension resistance and vertical stirrups will be used wherever web reinforcement is required. In estimating the spacing of stirrups, it is assumed that the concrete can take care of the shear up to a unit value of 40 lb. per sq. in. The stirrups, therefore, need only be designed to take the shear in excess of this amount. Where irregular loads occur, the following method is convenient for determining the spacing of stirrups.

The total shear per linear inch at any section to be taken by the web reinforcement $= \dfrac{(V - V_1)}{jd}$, in which V is the total shear existing at that section, and V_1 the amount of shear that may be taken by the concrete itself.

The spacing,

$$s = \frac{A_s f_s}{\dfrac{V - V_1}{jd}} = \frac{A_s f_s jd}{V - V_1}$$

The amount of reduction in the shear to be resisted by the web reinforcement, that is, the reduction of $(V - V_1)$, over the distance provided for by the first stirrup, s_1 in., equals s_1 times the load per linear inch. The next stirrup may be spaced a distance s_2 in. from the preceding one, s_2 being determined from an equation as above, substituting for $(V - V_1)$ there used, the reduced value of $(V - V_1)$, the actual value existing at the section under investigation. Thus, at the left end of B-8,

$$V - V_1 = 9450 - (40 \times 10 \times .875 \times 17) = 3500 \text{ lb.}$$

Assuming ¼-in. round U-stirrups

$$s_1 = \frac{2 \times .0491 \times 16{,}000 \times .875 \times 17}{3500} = 6.7 \text{ in.}$$

The load per linear inch on this portion of the beam equals $^{430}/_{12} = 36$ lb.

The spacing of the next stirrup

$$s_2 = \frac{2 \times .0491 \times 16{,}000 \times .875 \times 17}{3500 - (6.7 \times 36)} = 7.2 \text{ in.}$$

In a similar manner the remaining spacings are computed, until the denominator of the equation becomes zero. In the present beam, the maximum allowable spacing = 8 in. Placing the first stirrup 2 in. from the edge of beam *B*-5, the remaining stirrups are spaced 3 at 7 in., and 8 at 8 in.

The computations may be grouped into a compact form as shown by those for the right end of the beam, as follows:

$$V_1 = 40 \times 10 \times .875 \times 17 = 5950 \text{ lb.}$$

At the support

$$V - V_1 = 12{,}100 - 5950 = 6150 \text{ lb.}$$

The load per linear inch = $^{770}/_{12} = 64.2$ lb.

With ¼-in. round U-stirrups,

$$A_s f_s jd = 2 \times .0491 \times 16{,}000 \times .875 \times 17 = 23{,}400$$

$$s_1 = \frac{23{,}400}{6150} = 3.8 \text{ in.} \quad 6150 - 3.8 \times 64.2 = 5905$$

$$s_2 = \frac{23{,}400}{5905} = 4.0 \text{ in.} \quad 5905 - 4.0 \times 64.2 = 5648$$

$$s_3 = \frac{23{,}400}{5648} = 4.2 \text{ in.} \quad 5648 - 4.2 \times 64.2 = 5378$$

The spacing of the remaining stirrups is determined by continuing the computations as above. Placing the first stirrup 2 in. from the edge of beam *B*-7, the remaining ones are spaced as follows: 6 at 4 in., 3 at 5½ in. and 5 at 8 in.

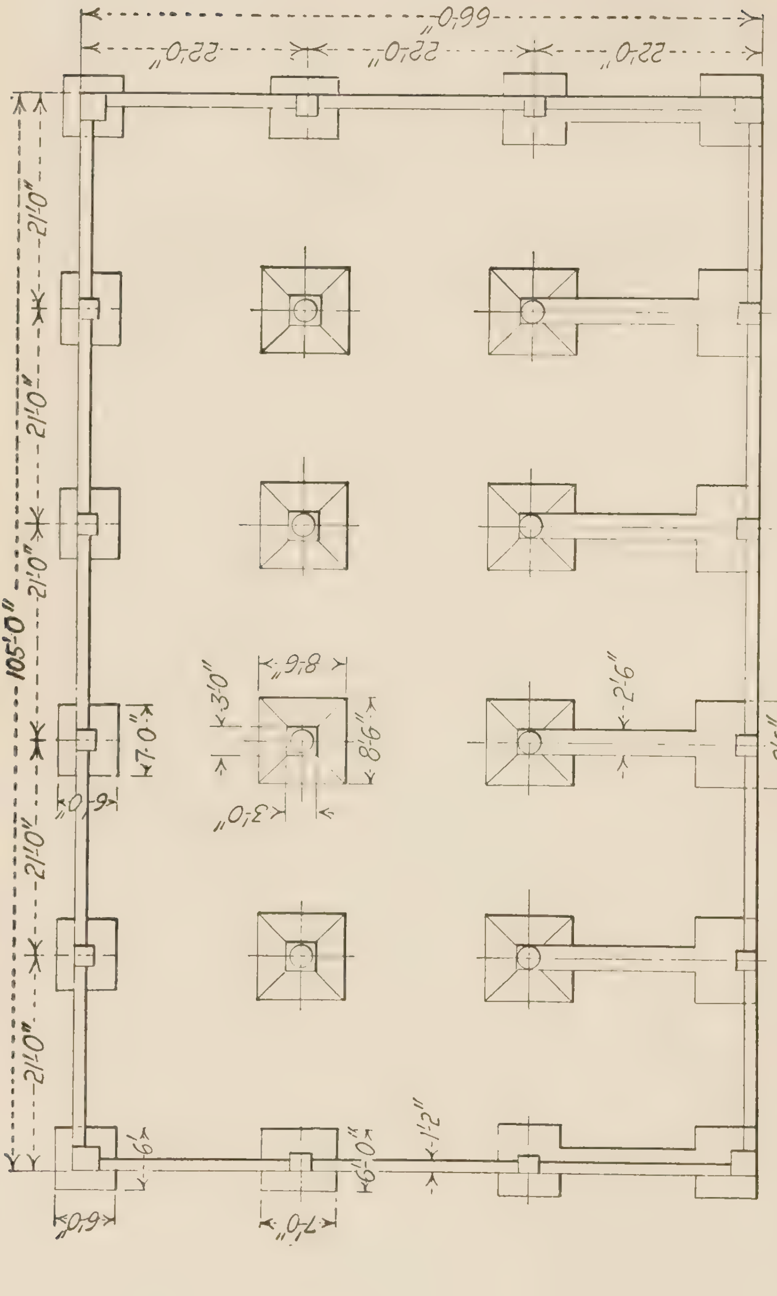
The method outlined above will be found especially convenient where concentrated loads are placed at intervals along the beam in addition to the uniform load.

Beam B-5. In addition to supporting the concentration from beam *B*-8 and its own weight, beam *B*-5 must be designed to

support the partition as shown and the required proportion of the floor load. The beam is a T-beam, with the Tee on one side only.

Beam B-7. This beam supports loads as stated for *B*-5, and, in addition, the concentrated load from *B*-9. It is a T-beam, with the Tee on one side only.

The complete framing details of the stairway beams are shown in Fig. 82, and the required steel details in Figs. 86 and 87. Figures 88 and 89 show an assembled drawing for a building similar to the above.



FOUNDATION PLAN

FIG. 77.

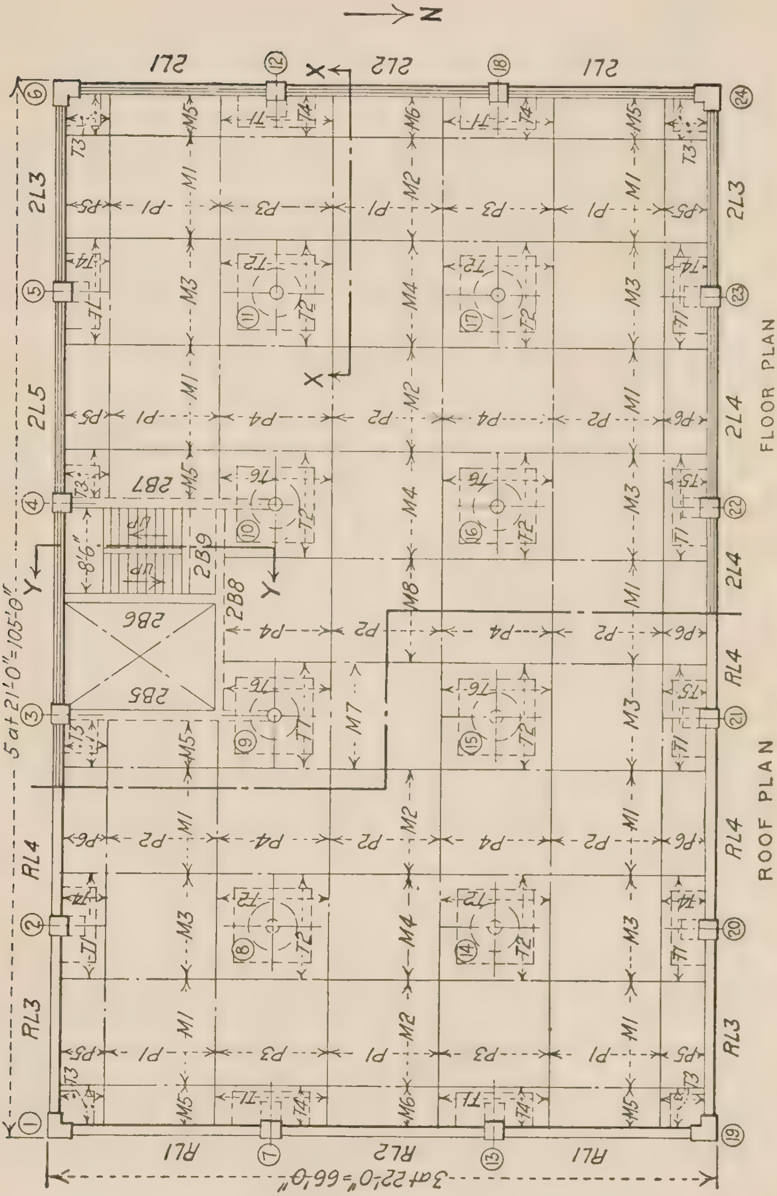
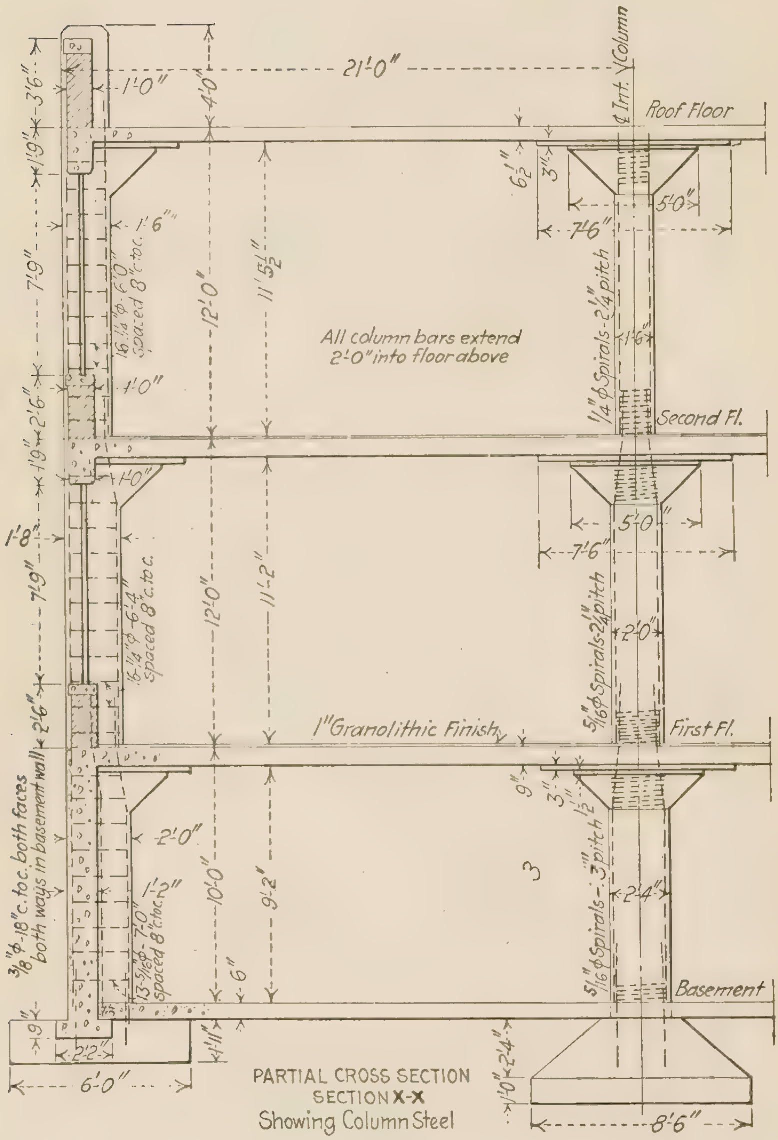
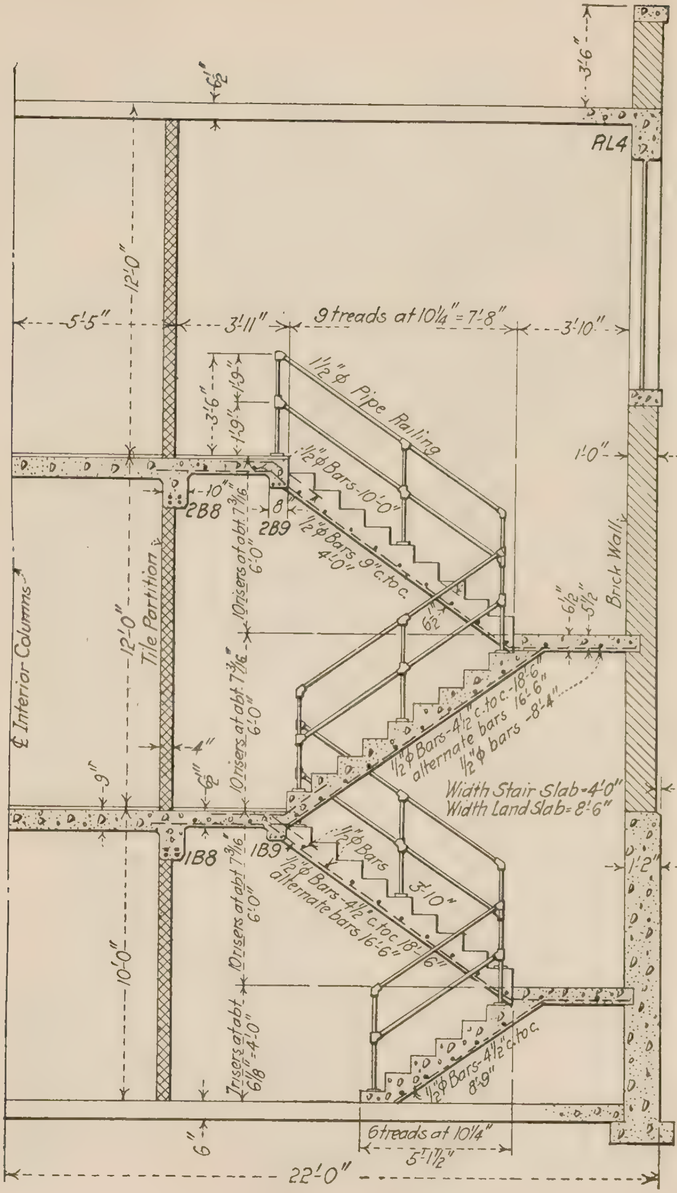


Fig. 78.



PARTIAL CROSS SECTION
SECTION X-X
Showing Column Steel

FIG. 79.



DETAIL OF STAIRWAY
Section Y-Y

Fig. 80.

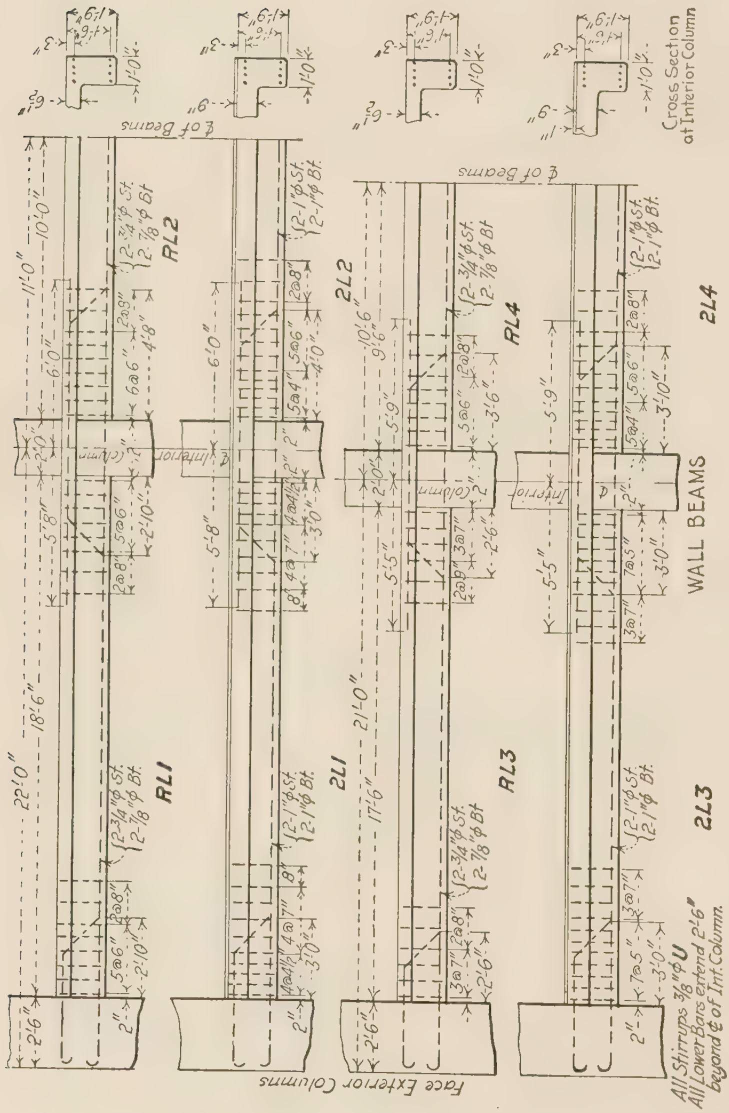
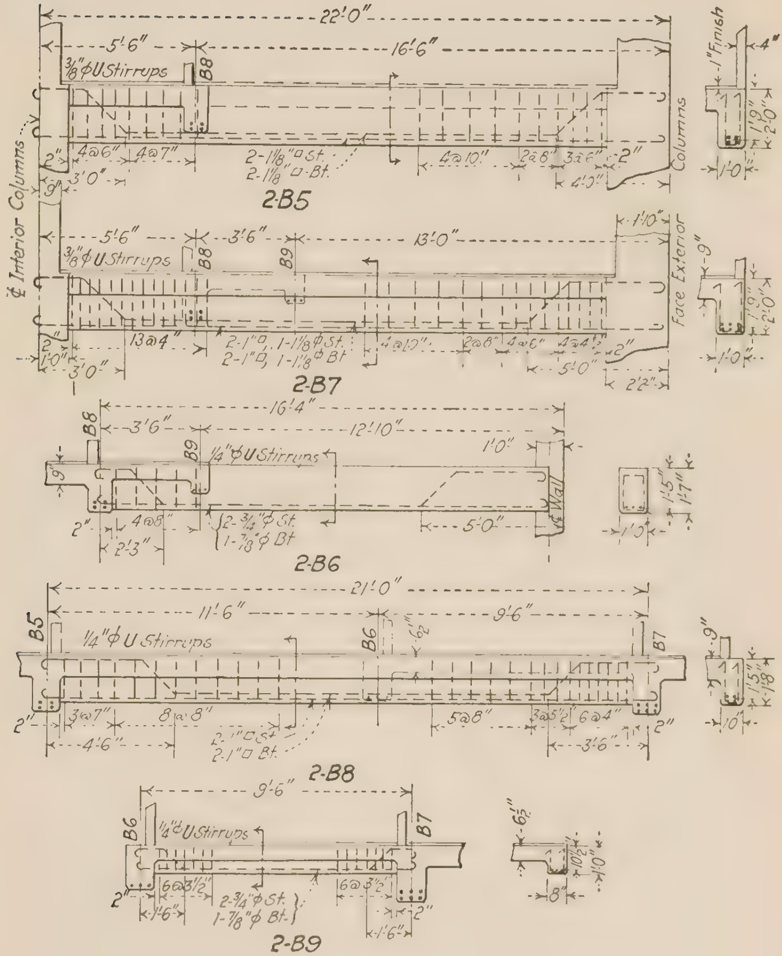

RL1
RL2
2L1
2L2
RL3
RL4
2L3
2L4
WALL BEAMS
Cross Section at Interior Column
℄ of Beams
Face Exterior Columns
℄ Interior Column
All Stirrups 3/8" φ U
All Lower Bars extend 2'-6" beyond ℄ of Int. Column.

FIG. 81.



DETAIL OF BEAMS FRAMING
Stair Well

Fig. 82.

FLOOR BAND SCHEDULE

Mk.	No. of Bands	Bars per Band		
P1	8	9-½"φ 9-½"φ	17'-3" 27'-0"	5'7" 11'7"
P2	6	7-½"φ 7-½"φ	23'-4" 23'-4"	11'7" 11'7"
P3	4	10-¾"φ 2-¾"φ	27'-10" 27'-0"	5'7" 11'7" 11'7"
P4	6	5-¾"φ 5-¾"φ	23'-4" 23'-4"	11'7" 11'7"
P5	6	5-¾"φ	27'-10"	5'7" 11'7"
P6	3	2-¾"φ 2-¾"φ	23'-4" 23'-4"	11'7" 11'7"
T1	10	9-¾"φ	10'-6"	
T2	11	7-¾"φ	13'-0"	Straight in top
T3	12	4-¾"φ	10'-6"	
T4	8	4-¾"φ	13'-0"	Over
T5	2	5-¾"φ	13'-0"	Column Head
T6	4	10-¾"φ	13'-0"	
T7	1	3-¾"φ 5-¾"φ	13'-0" 12'-4"	

Mk.	No. of Bands	Bars per Band		
M1	9	9-½"φ 9-½"φ	18'-0" 28'-4"	5'10" 12'1"
M2	4	7-½"φ 7-½"φ	24'-4" 24'-4"	12'1" 12'1"
M3	6	10-¾"φ 2-¾"φ	29'-2" 28'-4"	5'10" 12'1" 12'1"
M4	3	5-¾"φ 5-¾"φ	24'-4" 24'-4"	12'1" 12'1"
M5	6	5-¾"φ	29'-2"	5'10" 12'1"
M6	2	2-¾"φ 2-¾"φ	24'-4" 24'-4"	12'1" 12'1"
M7	1	3-¾"φ 5-¾"φ 2-¾"φ	23'-7" 24'-4" 24'-4"	12'1" 11'4" 12'1"
M8	1	8-½"φ 8-½"φ	24'-4" 23'-9"	12'1" 11'4"

***NOTE:** See **FIG. 65-a** for alternate method of bending floor slab steel*

Fig. 83.

ALTERNATE FLOOR BAND SCHEDULE

Mk.	No of Bands	Bars per Band	Bent Bar Detail
P1	8	9-1/2φ 27'-9" bt. 9-1/2φ 16-6" st.	5'7" 11'7"
P2	6	7-1/2φ 34'-0" bt. 7-1/2φ 12'-6" st.	11'7" 11'7"
P3	4	6-3/4φ 27'-9" bt. 6-3/4φ 16'-6" st.	5'7" 11'7"
P4	6	5-3/4φ 34'-0" bt. 5-3/4φ 13'-0" st.	11'7" 11'7"
P5	6	3-3/4φ 27'-9" bt. 3-3/4φ 16'-6" st.	5'7" 11'7"
P6	3	3-3/4φ 34'-0" bt. 2-3/4φ 13'-0" st.	11'7" 11'7"
T1	10	13-3/4φ 10'-6"	
T2	11	13-3/4φ 13'-0"	Straight
T3	12	7-3/4φ 10'-6"	
T4	8	6-3/4φ 13'-0"	In Top
T5	2	4-3/4φ 13'-0"	Over
T6	4	10-3/4φ 13'-0"	Column Head
T7	1	6-3/4φ 13'-0" 7-3/4φ 12'-4"	

Mk	No. of Bands	Bars per Band	Bent Bar Detail
M1	9	9-1/2φ 29'-0" bt. 9-1/2φ 17'-3" st.	5'10" 12'1"
M2	4	7-1/2φ 35'-3" bt. 7-1/2φ 13'-3" st.	12'1" 12'1"
M3	6	6-3/4φ 29'-0" bt. 6-3/4φ 17'-3" st.	5'10" 12'1"
M4	3	5-3/4φ 35'-3" bt. 5-3/4φ 13'-6" st.	12'1" 12'1"
M5	6	3-3/4φ 29'-0" bt. 3-3/4φ 17'-3" st.	5'10" 12'1"
M6	2	3-3/4φ 35'-3" bt. 2-3/4φ 13'-6" st.	12'1" 12'1"
M7	1	2-3/4φ 35'-3" bt. 3-3/4φ 34'-5" bt. 5-3/4φ 13'-6" st.	12'1" 12'1" 12'1" 11'4"
M8	1	8-1/2φ 34'-5" bt 8-1/2φ 13'-6" st.	12'1" 11'4"

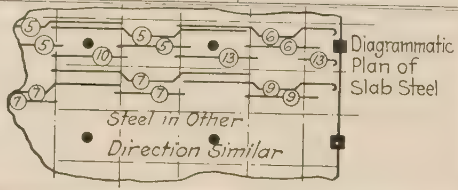


Fig. 83a.

ROOF BAND SCHEDULE

Mk.	No. of Bands	Bars per Band		
P1	6	8-1/2" φ 6-1/2" φ	17'-3" 27'-0"	5'-7" 11'-7"
P2	9	6-1/2" φ 6-1/2" φ	23'-4" 23'-4"	11'-7" 11'-7"
P3	4	9-5/8" φ 4-5/8" φ	27'-10" 27'-0"	5'-7" 11'-7" 11'-7"
P4	6	6-5/8" φ 6-5/8" φ	23'-4" 23'-4"	11'-7" 11'-7"
P5	4	4-5/8" φ 2-5/8" φ	27'-10" 27'-0"	5'-7" 11'-7" 11'-7"
P6	6	3-5/8" φ 3-5/8" φ	23'-4" 23'-4"	11'-7" 11'-7"
T1	12	9-5/8" φ	10'-6"	
T2	12	3-5/8" φ	13'-0"	Straight in top
T3	8	4-5/8" φ	10'-6"	
T4	8	1-5/8" φ	13'-0"	Over
T5	4	3-5/8" φ	13'-0"	
T6	4	7-5/8" φ	13'-0"	Column Head

Mk.	No. of Bands	Bars per Band		
M1	10	8-1/2" φ 6-1/2" φ	18'-0" 28'-4"	5'-10" 12'-1"
M2	5	6-1/2" φ 6-1/2" φ	24'-4" 24'-4"	12'-1" 12'-1"
M3	8	9-5/8" φ 4-5/8" φ	29'-2" 28'-4"	5'-10" 12'-1" 12'-1"
M4	4	6-5/8" φ 6-5/8" φ	24'-4" 24'-4"	12'-1" 12'-1"
M5	4	4-5/8" φ 2-5/8" φ	29'-2" 28'-4"	5'-10" 12'-1" 12'-1"
M6	2	3-5/8" φ 3-5/8" φ	24'-4" 24'-4"	12'-1" 12'-1"

Fig. 84.

COLUMN SCHEDULE

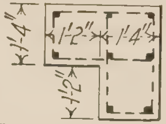
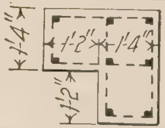
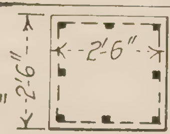
Column Mark	1, 6, 19, 24		2-5, 7, 12, 13, 18, 20-23	8-11, 14-17
No. of Columns	4		12	8
Second Floor	2'-6" x 2'-6" 8-1"φ - 12'-6" 32-1/4"φ Ties- 6'-6"@ 8"		1'-6" x 2'-0" 6-3/4"φ - 12'-6" 16-1/4"φ Ties - 6'-0"@ 8"	1'-6" Diameter 5-5/8"φ - 11'-10" 1/4"φ Spirals @ 2 1/4"
First Floor	2'-6" x 2'-6" 8-1"φ - 14'-0" 32-1/4"φ Ties - 6'-6"@ 8"		1'-8" x 2'-0" 6-7/8"φ - 14'-0" 16-1/4"φ Ties - 6'-4"@ 8"	2'-0" Diameter 10-1"φ - 14'-0" 5/16"φ Spirals @ 2 1/4"
Basement	2'-6" x 2'-6" 8-1"φ - 12'-0" 13-5/16"φ Ties - 9'-0"@ 8"		2'-0" x 2'-0" 8-1"φ - 12'-0" 13-5/16"φ Ties - 7'-0"@ 8"	2'-4" Diameter 10-1"φ - 12'-0" 5/16"φ Spirals @ 3"
Dowels	8-1"φ - 4'-0"		8-1"φ - 4'-0"	10-1"φ - 4'-0"

FIG. 85.

BEAM SCHEDULE

Mk.	Cross Section	No. of Beams	Longitudinal Steel per Beam: Bent Bars		Longitudinal Steel per Beam: Straight Bars		Stirrups: No., Size, Length		Stirrups: Spacing Each End
RL1	12" x 21"	4	2 - 7/8" ϕ	29'-4"	* 2 - 3/4" ϕ	24'-10"	16 - 3/8" ϕ	4'-6"	5 @ 6", 2 @ 8"
RL2	12" x 21"	2	2 - 7/8" ϕ	34'-4"	2 - 3/4" ϕ	27'-0"	18 - 3/8" ϕ	4'-6"	6 @ 6", 2 @ 8"
RL3	12" x 21"	4	2 - 7/8" ϕ	28'-1"	* 2 - 3/4" ϕ	23'-10"	12 - 3/8" ϕ	4'-6"	3 @ 7", 2 @ 8"
RL4	12" x 21"	6	2 - 7/8" ϕ	32'-10"	2 - 3/4" ϕ	26'-0"	16 - 3/8" ϕ	4'-6"	5 @ 6", 2 @ 8"
2L1	12" x 21"	4	2 - 1" ϕ	29'-4"	* 2 - 1" ϕ	24'-10"	20 - 3/8" ϕ	4'-6"	4 @ 4 1/2", 4 @ 7", 1 @ 8"
2L2	12" x 21"	2	2 - 1" ϕ	34'-4"	2 - 1" ϕ	27'-0"	26 - 3/8" ϕ	4'-6"	5 @ 4", 5 @ 6", 2 @ 8"
2L3	12" x 21"	4	2 - 1" ϕ	28'-1"	* 2 - 1" ϕ	23'-10"	22 - 3/8" ϕ	4'-6"	7 @ 5", 3 @ 7"
2L4	12" x 21"	3	2 - 1" ϕ	32'-10"	2 - 1" ϕ	26'-0"	26 - 3/8" ϕ	4'-6"	5 @ 4", 5 @ 6" 2 @ 8"
2L5	12" x 21"	2	2 - 1" ϕ	28'-9"	* 2 - 1" ϕ	24'-10"	26 - 3/8" ϕ	4'-6"	5 @ 4", 5 @ 6", 2 @ 8"
2B5	12" x 24"	2	2 - 1 1/8" □	24'-10"	# 2 - 1 1/8" □	23'-8"	19 - 3/8" ϕ	5'-0"	S. 4 @ 6", 4 @ 7" N. 3 @ 6", 2 @ 8", 4 @ 10"
2B6	12" x 19"	2	1 - 7/8" ϕ	17'-11"	# 2 - 3/4" ϕ	16'-9"	5 - 1/4" ϕ	4'-8"	S. 4 @ 8" N. none
2B7	12" x 24"	2	2 - 1" □ 1 - 1 1/8" □	24'-10" 24'-10"	# 2 - 1" ϕ # 1 - 1 1/8" □	23'-8" 23'-8"	28 - 3/8" ϕ	5'-0"	S. 13 @ 4", N. 4 @ 4 1/2, 4 @ 6", 2 @ 8", 4 @ 10"
2B8	10" x 20"	2	2 - 1" □	23'-8"	# 2 - 1" □	22'-8"	27 - 1/4" ϕ	4'-2"	W. 3 @ 7", 8 @ 8" E. 6 @ 4", 3 @ 5 1/2", 5 @ 8"
2B9	8" x 12"	2	1 - 7/8" ϕ	11'-10"	# 2 - 3/4" ϕ	11'-2"	14 - 1/4" ϕ	2'-10"	W. 6 @ 3 1/2" E. 6 @ 3 1/2"

* *Hooked One End* · # *Hooked Each End*

FIG. 86.

BEAM STEEL BENDING SCHEDULE

Mk.	Total No. of Bars	Size and Length					H				H Out to Out
RL1	8	7/8″ ϕ	29′-4″	6″	3′-11″	1′-3″	12′-10″	1′-3″	8′-7″		1′-3″
RL2	4	7/8″ ϕ	34′-4″		10′-1″	1′-3″	10′-8″	1′-3″	10′-1″		1′-3″
RL3	8	7/8″ ϕ	28′-1″	6″	3′-7″	1′-3″	12′-6″	1′-3″	8′-0″		1′-3″
RL4	12	7/8″ ϕ	32′-10″		8′-8″	1′-3″	12′-0″	1′-3″	8′-8″		1′-3″
2L1	8	1″ ϕ	29′-4″	6″	4′-1″	1′-3″	12′-6″	1′-3″	8′-9″		1′-3″
2L2	4	1″ ϕ	34′-4″		9′-5″	1′-3″	12′-0″	1′-3″	9′-5″		1′-3″
2L3	8	1″ ϕ	28′-1″	6″	4′-1″	1′-3″	11′-6″	1′-3″	8′-6″		1′-3″
2L4	6	1″ ϕ	32′-10″		9′-0″	1′-3″	11′-4″	1′-3″	9′-0″		1′-3″
2L5	4	1″ ϕ	28′-9″	6″	4′-5″	1′-3″	11′-4″	1′-3″	9′-0″		1′-3″
2B5	4	1 1/8″ □	24′-10″	6″	2′-4″	1′-6″	15′-0″	1′-6″	2′-4″	6″	1′-6″
2B6	2	7/8″ ϕ	17′-11″	6″	1′-3″	1′-3″	9′-1″	1′-3″	3′-1″	6″	1′-3″
2B7	4 2	1″ □ 1 1/8″ □	24′-10″ 24′-10″	6″ 6″	2′-4″ 2′-4″	1′-6″ 1′-6″	14′-0″ 14′-0″	1′-6″ 1′-6″	3′-4″ 3′-4″	6″ 6″	1′-6″ 1′-6″
2B8	4	1″ □	23′-8″	6″	3′-8″	1′-2″	13′-0″	1′-2″	2′-8″	6″	1′-2″
2B9	2	7/8″ ϕ	11′-10″	6″	1′-1″	9″	6′-6″	9″	1′-1″	6″	9″

Fig. 87.

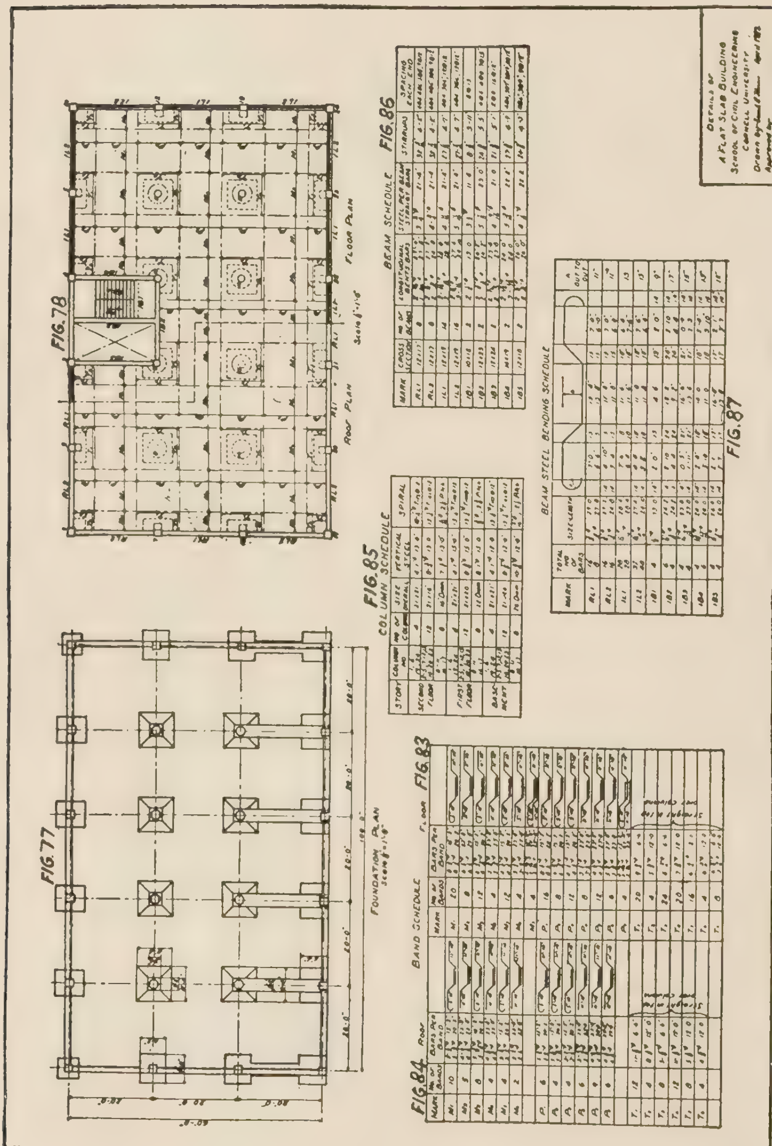


FIG. 77
FOUNDATION PLAN
FIG. 78
ROOF PLAN
FLOOR PLAN
FIG. 83
FIG. 84
BAND SCHEDULE
FIG. 85
COLUMN SCHEDULE
FIG. 86
BEAM SCHEDULE
FIG. 87
BEAM STEEL BENDING SCHEDULE
DETAILS OF
A FLAT SLAB BUILDING
SCHOOL OF CIVIL ENGINEERING
CORNELL UNIVERSITY

FIG. 88.

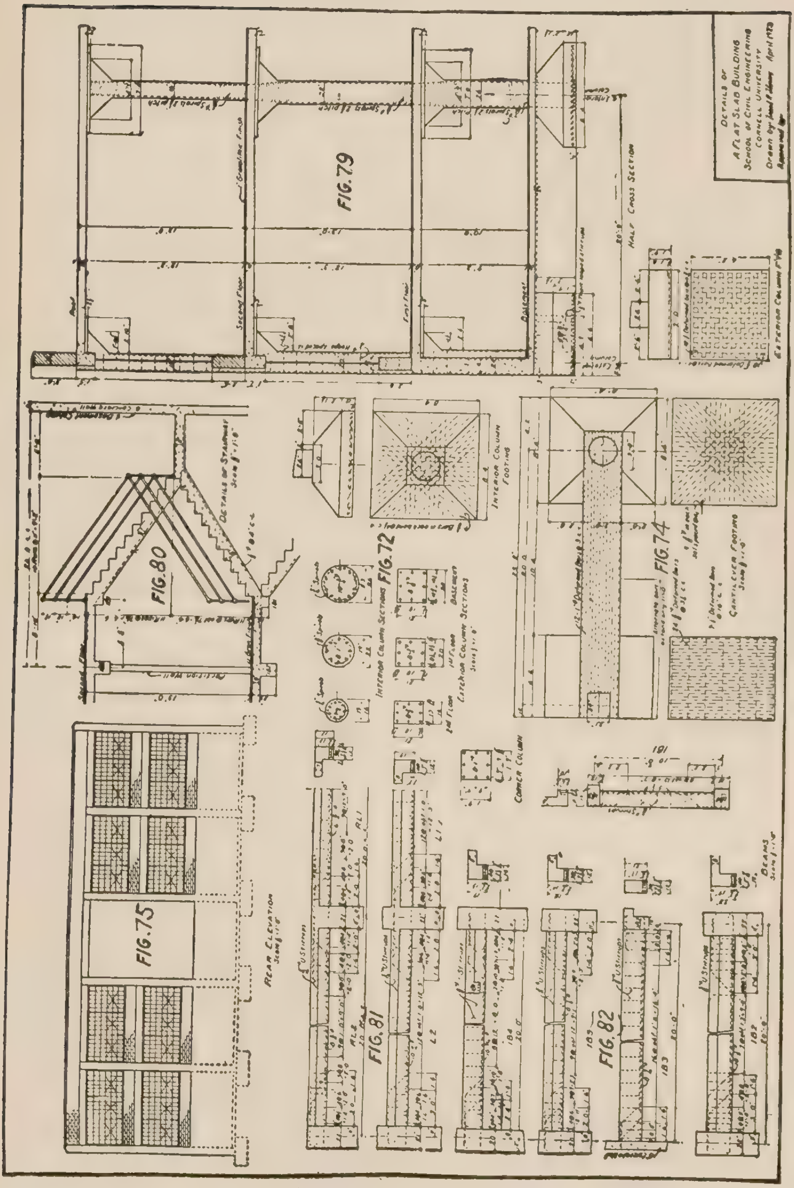

Details of
A Flat Slab Building
School of Civil Engineering
Cornell University
FIG. 79
Half Cross Section
FIG. 80
Details of Stairway
FIG. 72
Interior Column Footing
Interior Column Sections
Exterior Column Sections
FIG. 74
Cantilever Footing
Exterior Column Ftg
Corner Column
FIG. 75
Rear Elevation
FIG. 81
FIG. 82
Beams

FIG. 89.

CHAPTER IX

RETAINING WALLS

179. Introductory. A pile of earth, cinders, or other material possessing more or less frictional stability, will, when deposited loosely in an unrestrained position, assume a definite slope. The steepness of this depends upon the internal friction of the material and other conditions, such as moisture content, etc. A mound of earth whose sides are permitted to assume this natural slope will, when thoroughly compacted, maintain its own integrity, and support external loads to a maximum amount which depends, among other things, upon the bearing qualities of the soil.

In engineering construction it frequently becomes necessary to prevent the sides of such a pile of earth from assuming this natural slope. Such a condition occurs when the width of a cut or embankment is limited either by restrictions of economy or right of ownership. The most common examples of the latter limitation are found in railway and highway construction where the width of the right of way is fixed. In such cases it is essential that the earth be held in position by means of a wall capable of resisting the lateral pressure caused by the conditions of restraint.

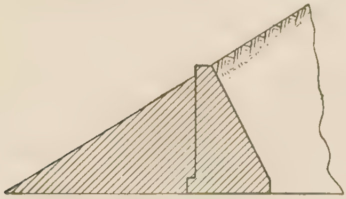


FIG. 90.

A wall whose express purpose is to hold in position a bank of earth or similar material is termed a retaining wall. The first step toward the design of a retaining wall is to determine its location. If the wall is to run along a fixed property line, such as a highway or a railroad, this provides definite placing. As is often the case, the amount of land available for the construction of a given cut or fill may be unlimited, but the cost of cutting or filling sufficiently to allow the natural slope of the earth to

obtain may be excessive. Wherever it is found that a retaining wall of the necessary height and section is cheaper than the additional cut or fill that it replaces, economy favors the construction. In Fig. 90, the wall replaces the shaded volume of fill. A few trials will show at what point the wall should be placed to obtain the minimum cost.

The section of wall to be chosen will be determined by a consideration of economy, ease of construction, and other factors interposed by existing conditions.

180. Types. Masonry retaining walls may be divided into two general classes: (1) the gravity wall, which retains the bank of earth entirely by its own weight; (2) the reinforced concrete wall, which utilizes the weight of the earth behind it in resisting the overturning moment of the retained material. In this latter class are included the cantilever wall, a type of construction consisting of a vertical arm supported upon a horizontal base slab, the vertical arm acting as a free cantilever in overcoming the pressure from the earth; and the counterfort wall, the vertical slab of which is anchored or tied to the base slab by means of counterforts or buttresses—triangular cross walls extending from the top of the vertical slab to the extreme point of the base slab at regular intervals throughout the length of the wall. The vertical slab of the reinforced walls may be placed at the front, at the rear, or at any point along the base slab, the exact location depending uoon limitations of economy and construction. Where conditions permit, a toe extension of from $\frac{1}{3}$ to $\frac{1}{2}l$ will produce a more economical design than would result if the vertical arm were placed at the front edge of the base slab. Paaswell[1] proves that for a given location of the resultant pressure, the most economical width of base occurs when the vertical arm is placed over the assumed point of application of the resultant pressure.

The back of a gravity type wall may be vertical, or may slope toward or away from the filling. The most economical section is obtained when the back slopes toward the filling. On account of difficulties of construction, however, the use of this section is restricted to comparatively isolated cases where unusual founda-

[1] Paaswell, "Retaining Walls," p. 82.

tion conditions exist. In cold localities, where there is danger of upheaval by frost, economy may well be sacrificed to security, and the back be given a slight batter forward. Where this batter is of an appreciable amount, added stability may be obtained by constructing the back as a series of steps, thus utilizing more fully the relieving weight of the earth directly over the base of the wall.

The section of wall to be chosen will be determined by a consideration of economy, ease of construction, foundation requirements, and other factors imposed by existing conditions. In comparing the relative economy of gravity walls and reinforced concrete walls, the added cost of construction in the case of the latter must be included. A careful study of the different types leads to the conclusion that unless affected by unusual conditions, the gravity type will prove most economical for low walls, the cantilever type for walls of medium height, and the counterfort type for the higher walls. The critical height, or height of separation between the various types, is not clearly defined, since it depends upon too many economic as well as constructive conditions. In general it is found uneconomical to use the counterfort construction for walls less than 18 ft. in height.

181. Conditions of Loading. There are three general conditions of loading that need consideration: (1) walls with no surcharge, the top surface of the fill being horizontal and level with the top of the wall; (2) walls with an inclined surcharge, the top surface of the fill extending upward and back from the top of the back of the wall; (3) walls with a horizontal surcharge extending some distance above the top of the wall.

The angle of inclination δ in Case 2 is usually taken as the angle of repose ϕ of the retained material. For ordinary conditions, this may be assumed as 33 degrees 42 minutes, which corresponds to a slope of $1\frac{1}{2}:1$. For dry sand or similar filling a slope of $1:1$, angle $= 45$ degrees, may be used.

Case 3 includes loadings in which the actual surface of earth does not extend above the top of the wall, but supports an external load such as a building, railroad tracks, etc. The loads are converted into an equivalent height of earth above the top of the wall by dividing the weight of the additional load per square

foot by the weight of the earth per cubic foot, and the pressure estimated for this equivalent height of surcharge of earth.

When the additional load from the fill consists of the weight from one or more railroad tracks, it is best to follow the recommendations of the American Railway Engineering Association as given below.

(*a*) In calculating the surcharge due to the track, the entire load shall be taken as uniformly distributed over a width of 14 ft.

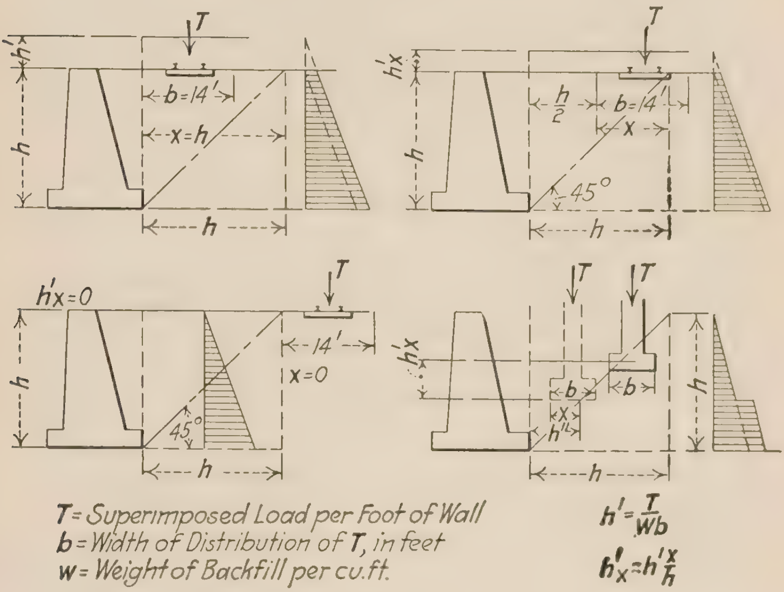


FIG. 91.

for a single track or tracks spaced more than 14 ft. centers, and the distance center to center of tracks where tracks are spaced less than 14 ft.

(*b*) In calculating the pressure on a retaining wall where the filling carries permanent tracks or structures, the full effect of the loaded surcharge shall be considered where the edge of the distributed load or structure is vertically above the back edge of the heel of the wall. The effect of the loaded surcharge may be neglected where the edge of the distributed load or structure is at a distance from the vertical line through the back edge of the

heel of the wall equal to h, the height of the wall. For intermediate positions the equivalent uniform surcharge load is to be taken as proportional. For example, for a track with the edge of the distributed load at a distance $\frac{h}{2}$ from the vertical line through the back edge of the heel of the wall, the equivalent uniform surcharge load is one-half of the normal distributed load distributed over the filling. Figure 91 explains the distribution. The height of surcharge loading will be equal to the load per linear foot divided by bw (b = 14 ft. for a single-track railway). Where the edge of the distributed load cannot come nearer to the vertical line through the back edge of the heel of the wall than $h - x$, the equivalent uniformly distributed load in terms of the height is

$$h'_x = h'\frac{x}{h}$$

The terms of this equation are explained in Fig. 91.

182. Determination of Earth Thrust. The first essential in any design is the determination of the force to be resisted. The principal force governing the dimensions of a retaining wall is the pressure exerted by the retained material in its attempt to assume its natural slope. In order fully to determine the pressure of the filling against the wall, the resultant must be known in amount, in line of action, and in point of application.

Many theories have been advanced which lead to a purely academic determination of earth thrust. These mathematical discussions of the action of earth masses premise an ideal, incompressible, homogeneous material, without cohesion, possessing frictional resistance between its particles, and of indefinite extent in the mass. Such a fill is rarely found in practice. The degree of exactness of the thrust as determined by any of the theoretical methods will depend upon the difference between the actual conditions and the theoretical.

While refinements in the theory of earth pressure are, therefore, unwarranted from a practical standpoint, such academic thrust determinations, when modified in accordance with the conclusions drawn from actual tests and the results of engineering experience, may become the basis for a rational working formula.

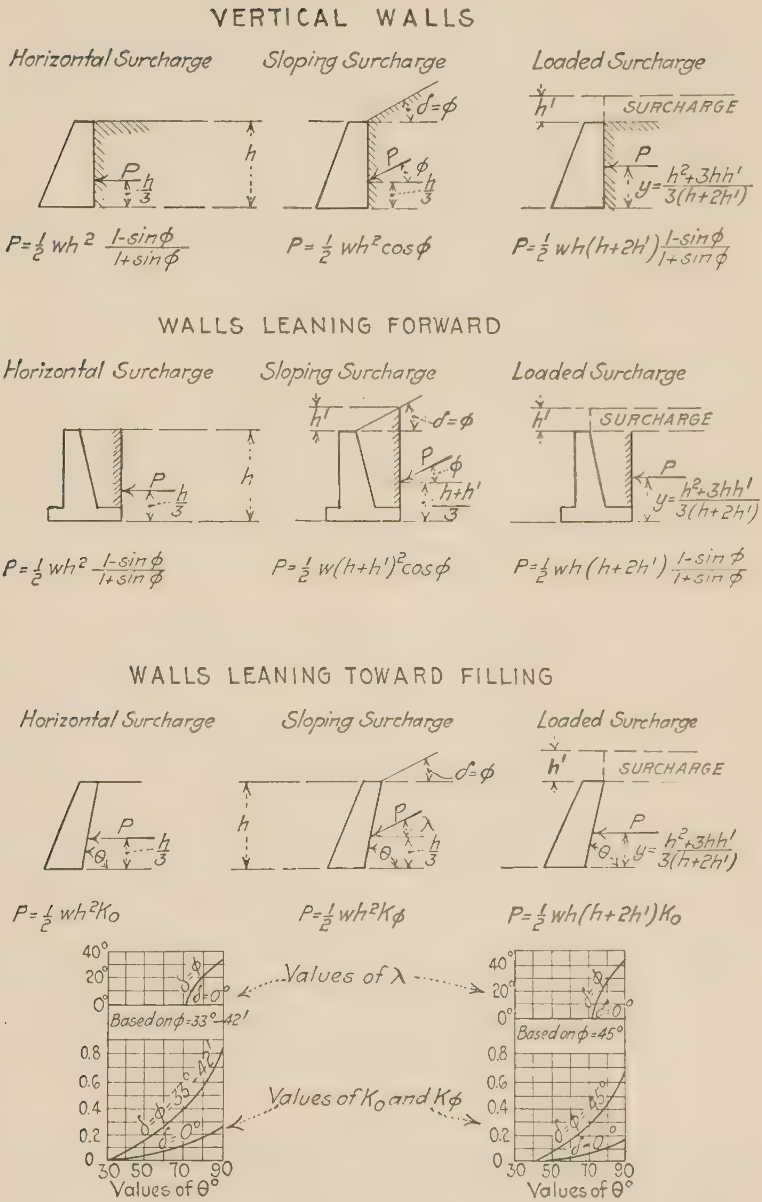

VERTICAL WALLS
Horizontal Surcharge
Sloping Surcharge
Loaded Surcharge
SURCHARGE
$P=\frac{1}{2}wh^2\frac{1-\sin\phi}{1+\sin\phi}$
$P=\frac{1}{2}wh^2\cos\phi$
$P=\frac{1}{2}wh(h+2h')\frac{1-\sin\phi}{1+\sin\phi}$
$y=\frac{h^2+3hh'}{3(h+2h')}$
WALLS LEANING FORWARD
Horizontal Surcharge
Sloping Surcharge
Loaded Surcharge
SURCHARGE
$P=\frac{1}{2}wh^2\frac{1-\sin\phi}{1+\sin\phi}$
$P=\frac{1}{2}w(h+h')^2\cos\phi$
$P=\frac{1}{2}wh(h+2h')\frac{1-\sin\phi}{1+\sin\phi}$
WALLS LEANING TOWARD FILLING
Horizontal Surcharge
Sloping Surcharge
Loaded Surcharge
SURCHARGE
$P=\frac{1}{2}wh^2K_0$
$P=\frac{1}{2}wh^2K_\phi$
$P=\frac{1}{2}wh(h+2h')K_0$
Values of λ
Based on $\phi=33°-42'$
Based on $\phi=45°$
Values of K_0 and K_ϕ
Values of $\theta°$

FIG. 92.

The subcommittee of the American Railway Engineering Association on Design of Plain and Reinforced Retaining Walls and Abutments, in its report of 1917, comments upon earth pressures as follows: "Actual tests on an extensive scale will be required to produce any results of real value. No such tests have yet been made, and in the absence of such definite information as they might be expected to produce, and believing that the intelligent use of theoretical formulas leads to economical and proper design, this committee therefore recommends that Rankine's formulas, which consider that the filling is a granular mass of indefinite extent, without cohesion, be used in the designing of retaining walls."

Rankine's development of earth pressure, which starts out with an infinitesimal prism and leads to an expression for the thrust of the entire earth mass upon a given surface, leads to the general equation,

$$P = C \times \frac{wh_1^2}{2} \tag{85}$$

in which P is the total thrust upon the back of the wall, w the weight of the earth per cubic foot, h_1 the height of the earth column in feet, and C a constant depending upon the angle of inclination of the back of the wall, the conditions of loading, and the physical properties of the earth fill. The original development by Rankine includes formulas for vertical walls only. This theory has been expanded by Ketchum[2] to include walls leaning away from the filling and walls leaning toward the filling. These latter equations, in addition to the original vertical wall expressions, are given in the report of the Committee mentioned above, in substantially the form shown in Fig. 92.

The amount of the pressure on any given horizontal strip 1 ft. in height at a distance x ft. below the surface of the earth is given by the equation

$$P_1 = Cwx. \tag{86}$$

The pressure distribution along the back of the wall for cases 1 and 2 of Art. 181 is shown in Fig. 93*a*, and for Case 3 in Fig. 93*b*.

[2] KETCHUM, "Walls, Bins, and Grain Elevators."

In determining foundation pressures in a wall of a cross-section as shown in Fig. 94, the earth fill vertically above the base slab is considered as a resisting pressure equivalent to the same weight of masonry, and the total overturning pressure is the total earth

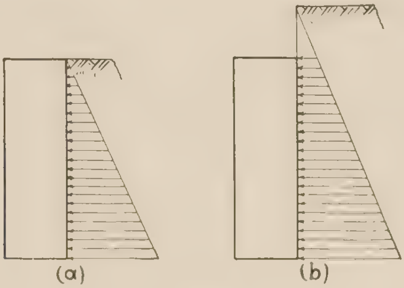


FIG. 93.

thrust on a vertical plane at the heel of the wall. The height CD is therefore used in equation (85) for determining the earth thrust. This applies also to walls of gravity section in which the back slopes away from the fill. In determining the bending

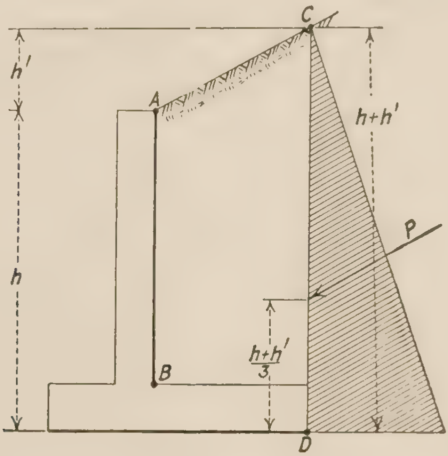


FIG. 94.

moment on the vertical arm AB (Fig. 94), the total thrust is due to a column of earth of a height equal to AB.

183. Line of Action of Earth Pressure. The line of action of the total thrust upon a wall with a vertical back exposed to the

action of the earth is parallel to the top surface of the filling. In a wall whose back slopes away from the fill, the total thrust upon a vertical plane through the heel of the wall acts parallel to the top surface of the earth. For walls leaning toward the filling, the resultant pressure P will be horizontal for a wall without surcharge or with a horizontal loaded surcharge, and will make an angle λ with the horizontal for a wall with a sloping surcharge. The values of λ will vary from the angle of surcharge, where the wall is vertical, to zero, where Rankine's theory shows that the resultant pressure is horizontal. Values of λ are given in Fig. 92.

184. Point of Application of Resultant Earth Pressure. For walls with no surcharge, or a sloping surcharge, the point of application of the total earth thrust is usually assumed at a point in the plane against which the earth is acting and at a distance of one-third its height, measured from the base of the plane. For walls with a loaded surcharge, the point of application is taken at the center of gravity of the pressure quadrilateral shown in Fig. 93*b*. The location of the point of application of the resultant thrust for the various conditions of loading is given in Fig. 92.

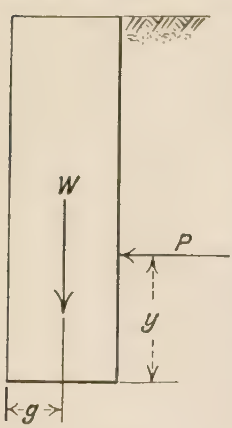


Fig. 95.

185. Factors Affecting the Design. Following a determination of the earth's thrust, an investigation must be made of all possible modes of failure, and each element of the construction so proportioned as to make such failures impossible. A gravity type of wall may fail by sliding along the plane of the base, by overturning, or by settlement at the toe, caused by crushing of the soil there. An extreme case of this will also cause overturning. A reinforced concrete wall may fail in any of the ways mentioned above. In addition, any of the thin sections which together furnish the necessary strength and rigidity might yield in a manner similar to a corresponding element in other constructions.

186. Overturning and Crushing. When the overturning moment Py (Fig. 95) becomes equal to the stability moment

Wg, the wall is at the point of incipient overturning. This condition exists when the resultant of the overturning and resisting loads passes through the toe of the wall. As long as the resultant load falls within the base, the wall is safe against overturning. The desirable location of the point of intersection of the resultant load and the base depends upon a consideration of foundation pressures.

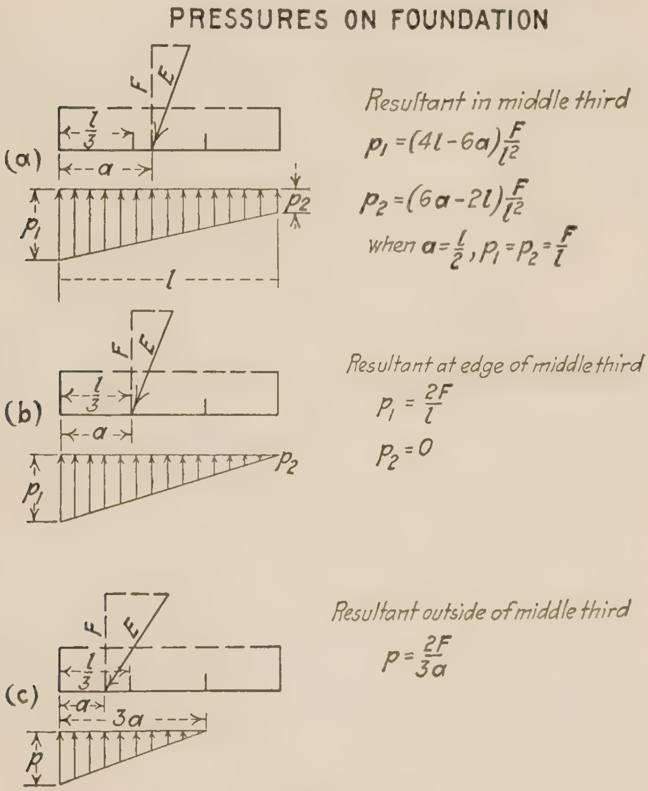


Fig. 96.

In Fig. 96*a*, let *E* represent the resultant of the total earth pressure and the resisting weight on a 1-ft. strip of wall, and let *F* represent its vertical component. The point of intersection of *E* with the base of the wall is located by the distance *a*. Under these conditions the column of earth directly under the base

sustains an eccentric load F at a distance of $\frac{l}{2} - a$ from the gravity axis of the column. Applications of the principles of flexure and direct stress as outlined in Chap. IV lead to the general expression for the values of p_1 and p_2, the unit pressures at the toe and heel of the wall, respectively, as follows:

$$p_1 = (4l - 6a)\frac{F}{l^2} \tag{87}$$

$$p_2 = (6a - 2l)\frac{F}{l^2} \tag{88}$$

Examination of equations (87) and (88) shows that uniform soil pressure occurs only when $a = \frac{l}{2}$, that is, when no eccentricity of load exists. For this condition, $p_1 = p_2 = \frac{F}{l}$. Since an ideal design of foundation demands a uniform distribution of upward pressure, the desired location of the point of intersection of the resultant E with the base is at the middle of the base. The economics of retaining walls, however, usually forbids the fulfilment of this condition. Further examination of the equations indicates that when the intersection occurs within the middle third of the base, compression exists over the entire foundation—the footing is bearing on the soil along its entire length. If a is less than $\frac{l}{3}$, tension exists at the heel—the footing is not bearing on the soil along its entire length. If the construction is such that this tension cannot be provided for, the entire load will have to be resisted by the compression under the forward portion of the wall, that is, over a distance $3a$ from the toe (see Fig. 96*c*). The expression for the unit pressure at the toe under this condition becomes

$$p_1 = \frac{2F}{3a} \tag{89}$$

A larger toe pressure results from the use of equation (89) than would be the case if equation (87) were applicable.

Analysis of the above discussion leads to the conclusion that, since it is economically undesirable under ordinary conditions to proportion the wall so as to cause a uniform distribution of the pressure on the foundation bed, a satisfactory design results

when the line of action of the resultant pressure on the foundation bed intersects the base at any point within the middle third, provided the safe bearing pressure of the foundation material is not exceeded. When the wall rests upon a compressible material where settlement may be expected, the resultant thrust E should strike at the middle or back of the middle of the base so that the wall will settle toward the filling. Where the wall rests on solid rock, or is carried on piles, the resultant thrust E may strike slightly outside the middle third, provided the wall is sufficiently safe against overturning, and also provided the maximum allowable pressure is not exceeded.

The requirements of foundation pressures, therefore, require the wall to be safe against incipient overturning; a wall proportioned to distribute properly the load over the foundation will furnish a factor of safety, against the tendency to topple over, greater than 1. To allow for obvious exigencies, this condition should always be investigated and a suitable factor of safety applied. Since the wall is designed for the greatest load that is anticipated, this factor need not be large. A value of 2 is satisfactory for ordinary design. An expression for the overturning safety factor may be derived as follows:

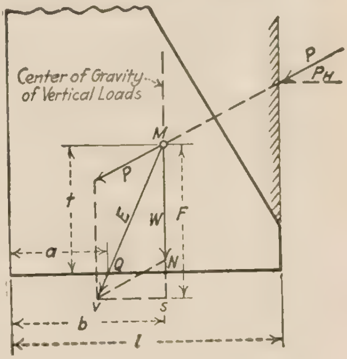


FIG. 97.

In Fig. 97 let F = vertical component of resultant E

P_H = horizontal component of earth thrust

l = length of base

n = factor of safety against overturning

a = distance from toe to point of intersection of resultant with base

In the triangles MNQ and MSV

$$\frac{t}{b-a} = \frac{F}{P_H}$$

$$\frac{F}{P_H \times t} = \frac{1}{b-a}$$

Multiplying each side by b

$$\frac{Fb}{P_H \times t} = \frac{b}{b - a} = n \qquad (90)$$

An approximate value for n may be found by substituting for b in the above equation its near equivalent $\frac{l}{2}$.

$$n = \frac{l}{l - 2a} \qquad (91)$$

187. Sliding. In order to prevent sliding of the wall along the base, the frictional resistance of the base against the foundation material must be greater than the horizontal component of the thrust on the back of the wall. The frictional resistance of the base is equal to the resisting weight multiplied by the coefficient of friction of the masonry on the soil. The coefficient of friction of masonry on dry clay varies from 0.5 to 0.6; on wet clay 0.33; on sand 0.4; on gravel 0.6. A factor of safety of $1\frac{1}{2}$ is usually considered satisfactory.

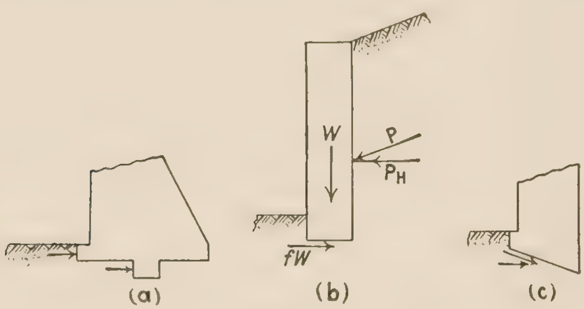


Fig. 98.

In case an adverse condition of sliding exists, the base may be widened, thus increasing the weight of the wall; narrow shallow trenches may be dug in the foundation, forming projections which will materially increase the resistance to sliding; the base may be inclined upward toward the toe; or the forward portion of the trench may be filled with masonry so that the wall butts directly against the original earth. Figure 98 illustrates the principles involved in sliding resistance.

188. Details of Construction. The front of the wall is usually built with a batter of from ½ to 1 in. in 12 in. A coping, projecting a short distance beyond the wall, adds to the architectural appearance and, to a certain extent, protects the masonry in the body of the wall from dripping water. The base of the foundation should be a sufficient distance below the surface of the ground to insure against the dangers of action by frost, a minimum of 2½ ft. being sufficient in ordinary climates. Expansion joints should be provided at intervals along the wall, preferably not further apart than 30 ft. In the reinforced walls where cracks would not only be unsightly but also detrimental to the integrity of the wall, additional steel should be placed at right angles to the main reinforcement to provide for temperature and shrinkage stresses. An amount varying from .1 to .33 per cent of the cross-section area is usually specified. Proper drainage of the fill behind the wall may be effected by inserting 4-in. drain tiles through the wall near the bottom at intervals of 10 to 15 ft., and piling crushed stone, gravel, or other coarse material around these "weep holes" at the back. At least one drain should be provided for each pocket formed by counterforts.

189. Application of Fundamental Principles. It should be remembered that[3] "no theoretical formulas can be more than an aid to the judgment of the experienced designer. The main value of such formulas is in obtaining economical proportions, in obtaining a proper distribution of the stresses, and in making experience already gained more valuable." A careful study should be made of the conditions in the design of each wall and modifications of the above discussion made wherever required by the peculiarities of the problem under consideration.

The foregoing fundamental considerations will be elaborated upon, and the additional computations required to proportion properly the elements composing the various types of reinforced concrete walls will be explained in the following typical designs.

190. Design of Gravity Wall. A gravity wall 16 ft.-0 in. high is to sustain a bank of earth with a loaded horizontal surcharge equivalent to 4 ft. of filling above the top of the wall. The safe

[3] Report of Committee on Masonry, *Bulletin*, Amer. Ry. Eng. Ass'n., February, 1917.

bearing pressure on the clay foundation bed is 2 tons per sq. ft. The weight of the retained fill is 100 lb. per cu. ft., and the angle of repose 33 degrees 42 minutes. Determine the required section of wall.

The ordinary procedure in the design of a gravity type wall is to select a tentative section, the dimensions of which are governed by the judgment and experience of the designer. This tentative section is then analyzed in accordance with the principles outlined above, and modifications in the assumed dimensions made where necessary.

In the present case, the tentative dimensions are shown in Fig. 99. Investigation must be made with and without the portion of surcharge directly over the base included in the resisting weight W_3. Assuming that the former condition obtains, the analysis is as follows:

From Fig. 92, the total pressure per foot of wall against the vertical plane through the heel of the wall

$$P = \frac{1}{2}wh(h + 2h') \times \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$= \frac{1}{2} \times 100 \times 16(16 + 8)(.286) = 5500 \text{ lb.}$$

The distance of the point of application of P above the bottom of the plane is

$$y = \frac{h^2 + 3h'h}{3(h + 2h')}$$

$$= \frac{16^2 + 3 \times 4 \times 16}{3(16 + 8)} = 6.24 \text{ ft.}$$

$$W_1 = \frac{8.5 + 1.5}{2} \times 13 \times 150 = 9750 \text{ lb.}$$

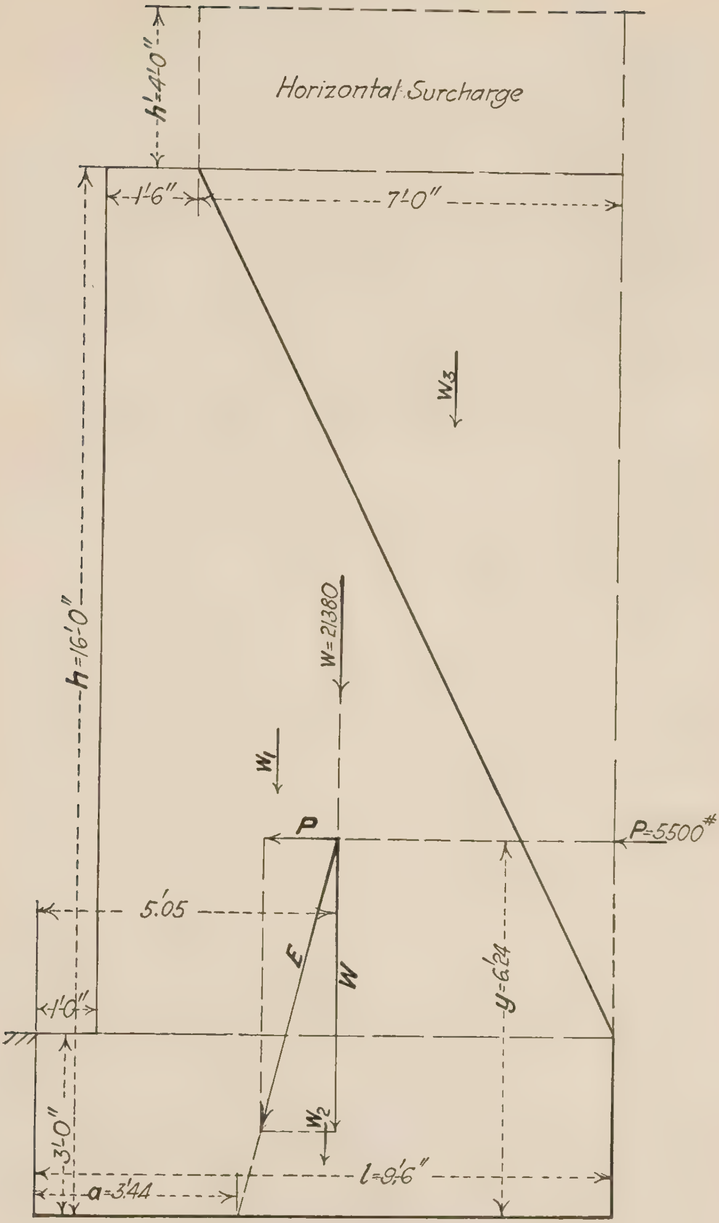
$$W_2 = 9.5 \times 3 \times 150 = 4280 \text{ lb.}$$

$$W_3 = \frac{17 + 4}{2} \times 7.0 \times 100 = 7350 \text{ lb.}$$

$$W = 21{,}380 \text{ lb.}$$

By taking moments about the toe of the wall, the point of application of the total resisting load W is found to be 5.05 ft. from that point.

The resultant of P and W = 22,100 lb. and intersects the base 3.44 ft. from the toe, or 0.27 ft. inside the forward edge of the middle third.



GRAVITY RETAINING WALL

Fig. 99.

$$p_1 = (4 \times 9.5 - 6 \times 3.44)\frac{21{,}380}{9.5^2} = 4100 \text{ lb. per sq. ft.}$$

$$p_2 = (6 \times 3.44 - 2 \times 9.5)\frac{21{,}380}{9.5^2} = 389 \text{ lb. per sq. ft.}$$

Investigation of the same section, assuming that the surcharge directly over the base is not included in the resisting load W_3, shows that the total load $W = 18{,}580$ lb., and its point of application is 4.90 ft. from the toe of the wall. The point of application of the resultant of P and W is 3.05 ft. from the toe. This is but 0.12 ft. outside the middle third, and will be assumed satisfactory. The toe pressure for this case is,

$$p_1 = \frac{2 \times 18{,}580}{3 \times 3.05} = 4050 \text{ lb. per sq. ft.}$$

For Case 2, which is the most severe condition, the overturning moment is

$$5500 \times 6.24 = 34{,}300 \text{ ft.-lb.}$$

and the resisting moment

$$18{,}580 \times 4.90 = 91{,}200 \text{ ft.-lb.}$$

$$\text{The factor of safety against overturning} = \frac{91{,}200}{34{,}300} = 2.66$$

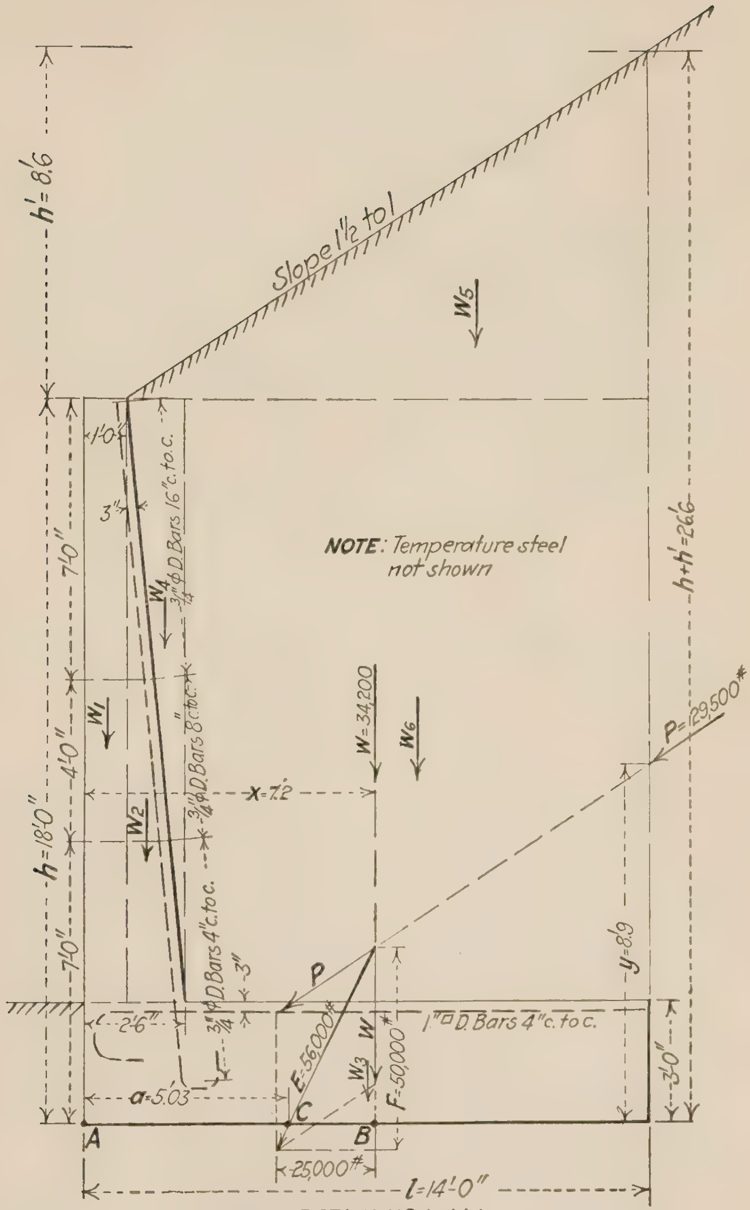
The force producing sliding $= 5500$ lb., and the force resisting sliding, assuming the coefficient of friction along the base as .5,

$$.5 \times 18{,}580 = 9290 \text{ lb.}$$

$$\text{The factor of safety against sliding} = \frac{9290}{5500} = 1.69$$

191. Design of Cantilever Wall. Design a reinforced concrete wall of the cantilever type 18 ft.-0 in. in height, to retain a bank of earth with a surcharge whose slope is $1\frac{1}{2}:1$. The wall is to be placed along the easement line, beyond which no encroachment is permissible. The soil is a firm clay with an allowable pressure of $3\frac{1}{2}$ tons per sq. ft. The weight of the retained fill is 100 lb. per cu. ft. The allowable unit stresses are as follows: $f_c = 800$, $f_s = 16{,}000$, $u = 100$, $v = 40$, and $n = 15$. The coefficient of friction between the concrete base and the subsoil equals .5.

Owing to the conditions, an L-shaped wall with a vertical face is necessary. The tentative dimensions are shown in Fig. 100.



CANTILEVER RETAINING WALL

FIG. 100.

The overturning pressure on a vertical plane through the heel of the wall from Fig. 92 is

$$P = \tfrac{1}{2} \cos \phi \times w(h + h')^2$$
$$= \tfrac{1}{2} \times .832 \times 100 \times (26.6)^2 = 29{,}500 \text{ lb.}$$

and its point of application $\frac{26.6}{3} = 8.9$ ft. from the base of the plane.

The total relieving weight W, consisting of the weight of the wall and the earth directly over the base, is found as in the preceding example to be 34,200 lb., and its point of application 7.2 ft. from the front face of the wall. The resultant of P and W, found graphically, is 56,000 lb. and its point of intersection with the base 5.03 ft. from the toe, or $4\frac{1}{4}$ in. inside the forward edge of the middle third.

$$p_1 = (4 \times 14 - 6 \times 5.03) \times \frac{50{,}000}{\overline{14}^2} = 6600 \text{ lb. per sq. ft.}$$

$$p_2 = (6 \times 5.03 - 2 \times 14) \times \frac{50{,}000}{\overline{14}^2} = 560 \text{ lb. per sq. ft.}$$

The factor of safety against overturning, by equation (90), Art. 186, equals

$$\frac{AB}{CB} = \frac{7.2}{7.2 - 5.03} = 3.3$$

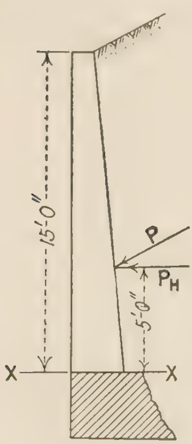


Fig. 101.

In order to furnish a sliding safety factor of $1\frac{1}{2}$, the friction force required equals $25{,}000 \times 1\frac{1}{2} = 37{,}500$ lb. The force furnished by the friction on the base equals $.5 \times 50{,}000 = 25{,}000$ lb. Assuming that the concrete is poured directly against the original earth in front, the sliding resistance furnished by the earth at the toe is $3 \times 7000 = 21{,}000$ lb. The total resistance is 46,000 lb., which is ample.

Vertical Arm. Figure 101 represents the force acting on this member.

$$P = \frac{.832}{2} \times 100 \times \overline{15}^2 = 9360 \text{ lb.}$$
$$P_H = 9360 \times .832 = 7800 \text{ lb.}$$
$$M_{x-x} = 7800 \times 5 \times 12 = 468{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{468{,}000}{146.7 \times 12}} = 16.3 \text{ in.}$$

$$V_{x-x} = P_H = 7800 \text{ lb.}$$

$$d = \frac{7800}{40 \times 12 \times \frac{7}{8}} = 18.6 \text{ in.}$$

The thickness of 30 in. selected gives a value of $d = 27$ in., allowing 3 in. insulation. A smaller value of d would require excessive reinforcement, which would add greatly to the cost of handling and placing. Assuming $j = .875$,

$$A_s = \frac{468{,}000}{16{,}000 \times .875 \times 27} = 1.24 \text{ sq. in. per ft. of wall.}$$

¾-in. round deformed bars 4 in. center to center are selected.

With this arrangement, $p = \frac{.4418}{4 \times 27} = .0041$, and from Table V, $j = .902$. The revised required steel area $=$ 1.20 sq. in. per ft. of wall, and the 4-in. spacing is still necessary.

$$u = \frac{7800}{1\frac{2}{4} \times 2.356 \times .9 \times 27} = 45 \text{ lb. per sq. in.}$$

The rods must continue into the base a distance beyond the top of the base slab sufficient to develop their strength in bond, or

$$\frac{16{,}000}{4 \times 100} \times \frac{3}{4} = 30 \text{ in.}$$

Investigation must be made of the depth and area of steel required at intermediate planes in the height of the vertical arm. This may be accomplished by considering the wall above the plane in question as a free cantilever and analyzing in a manner similar to that followed above for the entire vertical arm. The actual effective depth furnished at the plane should be used in solving for the steel area. The depth required at any section should be less than that furnished.

On account of the decreasing pressure, the number of bars required per foot of wall at any horizontal plane decreases as the top of the vertical arm is approached. Therefore, alternate bars may be discontinued a safe distance beyond the points of theoretical cut-off. The points at which these bars are no longer required may best be found by computing the steel area required at two or more intermediate heights, and plotting required steel

areas against height of vertical arm. In the present case, every other bar is discontinued at a distance of 4 ft. from the top of the base slab, and every other remaining bar is cut off at a point 8 ft. from the top of the base slab, both of these being well above the theoretical points of cut-off. This arrangement gives a spacing of 16 in. for the bars near the top of the wall. This represents practically the maximum spacing desirable for these bars.

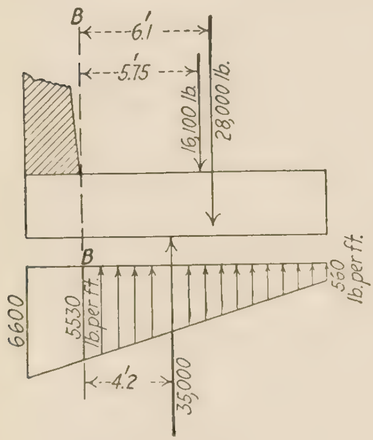


Fig. 102.

Base Slab. Figure 102 represents the forces acting on the base slab. The maximum moment occurs along the plane *B-B*, and is equal to the difference in moments of the upward foundation pressure and the downward load from the filling above the portion of the base slab behind the plane *B-B*. The vertical component of the resultant earth pressure on the wall, assumed as uniformly distributed over the rear portion of the base slab, should also be included in computing the downward moment. The total downward load of the filling and base slab itself to the rear of *B-B* is

$$\frac{16 + 23.6}{2} \times 11.5 \times 100 + 3 \times 11.5 \times 150 = 28{,}000 \text{ lb.}$$

Its point of application is 6.1 ft. from B. The vertical component of the thrust $P = 29{,}500 \times .554 = 16{,}100$ lb., and is assumed as uniformly distributed over the base slab to the rear of plane *B-B*.

The total upward pressure from the foundation (see Fig. 102) is

$$\frac{560 + 5530}{2} \times 11.5 = 35{,}000 \text{ lb.}$$

and its center of gravity is

$$\frac{11.5(5530 + 2 \times 560)}{3(5530 + 560)} = 4.2 \text{ ft. from } B$$

$$M_{B-B} = 28{,}000 \times 6.1 + 16{,}100 \times 5.75 - 35{,}000 \times 4.2 = 116{,}700 \text{ ft.-lb., or } 1{,}400{,}000 \text{ in.-lb.}$$

$$V_{B-B} = 28{,}000 + 16{,}100 - 35{,}000 = 9100 \text{ lb.}$$

$$d \text{ for moment} = \sqrt{\frac{1{,}400{,}000}{146.7 \times 12}} = 28.2 \text{ in.}$$

$$d \text{ for shear} = \frac{9100}{40 \times \frac{7}{8} \times 12} = 21.6 \text{ in.}$$

Allowing 3 in. for insulation, the effective depth furnished is 33 in. A smaller value would require excessive steel.

$$A_s = \frac{1{,}400{,}000}{16{,}000 \times .874 \times 33} = 3.0 \text{ sq. in. per ft. of wall.}$$

This is furnished by 1-in. square deformed bars 4 in. center to center.[4]

$$u = \frac{9100}{1\frac{3}{4} \times 4 \times \frac{7}{8} \times 33} = 26 \text{ lb. per sq. in.}$$

$$l_1 = \frac{16{,}000}{4 \times 100} \times 1 = 40 \text{ in.}$$

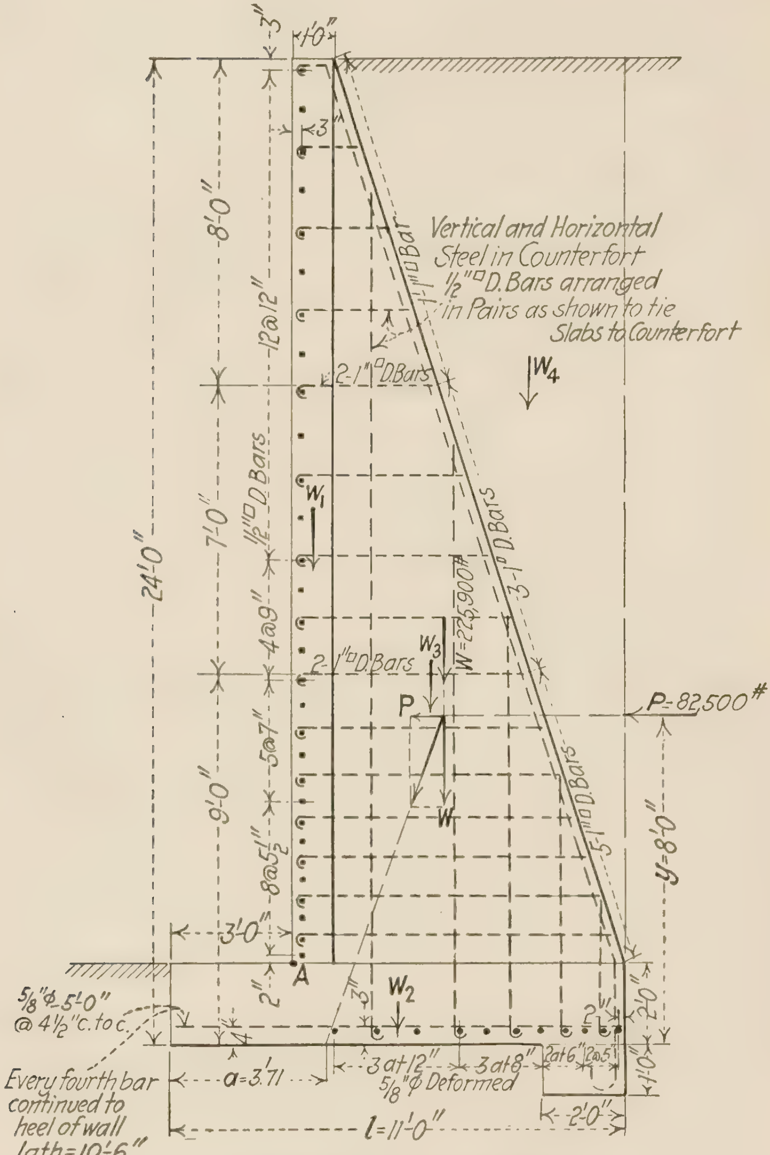
A hook on the wall end of each bar provides for the deficiency in length of embedment.

Since the downward moment is greater than the upward moment, the tension face is uppermost, and the steel must be placed along that face. Alternate bars in the base slab could be discontinued as outlined in the design of the vertical slab.

In the vertical slab the principal temperature reinforcement is horizontal, and most of it is placed along the exposed face. On the front face, ½-in. round horizontal bars, 12 in. center to center vertically, are used, and wired to ½-in. round vertical bars 36 in. center to center. On the rear face, ½-in. round horizontal bars 24 in. center to center are wired to the main slab reinforcement.

In the base slab, ½-in. round bars are wired to the main reinforcing steel, and placed 12 in. center to center to prevent the formation of cracks which would permit seepage of water into the slab, with the resulting damage to the reinforcing bars.

[4] This amount of steel is rather large for efficient handling. A more satisfactory design would result by increasing the thickness of the base slab so that the spacing of the 1-in. square bars could be increased to approximately 6 in.



COUNTERFORT RETAINING WALL

Fig. 103.

192. Design of Counterfort Wall. Design a reinforced concrete wall of the counterfort type, 24 ft.-0 in. in height, to support an earth fill, the upper surface of which is horizontal and level with the top of the wall. The weight of the filling is 100 lb. per cu. ft., and the angle of repose 33 degrees 42 minutes (slope $1\frac{1}{2}:1$). The allowable pressure on the soil is 2 tons per sq. ft., and the coefficient of friction between the base and the subsoil is .4. A spacing of 10 ft. for the counterforts for walls of medium height will usually be economical. The ordinary range of counterfort spacing is from 8 ft. for the low walls to 12 ft. for the higher walls.

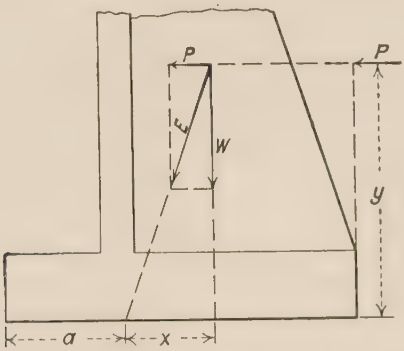


Fig. 104.

The thickness of counterfort varies from 12 to 18 in. In the present case a thickness of 18 in. and a spacing of 10 ft. will be used. The remaining dimensions are assumed as shown in Fig. 103. In estimating foundation pressures, a length of wall of 10 ft. is considered to allow for the effect of the counterfort. Allowable unit stresses are as follows: $f_s = 650$, $f_s = 16{,}000$, $u = 100$. $v = 40$, and $n = 15$.

From Fig. 92, $P = .143 \times 100 \times \overline{24}^2 \times 10 = 82{,}500$ lb.

$$y = \tfrac{1}{3} \times 24 = 8 \text{ ft.}$$

As in the preceding designs

$$\begin{aligned}
W_1 &= 10 \times 1 \times 22 \times 150 &&= 33{,}000 \text{ lb.}\\
W_2 &= 2 \times 11 \times 10 \times 150 &&= 33{,}000 \text{ lb.}\\
W_3 &= \tfrac{1}{2} \times 7 \times 22 \times 1.5 \times 150 &&= 17{,}300 \text{ lb.}\\
W_4 &= \tfrac{1}{2} \times 7 \times 22 \times 1.5 \times 100 &&= 11{,}600 \text{ lb.}\\
W_5 &= 7 \times 8.5 \times 22 \times 100 &&= 131{,}000 \text{ lb.}\\
W &= && \ 225{,}900 \text{ lb.}
\end{aligned}$$

The point of application of the total relieving weight W is found by taking moments about the toe of the base slab to be at a distance of 6.64 ft. from the toe. The resultant pressure on the base is

$$\sqrt{(225{,}900)^2 + (82{,}500)^2} = 240{,}000 \text{ lb.},$$

and its point of intersection with the base is determined analytically as follows: In Fig. 104 let x be the horizontal distance from the line of action of W to the point where the resultant E intersects the base. Then, by similar triangles,

$$\frac{x}{y} = \frac{P}{W}$$

$$x = \frac{82{,}500 \times 8}{225{,}900} = 2.93 \text{ ft.}$$

The distance $a = 6.64 - 2.93 = 3.71$ ft. The resultant pressure on the base intersects the base .04 ft. inside the middle third.

$$p_1 = \frac{1}{10}(4 \times 11 - 6 \times 3.71)\frac{225{,}900}{11^2} = 4050 \text{ lb. per sq. ft.}$$

$$p_2 = \frac{1}{10}(6 \times 3.71 - 2 \times 11)\frac{225{,}900}{11^2} = 50 \text{ lb. per sq. ft.}$$

The factor of safety against overturning is

$$\frac{225{,}900 \times 6.64}{82{,}500 \times 8} = 2.27$$

The factor of safety against sliding, including in the sliding resistance the resistance offered by the soil in front of the 12-in. key wall shown in Fig. 103, is

$$\frac{225{,}900 \times .4 + 10 \times 4000}{82{,}500} = 1.58$$

Vertical Slab. The vertical slab is designed as a simple slab supported by the counterforts. The thickness of slab required is governed by the pressure at the top of the base slab. For a horizontal strip of vertical slab 12 in. high, 22 ft. down from the top of the wall, the pressure per linear foot, from equation (86), Art. 182, is

$$P_1 = wxC = wx\frac{(1 - \sin \phi)}{(1 + \sin \phi)}$$
$$= 100 \times 22 \times .286 = 630 \text{ lb.}$$

The maximum bending moment in the strip, assuming the length as the clear distance between counterforts, is

$$M = \tfrac{1}{8} \times 630 \times \overline{8.5}^2 \times 12 = 68{,}400 \text{ in.-lb.}$$

$$d = \sqrt{\frac{68{,}400}{107.7 \times 12}} = 7.3 \text{ in.}$$

$$V = \tfrac{1}{2} \times 630 \times 8.5 = 2680 \text{ lb.}$$

$$d = \frac{2680}{40 \times \tfrac{7}{8} \times 12} = 6.4 \text{ in.}$$

The thickness of 12 in. adopted, furnishing an effective depth of 9 in., is satisfactory. A thinner section would be impractical because of the difficulty in placing the concrete in the forms.

With $K = \dfrac{68{,}400}{12 \times 9^2} = 70.5$, Table IV gives $j = .895$

$$A_s = \frac{68{,}400}{16{,}000 \times .895 \times 9} = 0.53 \text{ sq. in. per ft.}$$

½-in. square bars, 5.5 in. center to center, are selected.

$$u = \frac{2680}{2 \times \dfrac{12}{5.5} \times 895 \times 9} = 76 \text{ lb. per sq. in.}$$

A similar investigation for strips of the vertical arm at heights of 18, 12, and 6 ft. give spacings of ½-in. square bars required of 7, 12, and 24 in., respectively. The rods are placed as shown in Fig. 103.

Heel Slab. The rear portion of the base slab is designed as a simple beam supported by the counterforts. The load on the slab is equal to the difference between the upward soil pressure and the downward load from the earth above it. The load distribution is shown in Fig. 105. The slab is analyzed in strips 2 ft. wide. The strengthening action of its monolithic construction fully warrants such a procedure. The strip at the heel is subjected to a resultant load per foot of

$$(2200 + 300) \times 2 - \left(\frac{50 + 780}{2}\right) \times 2 = 4170 \text{ lb.}$$

$$M = \tfrac{1}{8} \times 4170 \times \overline{8.5}^2 \times 12 = 453{,}000 \text{ in.-lb.}$$

$$V = 4170 \times \frac{8.5}{2} = 17{,}700 \text{ lb.}$$

The depth required is governed in this case by the shear.

$$d = \frac{17{,}700}{40 \times .9 \times 24} = 20.5 \text{ in.}$$

Allowing 3 in. insulation for the steel, an effective depth of 21 in. is furnished and the assumed thickness of slab is satisfactory.

$$A_s = \frac{1}{2} \times \frac{453{,}000}{16{,}000 \times .9 \times 21} = 0.75 \text{ sq. in. per ft.}$$

⅝-in. round bars 5 in. center to center furnish the necessary area.

$$u = \frac{17{,}700}{1.964 \times {}^{24}\!/_{5} \times .9 \times 21} = 99 \text{ lb. per sq. in.}$$

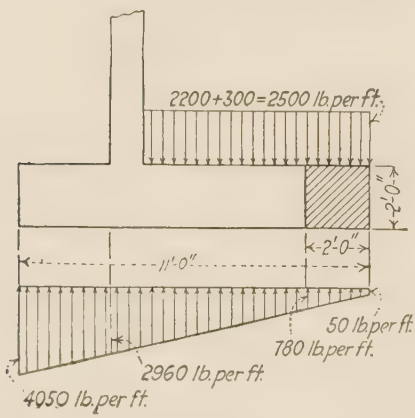


FIG. 105.

Deformed bars are necessary.

$$p = \frac{.3068}{5 \times 21} = .0029$$

From Table V, $j = .914$ and the revised steel area $= 0.74$ sq. in. per ft.

Since the downward load exceeds the upward pressure, the steel at the heel must be placed near the bottom of the slab. The point where the upward pressure becomes equal to the downward load is at a distance of x ft. from the heel, where

$$x = (2500 - 50) \times {}^{11}\!/_{4000} = 6.75 \text{ ft.}$$

The heel slab steel in front of this point is required in the top of the slab. In the present case, the top steel is omitted, since

the point of inflection is approximately under the rear face of the vertical slab.

Following an investigation of the other strips along the heel slab, the reinforcement is placed as shown in Fig. 103.

Toe Slab. The toe slab is designed as a free cantilever with a length of 3 ft.-0 in. The moment in the cantilever is due to an upward load as represented by the trapezoid of pressure underneath (Fig. 105) and a downward load equal to the weight of the toe slab itself. The resultant moment equals 183,800 in.-lb., and the resultant shear 9600 lb. With an effective depth of 20 in., the unit shear,

$$v = \frac{9600}{12 \times .9 \times 20} = 44 \text{ lb. per sq. in.,}$$

which may be considered satisfactory.

$$A_s = \frac{183{,}800}{16{,}000 \times .9 \times 20} = 0.64 \text{ sq. in. per ft.}$$

To satisfy requirements of bond, $\frac{5}{8}$-in. round deformed bars are spaced at $4\frac{1}{2}$ in. center to center, and every fourth bar continued to the heel of the wall to furnish transverse reinforcement in the heel slab.

$$u = \frac{9600}{1.964 \times \frac{12}{4.5} \times .9 \times 20} = 102 \text{ lb. per sq. in.}$$

Counterfort. The moment in the counterfort is due to the pressure of the earth on the vertical slab over a length of wall equal to the distance center to center of counterforts.

$$P = .143 \times 100 \times \overline{22}^2 \times 10 = 69{,}300 \text{ lb.}$$
$$y = \tfrac{1}{3} \times 22 = 7.33 \text{ ft.}$$
$$M = 69{,}300 \times 7.33 \times 12 = 6{,}110{,}000 \text{ in.-lb.}$$

The effective depth of the counterfort is the perpendicular distance from the point A (Fig. 103) to the reinforcing steel.

$$d = 8 \times \frac{22}{\sqrt{22^2 + 7^2}} \times 12 - 3 = 88 \text{ in.}$$

$$A_s = \frac{6{,}110{,}000}{16{,}000 \times .9 \times 88} = 4.83 \text{ sq. in.}$$

Five 1-in. square deformed bars are used.

$$p = \frac{5.0}{18 \times 88} = .0032,\ j = .912,\ A_s = 4.77$$

$$u = \frac{69{,}300}{4 \times 5 \times .912 \times 88} = 43 \text{ lb. per sq. in.}$$

The effective depth to be used in determining the unit shear on the base of the counterfort is equal to the horizontal distance from A (Fig. 103) to the reinforcing steel, that is,

$$d = (8 \times 12) - 3 = 93 \text{ in.}$$

$$v = \frac{69{,}300}{18 \times .912 \times 93} = 45 \text{ lb. per sq. in.}$$

This value of the unit shear is satisfactory because horizontal bars are provided to anchor the vertical slab and the counterfort together. These bars serve the added function of web reinforcement.

Following an investigation of the moment and shear at heights of 6, 12, and 18 ft., the rods are bent into the counterfort in pairs as shown in Fig. 103, and anchored to the vertical slab steel.

To provide for the pull of the vertical slab, horizontal ½-in. square bars are placed in pairs in the counterfort, one on either side, so arranged as to hook over the vertical slab reinforcement and extend to the rear of the counterfort. These bars are so spaced as to engage every other bar in the vertical slab. The amount of pull at the base of the vertical slab per foot of height is

$$.286 \times 100 \times 22 \times 8.5 = 5360 \text{ lb.}$$

The area of steel required per foot of height is

$$\frac{5360}{16{,}000} = .334 \text{ sq. in.}$$

The ½-in. square bars, 11 in. center to center, furnish 0.56 sq. in. Similar investigations at various heights show that the arrangement described above is satisfactory. Vertical ½-in. square bars are placed at intervals in the counterfort and anchored to the base slab reinforcement as shown in Fig. 103.

The main reinforcing bars must extend into the base slab a distance

$$l_1 = \frac{16{,}000}{4 \times 100} \times 1 = 40 \text{ in.}$$

A key wall 1 ft.-6 in. deep and 2 ft.-0 in. wide is constructed under the heel of the wall to provide the necessary embedment. The main bars are hooked as a further precaution.

In the above design the vertical slab and rear portion of the base slab have been assumed as simply supported and no provision made for continuity over the supports (the counterforts). Some designers prefer to consider these portions as partially or fully continuous, and to provide for the negative moment at the supports by bending every second or third bar to the opposite face of the slab. Where such an assumption is made, a moment coefficient of $1/10$ or $1/12$ may, of course, be used.

To provide for the negative moment, some bars may be bent to the opposite face at the counterforts, or additional straight bars may be placed in that face across the counterforts. The length of such bars should be from 0.5 to 0.6 of the spacing of counterforts. In the higher walls some provision should be made for continuity even though the slabs are designed as simply supported.

CHAPTER X

ARCHES

193. Advantages and Forms of Reinforcement. An arch with a parabolic axis and loaded with a fixed uniform load would require no steel reinforcement, for the line of pressure would coincide with the axis of the arch, no moment would be produced in any section, and the stresses would be wholly compressive. In the principal adaptation of the concrete arch, namely the concrete arch bridge, the live load is not fixed, and the most desirable form of arch departs considerably from the parabola. Provided the ratio of live load to dead load is so small that under no loading conditions the line of the resultant pressure departs from the middle third, compressive stresses only exist. Often, however, in order to accomplish this result without the use of reinforcement, rather heavy sections are required. With the use of steel reinforcement it is not necessary to keep the line of pressure within the middle third as the steel can care for tensile stresses just as in a reinforced concrete beam.

As in all concrete members whose main stress is compression, there is no theoretical economy in the use of the reinforcement. With the line of pressure within the middle third, the stress in the steel is compression and the working values low. Even when the line of pressure is outside the middle third, the tensile stress is usually not great, as the compressive stress in the concrete on the other side of the section is the governing factor. As in columns, the use of the reinforcement makes the arch a more secure and reliable structure and aids in preventing cracks due to settlement.

The forms of reinforcement vary. The smaller arches are now generally constructed with ordinary reinforcing bars on both sides, these bars being usually tied together in some manner in order to be more effective in aiding the concrete in resisting the compression. Some of the earlier arches used wire netting as reinforce-

ment and some of the so-called "system" types use certain forms of structural steel. A few of the larger arches of the present day are built with heavy structural steel reinforcement, or a combination of structural steel and ordinary reinforcing bars.

194. The Curve of the Intrados. The form of the curve of the intrados (see Fig. 106) is one of the main considerations in the design. This curve is usually circular, multi-centered, elliptical or parabolic. The multi-centered arch is perhaps the most common, as it gives a pleasing appearance and generally an economical design. It is usually either three centered or five

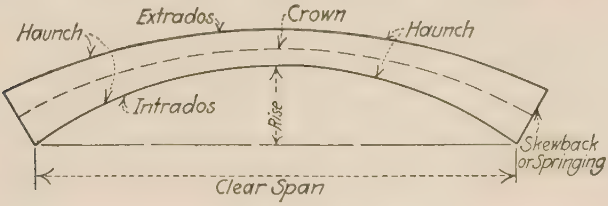


FIG. 106.

centered. The three-centered arch ordinarily gives the more economic design, but the five-centered arch has more graceful lines and is often necessary on account of clearance requirements. The formulas for the radii when the span and rise are known are as follows (see Fig. 107):

$$R_1 = \frac{x^2 + y^2}{2y}$$

$$R_2 = \frac{1}{2} \cdot \frac{\overline{BE}^2 + \overline{ED}^2}{ED \cos \theta_1 - BE \sin \theta_1}$$

$$R_3 = \frac{1}{2} \cdot \frac{\overline{AF}^2 + \overline{FB}^2}{FB \cos (\theta_1 + \theta_2) - AF \cos (\theta_1 + \theta_2)}$$

$$\sin \theta_1 = \frac{x}{R_1}$$

$$\sin \frac{1}{2}\theta_2 = \frac{\frac{1}{2}\sqrt{\overline{DE}^2 + \overline{BE}^2}}{R_2}$$

195. Spandrels. The space between the back of the arch (the curve of the extrados) and the roadway is known as the spandrel. This space may be entirely filled with earth which is retained by side walls supported by the arch ring. This type of

construction is known as the filled spandrel arch. The spandrel space may, on the other hand, be left more or less open, and the roadway supported on a series of transverse or longitudinal walls or on a complete superstructure of columns, girders, beams, and slabs. This is known as open spandrel construction. With filled spandrels, the side walls may be of the gravity type or they may be reinforced and tied together with cross walls. Solid filling

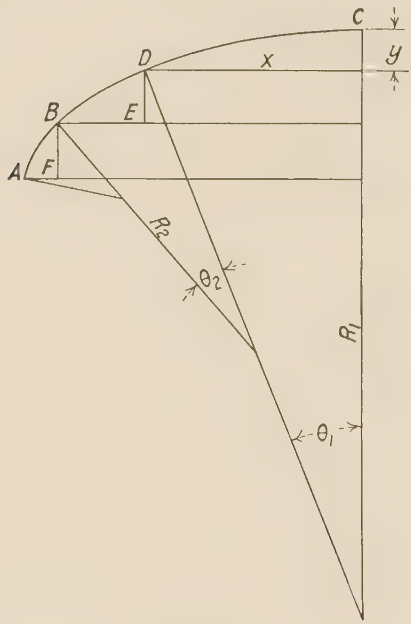


Fig. 107.

increases the dead load, but open spandrel construction requires a much larger amount of form work. For short span arches and for arches where the ratio of rise to span is small, the filled spandrel type will usually be found to be the more economical.

196. Loads. The principal load on an arch ring is the dead load, which consists of the weight of the arch ring itself, the spandrel construction, and the roadway. With open spandrel construction the dead loads (except that of the ring itself) and their points of application are definitely known. In such cases it is usual to assume the weight of the arch ring as concentrated

at the point of application of the superimposed loads. While in filled arches the pressure from the filling is really inclined, it is on the side of safety to neglect the horizontal component of the inclined force and design for the vertical loads only. Except in arches of comparatively high rise, the error is not great.

The depth of filling at the crown depends on the type of load which the arch is to sustain. For highway bridges it is rarely advisable to use less than 1 ft. below the pavement, while for railroad bridges not less than 2 ft. below the ballast should be used in order to distribute the load uniformly and absorb the shocks from rapidly moving traffic.

The weight of the earth filling is taken as either 100 or 120 lb. per cu. ft. The latter value is generally used where it is probable that at times the filling may be thoroughly saturated with water. Ballast is assumed to weigh 120 lb. per cu. ft. and pavement (except wood block) and concrete 150 lb. per cu. ft.

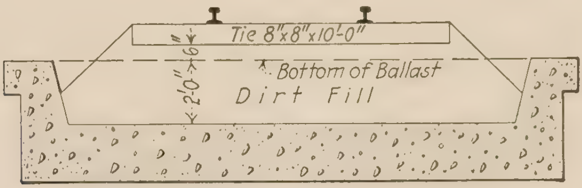


Fig. 108.

The live load used in a design should be the greatest that is likely to come upon the structure. For railroad bridges of 100-ft. span or less, the weight of the locomotive properly distributed should furnish the basis for design. For longer spans a somewhat lighter load should be used. Where the filling is sufficient thoroughly to distribute the load, an equivalent uniform live load may be used. For highway bridges the same live loads are used as for the design of slab, beam, and girder bridges (see Chap. XI) except that in filled arches no allowance for impact need be made.

197. The Arch Ring. The arch ring may have a width equal or nearly so to that of the roadway it supports, or a series of narrow rings or ribs may be constructed on which the roadway is supported. The former type is known as the barrel type and is

illustrated in Fig. 108. The latter is known as the ribbed type and is illustrated by Fig. 109. The former is always used with filled spandrel construction. It is usual to make such a design for a section of arch 1 ft. in width as in the design of slabs reinforced in one direction only. The weight of roadway, fill, and arch ring may be assumed concentrated at a number of selected points, without any great error. In the ribbed type, two ribs give the more simple design, and should preferably be used except for very wide bridges.

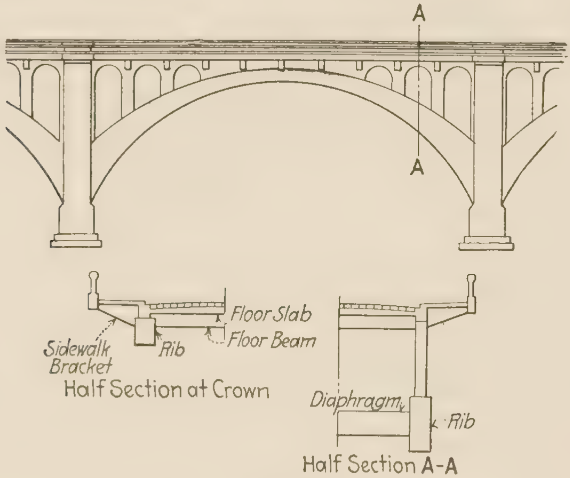


FIG. 109.

198. Crown Thickness. Various empirical formulas have been proposed for determining a trial thickness of arch ring at the crown. These depend upon the span, the rise, and the loads to be sustained. They give variable results, and none of them seems to fit a great variety of conditions. In flat arches the temperature and arch shortening stresses are of great importance, and no formula so far developed, to the authors' knowledge, gives sufficiently definite consideration to these factors. Figures 110 and 111, taken from an article by Joseph P. Schwada in the *Engineering News* of Nov. 9, 1916, seem to give as satisfactory a final thickness as any other method. It should be noted that these curves are for barrel arches for definite unit stresses and loads, and are based on a temperature variation of ± 20 deg.

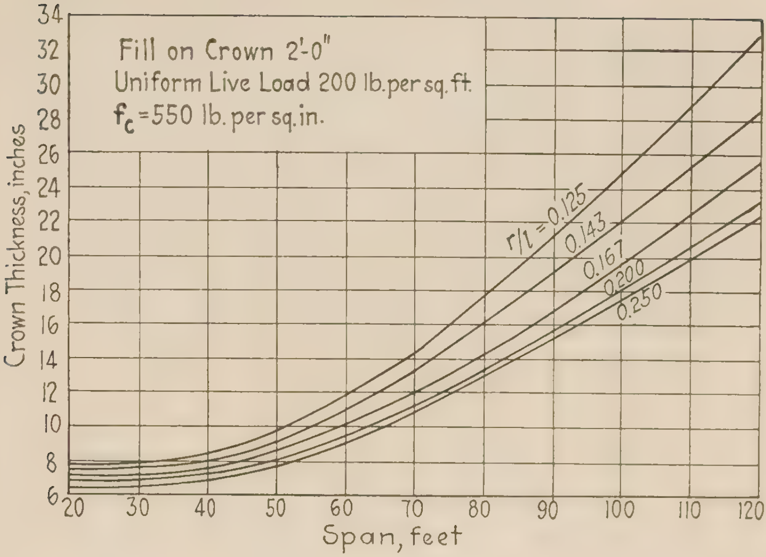


FIG. 110.

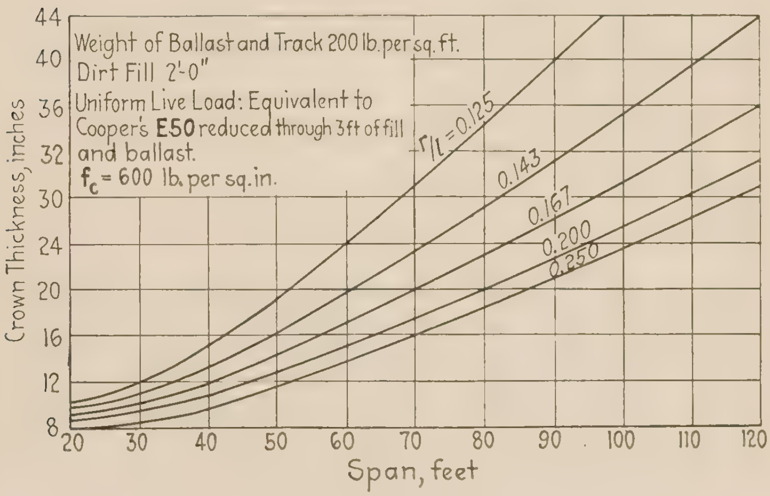


FIG. 111.

Fahrenheit. Allowance must be made for other unit stresses, loads, and ranges of temperature.

199. Analysis of the Arch by the Elastic Theory. Let Fig. 112 represent a curved beam, the curvature of the beam being large in proportion to its depth, so that the length of all fibers may be considered equal. Assuming ab as fixed and considering an elementary length Δs, the plane dc, in deflecting, rotates through an angle $\Delta\phi$ to the position $d'c'$. The change in length of a fiber at a distance e from the neutral axis is $e\Delta\phi$. The deformation per unit length is then $\frac{e\Delta\phi}{\Delta s} = \frac{f}{E}$. But $M = \frac{fI}{e} = \frac{E\Delta\phi I}{\Delta s}$

and
$$\Delta\phi = \frac{M\Delta s}{EI}.$$

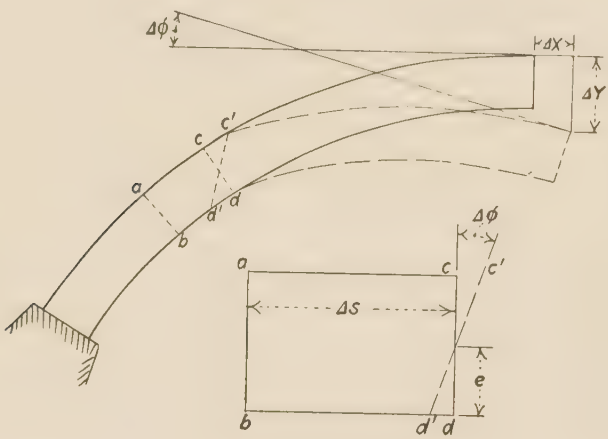


FIG. 112.

In the rotation of bending, it is assumed that each small block successively changes, beginning from the left end. In Fig. 113 from similar triangles since $r\Delta\phi$ is practically perpendicular to r, $\frac{r\Delta\phi}{r} = \frac{\Delta y}{x}$ so that $\Delta y = x\Delta\phi$ and since for a positive moment Δy is negative, substituting the value of $\Delta\phi$

$$\Delta y = -\frac{Mx\Delta s}{EI}$$

Similarly
$$\Delta x = \frac{My\Delta s}{EI}$$

A concrete arch with fixed ends is statically indeterminate since there are six unknown quantities, three at each support, namely the vertical and horizontal components of the reaction and the bending moment. Since the ends are fixed $\Delta x = 0$, $\Delta y = 0$, and $\Delta\phi = 0$. By considering the arch cut at the crown, the

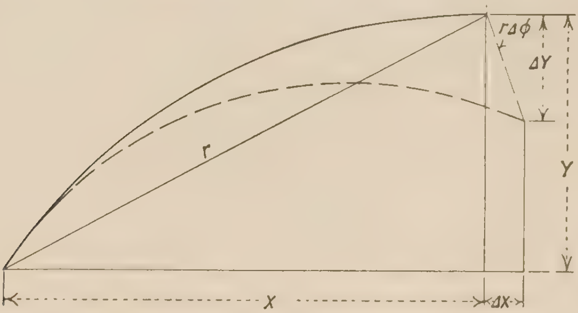


FIG. 113.

thrust, shear, and moment at that point may be determined, and these being known, each half of the arch becomes statically determinate.

With the origin of coordinates at the crown C (Fig. 114), the horizontal movement of C due to bending bears the same relation to each cantilever. Then from the theory developed above

$$\sum_C^A \Delta x = -\sum_C^B \Delta x$$

or

$$\sum_C^A \frac{My\Delta s}{EI} = -\sum_C^B \frac{My\Delta s}{EI} \tag{92}$$

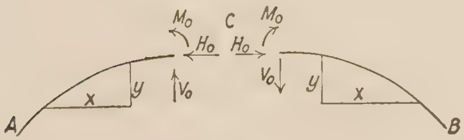


FIG. 114.

The changes in Δy are equal and in the same direction, so that

$$\sum_C^A \frac{Mx\Delta s}{EI} = \sum_C^B \frac{Mx\Delta s}{EI} \tag{93}$$

Also the changes in direction of the tangent to the axis at C are equal but opposite in direction, hence,

$$\sum_C^A \frac{M\Delta s}{EI} = -\sum_C^B \frac{M\Delta s}{EI} \tag{94}$$

Denoting $\sum_C^A M$ as $\sum M_L$ and $\sum_C^B M$ as $\sum M_R$, dividing the arch ring into divisions such that Δs is a constant[1] and eliminating

[1] If the arch is divided so that $\frac{I}{s}$ is a constant, the equations for H_o, V_o, and M_o are

$$H_o = \frac{n\Sigma my - \Sigma m\Sigma y}{2[(\Sigma y)^2 - n\Sigma y^2]}$$

$$V_o = \frac{\Sigma mx}{2\Sigma x^2}$$

$$M_o = \frac{\Sigma m + 2H_o\Sigma y}{2n}$$

In this case all ys are measured downward from the axis through the crown and are considered as negative; n equals the number of divisions in one-half of the arch.

For temperature changes

$$H_o = \mp\frac{EI}{s}\cdot\frac{\omega tln}{2[n\Sigma y^2 - (\Sigma y)^2]}$$

$$M_o = \frac{H_o\Sigma y}{n}$$

and

$$M = M_o + H_o y$$

the constant E

$$\sum M_L \frac{y}{I} = -\sum M_R \frac{y}{I} \tag{95}$$

$$\sum M_L \frac{x}{I} = \sum M_R \frac{x}{I} \tag{96}$$

$$\sum \frac{M_L}{I} = -\sum \frac{M_R}{I} \tag{97}$$

Considering the left half of the arch as a free body (Fig. 115), for any section M_o tends to produce counterclockwise rotation, which will be taken as positive. Similarly V_o is always positive. H_o produces compression and acts towards the cut section at the crown. Hence for any section

$$M = M_o + H_o y + V_o x - m \tag{98}$$

where m is the bending moment at the section due to the external loads.

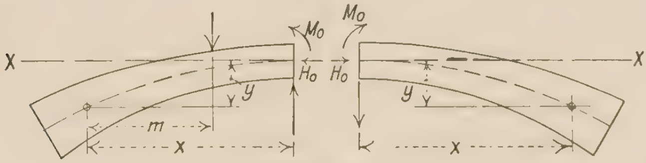


FIG. 115.

Similarly for the right half

$$M = M_o + H_o y - V_o x - m \tag{99}$$

For arch shortening

$$H_o = \frac{I}{s} \cdot \frac{c_a l n}{2[n \Sigma y^2 - (\Sigma y)^2]}$$

The graphical method of dividing the half axis into divisions so that $\frac{I}{s}$ is constant is shown in the accompanying figure. AB is drawn equal to one-half the arch axis. The curve CD is then drawn through points whose ordinates are I and whose abscissas are the corresponding distances along the arch axis measured from the springing. A length AE is then assumed, a perpendicular FG erected at its center, and the lines AG and GE drawn. Starting from E, lines are drawn parallel alternately to AG and GE. Only two or three trials are usually necessary to divide the axis into the required number of divisions. For most arches ten divisions are sufficient. The base of each triangle thus formed corresponds to s and its altitude to I. Since all the triangles are similar, $\frac{I}{s}$ is constant throughout. The center of all these divisions is now located on a drawing of the arch axis and their coordinates with the crown as the origin determined.

Substituting in equations (95,), (96), and (97), and combining the terms

$$2H_o\sum\frac{y^2}{I} + 2M_o\sum\frac{y}{I} - \sum\frac{m_Ly}{I} - \sum\frac{m_Ry}{I} = 0 \qquad (100)$$

$$2V_o\sum\frac{x^2}{I} - \sum\frac{m_Lx}{I} + \sum\frac{m_Ry}{I} = 0 \qquad (101)$$

$$2H_o\sum\frac{y}{I} + 2M_o\sum\frac{1}{I} - \sum\frac{m_L}{I} - \sum\frac{m_R}{I} = 0 \qquad (102)$$

Considering the application of load on the left half of the arch only, the terms containing m_R disappear. Combining equations (100) and (102),

$$H_o = \frac{\sum\frac{my}{I}\sum\frac{1}{I} - \sum\frac{m}{I}\sum\frac{y}{I}}{2\left[\sum\frac{y^2}{I}\sum\frac{1}{I} - \left(\sum\frac{y}{I}\right)^2\right]} \qquad (103)$$

$$M_o = \frac{\sum\frac{m}{I} - 2H_o\sum\frac{y}{I}}{2\sum\frac{1}{I}} \qquad (104)$$

while

$$V_o = \frac{\sum\frac{mx}{I}}{2\sum\frac{x^2}{I}} \qquad (105)$$

If the origin of coordinates[2] is shifted so that $\sum\frac{y}{I} = 0$, equations

[2] This position may be determined as follows:

Let y' = distance from axis at crown to axis taken so that $\sum\frac{y}{I} = 0$

y_c = distance from axis at crown to any division point

y = corresponding distance from new axis to the same division point

Then $$y' = y_c - y \text{ or } y = y_c - y'$$

and $$\sum\left(\frac{y_c - y'}{I}\right) = 0$$

or $$\sum\frac{y_c}{I} - \sum\frac{y'}{I} = 0$$

Therefore,

$$\sum\frac{y_c}{I} = \sum\frac{y'}{I} \text{ and since } y' \text{ is a constant}$$

$$y' = \frac{\sum\frac{y_c}{I}}{\sum\frac{1}{I}}$$

(103) and (104) become

$$H_o = \frac{\sum \frac{my}{I}}{2\sum \frac{y^2}{I}} \tag{106}$$

$$M_o = \frac{\sum \frac{m}{I}}{2\sum \frac{1}{I}} \tag{107}$$

The ordinate y when measured upward from the axis is taken as positive, when measured downward as negative.

For temperature stresses Δx is equal to the change in length of the half span $= \frac{\omega t l}{2}$, where ω = the coefficient for one degree of temperature change, t the number of degrees of temperature change, and l the span. Then from equation (92)

$$\sum \frac{M_L y}{I} \cdot \frac{\Delta s}{E} = \frac{\omega t l}{2}$$

Also since $\Delta\phi = 0$

$$\sum \frac{M_L}{I} = 0$$

There being no external loads, $m = 0$, and from symmetry $V_o = 0$, hence $M = M_o + H_o y$. Substituting this value of M in the above equations

$$M_o \sum \frac{y}{I} + H_o \sum \frac{y^2}{I} = \frac{\omega t l}{2} \cdot \frac{E}{s}$$

and

$$\sum \frac{M_o}{I} + \sum H_o \frac{y}{I} = 0$$

Equation (106) will give the same value of H_o as equation (103) while the value of M_o from equation (107) is different by $\frac{H_o \sum \frac{y}{I}}{\sum \frac{1}{I}} = H_o y'$ from that determined from equation (104). In the general expression for moment at any section, $M = M_o + H_o y + V_o x - m$ the sum of the first two terms is the same whichever position of the axis is taken as a basis for the summations.

But with
$$\sum \frac{y}{I} = 0$$

$$H_o = \pm \frac{\omega t l}{2\sum \frac{y^2}{I}} \cdot \frac{E}{s} \qquad (108)^3$$

$$M_o = 0$$

and
$$M = H_o y \qquad (109)$$

A thrust throughout the arch producing an average stress on the concrete equal to c_a lb. per sq. in. would shorten the arch span an amount equal to $\frac{c_a l}{E}$ if the arch and the abutments were not fixed. Since the abutments are fixed, and the arch cannot shorten, there is a tensile stress developed. The action is similar to that of a fall in temperature. The resulting H_0 may be found by substituting $\frac{c_a l}{E}$ for $\omega t l$ of equation (108). There results

$$H_o = + \frac{c_a l}{2s\sum \frac{y^2}{I}} \qquad (110)$$

and similarly
$$M = H_o y.$$

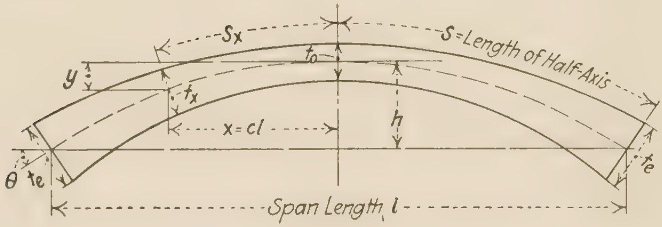


FIG. 116.

200. Approximate Methods of Analysis. Since the form and dimensions of an arch must be assumed before any calculations are made, it frequently happens that the first assumptions do not give an economical design, and all the calculations must be repeated. A complete analysis is a long and tedious operation and it is desirable to have a method for more nearly determining the dimensions in advance so that the final stresses will be close to the desired values. Victor A. Cochrane[4] has developed a series of approximate equations for determining stresses in an

[3] + for a fall in temperature
− for a rise in temperature

[4] *Proceedings*, Engineers' Society of Western Pennsylvania, vol. 32, p. 647.

TABLE OF FORMULAS FOR THRUSTS AND MOMENTS AT CROWN AND SPRINGING SECTIONS OF ARCHES

OPEN SPANDREL ARCHES

STRESSES PRODUCED BY	SECTION AT CROWN	SECTION AT SPRINGING
DEAD LOAD	$T_c = \frac{6+5r}{48r} w_c l$ M_c assumed $= 0$	$V_s = \frac{3+5r}{6} w_c l$ $T_s = \sqrt{T_c^2 + V_s^2}$ M_s assumed $= 0$
LIVE LOAD MAXIMUM POSITIVE MOMENT	$T_c = \frac{59.5+31.5r+5r^2}{1000r} wl$ $M_c = +\frac{67.5+80r+40r^2-(20+13r)U_s+3U_s^2}{10000} wl^2$	$T_s = \frac{93.3-42.5r+155r^2}{1000r} wl$ $T_c = (\frac{1}{2}Tm_2+Tm_1) - \frac{0.0063}{U_s r} wl$ $M_s = +\frac{322+27r-(18.1+13r)(3.79-0.6r-U_s)^2}{10000} wl^2$
LIVE LOAD MAXIMUM NEGATIVE MOMENT	$T_c = \frac{61+2U_s+(7+2U_s)r-70r^2}{1000r} wl$ $M_c = -\frac{55-9U_s+0.85U_s^2}{10000} wl$	$T_s = \frac{24.4+(211-2U_s)r+(120+30U_s)r^2}{1000r} wl$ $T_c = (\frac{1}{2}Tm_2 + \frac{0.0063}{U_s r}) wl$ $M_s = \frac{289-172r-(17.8-28r)(3.36+2.4r-U_s)^2}{10000} wl^2$
TEMPERATURE (FALL OF $t°$)	$T_c = -[18.4U_s-7.5-(20.5U_s-15)r-(10U_s-2)r^2]\frac{\omega t EI_0}{h^2}$ $M_c = -[46.25-17.5U_s+2.44U_s^2+5r+(25.5-3.6U_s)r^2]\frac{hT_c}{100}$	$T_s = (1.09-1.75r)T_c$ $M_s = M_c + hT_c$

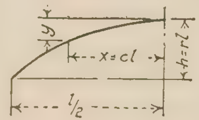
FILLED SPANDREL ARCHES

STRESSES PRODUCED BY	SECTION AT CROWN	SECTION AT SPRINGING
DEAD LOAD	$T_c = \frac{1+3r}{8r} w_c l$ M_c assumed $= 0$	$V_s = \frac{2+15r}{4} w_c l$ $T_s = \sqrt{T_c^2 + V_s^2}$ M_s assumed $= 0$
LIVE LOAD MAXIMUM POSITIVE MOMENT	$T_c = \frac{57.6+(189-8U_s)r-220r^2}{1000r} wl$ $M_c = +\frac{72+105r+220r^2-(17+10r)U_s+1.5U_s^2}{10000} wl^2$	$T_s = \frac{84+6.6U_s+(153-51U_s)r-(500-140U_s)r^2}{1000r} wl$ $T_c = [\frac{1}{2}Tm_2 + Tm_1 + \frac{0.0026(U_s-2)^2}{r}] wl$ $M_s = +\frac{316+1370r-2100r^2-(17-20r)(3.68+7.6r-U_s)^2}{10000} wl^2$
LIVE LOAD MAXIMUM NEGATIVE MOMENT	$T_c = \frac{57.8+2U_s+(10+30U_s)r-(380+30U_s)r^2}{1000r} wl$ $M_c = -\frac{49.4-100r-(3-26r)U_s-0.75U_s^2}{10000} wl^2$	$T_s = \frac{27.6+(125+6U_s)r+320r^2}{1000r} wl$ $T_c = [\frac{1}{2}Tm_2 - \frac{0.0026(U_s-2)^2}{r}] wl$ $M_s = -\frac{283-480r-9(4.22-2.8r-U_s)^2}{10000} wl^2$
TEMPERATURE (FALL OF $t°$)	$T_c = -[19.4U_s-7.5+(17U_s-31)r-140(U_s-1)r^2]\frac{\omega t EI_0}{h^2}$ $M_c = -(38.5-12.8U_s+1.6U_s^2)\frac{hT_c}{100}$	$T_s = (1.13-2.55r)T_c$ $M_s = M_c + hT_c$

AVERAGE STRESSES

FOR DEAD LOAD	$f_a = [1.030+2.5(r+0.05)^2 - \frac{(20r+8)U_s-(U_s-1)^2}{100}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. POS. MOMENT AT CROWN	$f_a = [0.920+2.6r^3-0.04U_s+\frac{(6.7+33r)(4-U_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. NEG. MOMENT AT CROWN	$f_a = [0.940+1.1(r+0.1)^2-0.055U_s+\frac{(6+155r^2)(4-U_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. POS. MOMENT AT SPRINGING	$f_a = [0.920+3.7r^3-0.04U_s+\frac{(7.5r+37r)(4-U_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. NEG. MOMENT AT SPRINGING	$f_a = [0.950+1.7r^2-0.05U_s+\frac{(4+48r)(4-U_s)^2}{1000}] f_{ac}$
FOR TEMPERATURE AND ARCH SHORTENING STRESSES	$f_a = [1.075-0.8r-(0.081-0.11r)U_s] f_{ac}$

These formulas are for arches having axes as determined by the following equations:



For Open Spandrel Arches:

$$y = \frac{8rl}{6+5r}(3c^2+10c^4r)$$

For Filled Spandrel Arches:

$$y = \frac{4rl}{1+3r}(c^2+24c^5r)$$

arch ring, which give values very close to those obtained by the exact method. While the authors' experience with these equations seems to indicate that the results obtained by their use are slightly smaller than those obtained by exact analysis, they are sufficiently accurate to warrant their use in preliminary calculations, and in less important structures might serve as a basis for the final designs. Some of the equations developed by Mr. Cochrane are given on page 377. The following notation is used.

l = span of arch axis in feet
h = rise of arch axis in feet
r = rise ratio $\frac{h}{l}$
y = ordinate of arch axis, the abscissa of which is cl
s = length of half axis, measured along axis from crown to springing
s_x = distance along axis from crown to a given section whose abscissa is $cl = x$
t_o = thickness of arch rib at crown
t_e = thickness of arch rib at springing
t_x = thickness of arch rib at point whose abscissa is x
$v = \frac{s_x}{s}$
u_s = ratio of thickness of springing to thickness at crown = $\frac{t_s}{t_o}$
M_c = moment at crown in foot-pounds
T_c = thrust at crown in pounds
M_s = moment at springing in foot-pounds
T_s = thrust at springing in pounds
T_{m_1} = coefficient for wl for thrust at crown corresponding to maximum positive moment at crown
T_{m_2} = coefficient of wl for thrust at crown corresponding to maximum negative moment at crown
V_s = approximate dead load vertical end reaction, or one-half dead weight of span in pounds
w_c = weight of arch at crown, plus average weight of superstructure at crown, in pounds per linear foot of span.

w = live load in pounds per linear foot of span (not necessarily the same for all positions of the live load)
ω = coefficient of linear expansion due to temperature changes
t = change in temperature in degrees Fahrenheit
E = modulus of elasticity of concrete in pounds per square foot
I_o = moment of inertia of arch rib at crown in bi-quadratic feet
f_a = average direct stress throughout arch in pounds per square foot
f_{ac} = direct stress at crown section in pounds per square foot

For open spandrel arches

$$y = \frac{8rl}{6 + 5r} \cdot (3c^2 + 10c^4r) \qquad (111)$$

$$\text{and } \tan\theta = \frac{8r}{6 + 5r} \cdot (3 + 5r) \qquad (112)$$

For filled spandrel arches

$$y = \frac{4rl}{1 + 3r} \cdot (c^2 + 24c^5r) \qquad (113)$$

$$\text{and } \tan\theta = \frac{4r}{1 + 3r} \cdot (1 + 7.5r) \qquad (114)$$

If the half axis is divided into ten equal sections, the ratio of the depth of the arch at the center of each section to the depth at the crown is given in the following table.

THICKNESSES OF TYPICAL ARCHES

Value of v	Values of u_s							
	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.05	1.007	1.006	1.005	1.004	1.003	1.002	1.001	1.000
.15	1.021	1.018	1.015	1.012	1.009	1.006	1.003	1.000
.25	1.035	1.030	1.025	1.020	1.015	1.010	1.005	1.000
.35	1.049	1.042	1.035	1.028	1.023	1.021	1.023	1.030
.45	1.063	1.054	1.048	1.048	1.057	1.070	1.083	1.101
.55	1.077	1.072	1.085	1.105	1.133	1.165	1.193	1.231
.65	1.095	1.125	1.168	1.215	1.269	1.328	1.385	1.455
.75	1.145	1.223	1.311	1.403	1.508	1.625	1.737	1.865
.85	1.245	1.393	1.547	1.700	1.862	2.025	2.185	2.355
.95	1.406	1.621	1.837	2.055	2.277	2.495	2.709	2.932
1.00	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250

The value of s, the length of the half axis, may be determined by scaling from the drawing or it may be taken by interpolation from the following table:

Lengths of the Half Arch Axis s in Terms of the Span Length l

Kinds of arches	Values of $\frac{s}{l}$ for rise-ratio r =				
	.10	.15	.20	.25	.30
Open spandrel arches	.513	.529	.551	.577	.607
Filled spandrel arches	.515	.534	.559	....	....

The formulas for moment, thrusts, and average stresses are given on page 377.

201. Form of Arch Axis. It is the usual practice to make the arch axis conform to the dead load equilibrium polygon through the crown and the springing. The positive live load moments (those producing compression in the upper fiber) are greater than the negative live load moments at both the crown and springing sections. At certain sections in the haunch the negative live load moments will be the greater. If the axis is made to conform to the equilibrium polygon for dead load plus one-half the live load over the whole span, the total maximum positive and negative moments due to live load and dead load will be equal. Unless however, the ratio of live to dead load is unusually large, there will be little difference between such an axis and the one conforming to the dead load equilibrium polygon.

The effect of the shortening of the arch axis is to produce positive bending moments at the crown and larger negative bending moments at the springing. Also the fall in temperature is often specified as greater than the rise which tends to produce larger positive than negative moments at the crown and larger negative than positive moments at the springing. In the haunch of flat arches the stresses produced by arch shortening and temperature changes are not nearly so great as those produced at the springing.

It is of benefit at the springing to make the arch axis conform to the dead load equilibrium polygon, since the arch shortening moments and the excess of fall over rise of temperature moments are the reverse of the larger live load moments, thus making the total positive and negative moments more nearly equal. The reverse is true at the crown, but since the springing section is much the larger it seems desirable to favor it rather than the crown section. The method of laying out the arch axis for dead load is outlined in Art. 203.

202. Procedure in Arch Design.

1. Assume a crown thickness from Fig. 110 or Fig. 111, or by comparison with a previous design, and compute the total dead load per linear foot of span at the crown.

2. Assume a vertical springing thickness of from two to three times the crown thickness. For flat, heavily loaded arches the lower limit should be assumed, while the upper limit will give best results for arches of high rise. Assume the amount of reinforcement at the crown and at the springing.

3. Make approximate computations for the length of the span and rise of the arch axis and determine the rise ratio, $\frac{h}{l}$.

4. By the equations of Art. 200 calculate the extreme fiber stresses due to the proper combinations of moments and thrusts using the methods and diagrams of Chap. IV. If the stresses are too small or too great, change the thickness at the crown or at the springing or both, and repeat the operation. It is usually not necessary to make approximate calculations for maximum negative moment at the crown or for maximum positive moment at the springing, especially if the fall in temperature is taken as being appreciably greater than the rise in temperature.

5. Lay out the arch axis according to equation (111) or equation (113). In flat arches y should be computed at the quarter point of the span and for three or four intermediate points between quarter point and springing. In arches of high rise, one additional point between quarter point and crown should be determined. From the first table of Art. 200 determine the thickness of the arch at several points, and draw the curves of the intrados, extrados, and neutral axis.

6. Compute the dead loads at the panel points or at suitable intervals and lay out an equilibrium polygon passing through the crown and springing.

7. Alter the shape of the arch axis so that it will fit the equilibrium polygon as nearly as practicable. Lay out the arch thickness again and determine the radii of the intrados, extrados, and neutral axis.

8. Analyze the arch so determined by the elastic theory for maximum stresses in the steel and in the concrete. In most arches the maximum stresses occur either at the crown or at the springing, although where the ratio of live to dead load is large the maxium stresses may be found in the haunch. For aesthetic reasons the arch ring must gradually increase in thickness from crown to springing. Such a ring has a thickness much greater than required over the greater part of the distance between crown and springing. For this reason an investigation of the crown and springing sections is usually sufficient.

203. Design of a Reinforced Concrete Arch.

Type—Filled spandrel

Clear span—70 ft.-0 in.

Rise of intrados—10 ft.-0 in.

Live loading—Cooper's E-60

Ballast 6 in. under ties

Minimum fill—2 ft.-0 in.

Unit stresses—for dead load, live load, and arch shortening

f_c—not greater than 650 lb.

f_s—not greater than 16,000 lb.

When temperature stresses are included an increase of 25 per cent in the above stresses is permissible.

Arch to be designed for a rise in temperature of 20 deg. Fahrenheit and for a fall in temperature of 30 deg. Fahrenheit.

Considering the weight of one locomotive distributed over 50 ft. of track and the load per foot distributed to the arch as shown in Fig. 108, the live load per foot section is about 600 lb. per ft. The rails and fastenings are assumed to weigh 150 lb. per ft. of track, and ballast and fill 120 lb. per cu. ft.

Ties are 8 in. $\times$ 8 in. $\times$ 10 ft.-0 in. In making computations for dead load it is assumed that the top of the ballast is level

with the base of rail and the weight of the ties neglected.

From Schwada's Curves (as given in Fig. 111), the crown thickness is assumed as 22 in. and the vertical springing thickness as two times the crown thickness, or 44 in.

In Fig. 117, assuming ae as a straight line, and $dc = \frac{1}{2}ce = 22$ in.

$h = 120 \text{ in.} + 11 \text{ in.} - 22 \text{ in.} \cos^2\theta$

$l = 840 \text{ in.} + 2(22 \text{ in.} \cos\theta \sin\theta)$

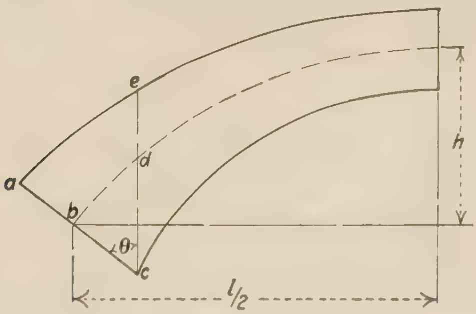


FIG. 117.

Assume $\theta = 38°$; $\sin 38° = .616$, $\cos 38° = .788$

$h = 117.3$ in.

$l = 861.4$ in.

$r = .136$

But $\tan\theta = \dfrac{4r}{1 + 3r} \cdot (1 + 7.5r) =$ (See page 379.)

$$\frac{4 \times .136}{1 + 3 \times .136} \cdot (1 + 7.5 \times .136) = .781$$

$\tan 38° = .781$.

Radial springing thickness $= 44 \times .788 = 34.7$

A springing thickness of 34 in. is assumed.

Reinforcement at crown $\frac{7}{8}$-in. round rods 6 in. center to center, one row 2 in. from extrados and one row 2 in. from intrados. Therefore $p = \dfrac{2.41}{12 \times 22} = .0091$ and $\dfrac{d'}{a} = \dfrac{2}{22} = .09$. At a point 10 ft.-0 in. from the springing, these rods are lapped with 1-in. round rods, the latter being carried through the springing section.

Therefore, at the springing, $p = \frac{3.14}{12 \times 34} = .0077$ and $\frac{d'}{a} = \frac{2}{34} = .06$. The dead load at the crown consists of

Fill	240 lb.
Ballast	140 lb.
Rails	10 lb.
Concrete	275 lb.
	665 lb.

Approximate Method of Testing Trial Arch

$$l = 71.8$$
$$h = 9.8$$
$$r = .136$$
$$u_s = \frac{34}{22} = 1.55$$
$$w_c = 665$$
$$w = 600$$
$$\omega = .000006$$
$$E = 288{,}000{,}000 \text{ lb. per sq. ft.}$$
$$t = +20 \text{ deg. Fahrenheit or } -30 \text{ deg. Fahrenheit}$$
$$\omega t E = 34{,}560 \text{ or } 51{,}840$$
$$I_o = \frac{1}{12} \times 1 \times \left(\frac{22}{12}\right)^3 + 14 \times \frac{2.41}{144} \times \left(\frac{9}{12}\right)^2 = .643$$
$$A_o = \frac{22}{12} + \frac{2.41}{144} \times 14 = 2.07$$
$$wl = 600 \times 71.8 = 43{,}100 \text{ lb.}$$
$$wl^2 = 3{,}093{,}000 \text{ ft.-lb.}$$

Section at Crown[5]

Dead Load

$$T_c = -\frac{1 + 3 \times .136}{8 \times .136} \times 665 \times 71.8 = -61{,}800$$

M_c *assumed* $= 0$

Live Load for Maximum Positive Moment

$$T_c = -\frac{57.6 + (189 - 8 \times 1.55) \times .136 - 220 \times .136^2}{1000 \times .136} \times 43{,}100$$
$$= -.570 \times 43{,}100 = -24{,}600$$

$$M_c = +\frac{72 + 105 \times .136 + 220 \times .136^2 - (17 + 10 \times .136) \times 1.55 + 1.5 \times 1.55^2}{10{,}000}$$
$$\times 3{,}093{,}000 = .00656 \times 3{,}093{,}000 = +20{,}300$$

[5] Mr. Cochrane's analysis considers a compressive stress to be of positive sign. The authors prefer the opposite convention and in the example here shown compression is indicated by a minus sign.

Live Load for Maximum Negative Moment

$$T_c = -\frac{57.8 + 2 \times 1.55 + (10 + 30 \times 1.55) \times .136 - (380 + 30 \times 1.55) \times .136^2}{1000 \times .136}$$
$$\times 43{,}100 = -.446 \times 43{,}100 = -19{,}230$$

Temperature (Fall of 30 deg.)

$$T_c = +[19.4 \times 1.55 - 7.5 + (17 \times 1.55 - 31) \times .136 - 140(1.55 - 1) \times .136^2]$$
$$\times \frac{51{,}840 \times .643}{9.8^2} = \frac{20.6 \times 51{,}840 \times .643}{9.8^2} = +7100$$

$$M_c = +(38.5 - 12.8 \times 1.55 + 1.6 \times 1.55^2) \times \frac{9.8 \times 7100}{100}$$
$$= \frac{22.5 \times 9.8 \times 7100}{100} = +15{,}700$$

Section at Springing

Dead Load

$$V_s = -\frac{2 + 15 \times .136}{4} \times 665 \times 71.8 = -48{,}200$$
$$T_s = \sqrt{61{,}800^2 + 48{,}200^2} = -78{,}400$$
$$M_s \text{ assumed} = 0$$

Live Load for Maximum Negative Moment

$$T_s = -\frac{27.6 + (125 + 6 \times 1.55) \times .136 + 320 \times .136^2}{1000 \times .136} \times 43{,}100$$
$$= .381 \times 43{,}100 = -16{,}400$$
$$T_c = -\left[.223 - \frac{.0026(1.55 - 2)^2}{.136}\right] \times 43{,}100 = .219 \times 43{,}100$$
$$= -9400$$
$$M_s = -\frac{283 - 480 \times .136 - 9(4.22 - 2.8 \times .136 - 1.55)^2 \times 3{,}093{,}000}{10{,}000}$$
$$= -.0171 \times 3{,}093{,}000 = -52{,}900$$

Temperature (Fall of 30 deg.)

$$T_s = (1.13 - 2.55 \times .136) \times 7100 = .78 \times 7100 = +5500$$
$$M_s = +15{,}700 - 9.8 \times 7100 = -53{,}900$$

Average Stresses

For Dead Load

$$f_a = -\left[1.030 + 2.5(.136 + .05)^2 - \frac{(20 \times .136 + 8) \times 1.55 - (1.55 - 1)^2}{100}\right]$$
$$\times \frac{61{,}800}{2.07} = \frac{.954 \times 61{,}800}{2.07} = -28{,}500$$

For Live Load Producing Maximum Positive Moment at Crown

$$f_a = -\left[.920 + 2.6 \times \overline{.136}^3 - .04 \times 1.55 + \frac{(6.7 + 33 \times .136)(4 - 1.55)^2}{1000}\right] \times \frac{24{,}600}{2.07}$$

$$= \frac{.932 \times 24{,}600}{2.07} = -11{,}100$$

For Live Load Producing Maximum Negative Moment at Springing

$$f_a = -\left[.950 + 1.7 \times \overline{.136}^2 - .05 \times 1.55 + \frac{(4 + 48 \times .136)(4 - 1.55)^2}{1000}\right]$$

$$\times \frac{9400}{2.07} = -\frac{.967 \times 9400}{2.07} = -4400$$

For Fall of Temperature (30 deg.) *and Arch Shortening Stresses*

$$f_a = [1.075 - .8 \times .136 - (.081 - .11 \times .136) \times 1.55] \times \frac{7100}{2.07}$$

$$= \frac{.864 \times 7100}{2.07} = +3000$$

SUMMARY FOR MAXIMUM POSITIVE MOMENT AT CROWN

	Thrust	Moment	Average stress
Dead load	−61,800	0	−28,500
Live load	−24,600	+20,300	−11,100
Arch shortening[6]	+ 5,100	+11,300	+ 2,200
	−81,300	+31,600	−37,400
Dead load + live load	−86,400	+20,300	−39,600
Temperature	+ 7,100	+15,700	+ 3,000
Arch shortening	+ 4,700	+10,500	+ 2,000
	−74,600	+46,500	−34,600

[6] See typical computation on p. 401.

SUMMARY FOR MAXIMUM NEGATIVE MOMENT AT SPRINGING

Dead load	−78,400	0	−28,500
Live load	−16,400	−52,900	− 4,400
Arch shortening	+ 3,300	−22,900	+ 1,800
	−91,500	−75,800	−31,100
Dead load + live load	−94,800	−52,900	−32,900
Temperature	+ 5,500	−53,900	+ 3,000
Arch shortening	+ 3,000	−29,400	+ 1,600
	−86,300	−136,200	−28,300

UNIT STRESSES

Section	Load	M	N	$\frac{e}{a}$	k	$\frac{M}{ba^2f_c}$	f_c
Crown	$D+L+S$	31,600	81,300	.212	.93	.113	580
	$D+L+S+T$	46,500	74,600	.340	.67	.127	760
Springing	$D+L+S$	75,800	91,500	.293	.75	.120	550
	$D+L+S+T$	136,200	86,300	.557	.48	.128	920

D = Dead Load, L = Live Load, S = Arch Shortening T = Temperature.

Analysis by the Elastic Theory. The shape of the arch axis may be determined from equation (113). The values of $cl = x$ and the corresponding values of y are tabulated below:

c	cl	c^5	$24c^5r$	c^2	$c^2 + 24c^5r$	y
.25	17.95	.00098	.0032	.0625	.0657	1.73
.30	21.54	.00243	.0079	.0900	.0979	2.72
.35	25.13	.00525	.0171	.1225	.1396	3.87
.40	28.72	.01024	.0334	.1600	.1934	5.37
.45	32.31	.01845	.0602	.2025	.2627	7.29
.50	35.90	.03125	.1020	.2500	.3520	9.77

The curve of the half arch axis is plotted, and half the radial thickness of various points along the axis laid off on either side of the axis. These thicknesses may be taken from the first table of Art. 200. The arch of Fig. 118 is laid out in this manner. In order to determine the approximate line of thrust due to dead

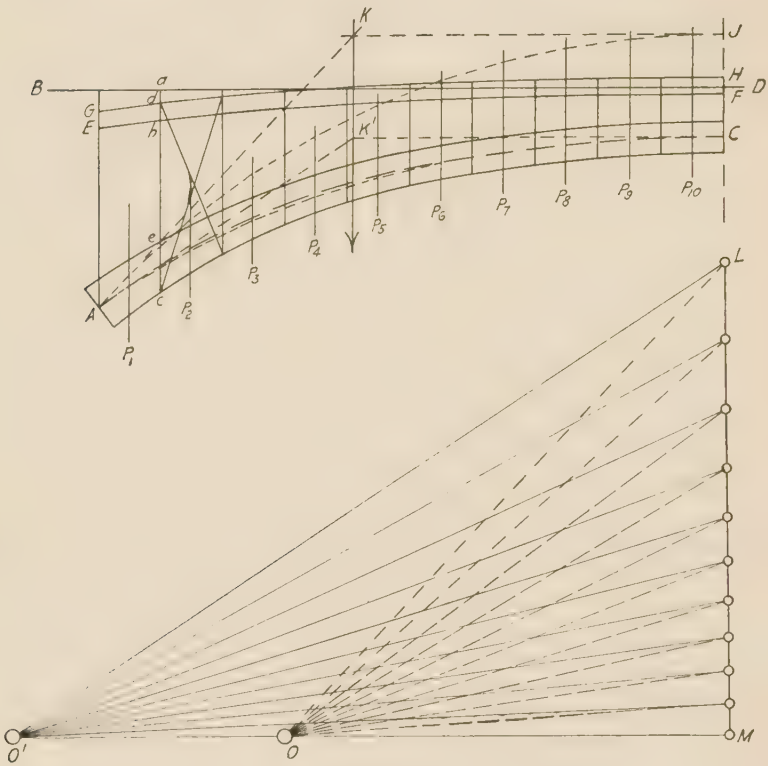


Fig. 118.

load, an equilibrium polygon for dead load may be drawn through the crown and springing. Such an equilibrium polygon closely represents the line of thrust produced by the dead load. The half span is divided into ten (or more) equal divisions. The horizontal line *BD* represents the top of the fill, and the line *EF* the reduced load line for fill. (The fill is assumed to weigh 120 lb. per cu. ft. and the concrete 150 lb. per cu. ft. Therefore, *be* is $\frac{120}{150}$ times *ae*.) The weight of the ballast, rails, and joints

per foot section of arch is reduced to equivalent weight of concrete and GH plotted parallel to EF. The dead load on the arch is then represented by the area between the line GH and the curve of the intrados. The load for each section is determined by measuring the ordinates AG, cd, etc., taking the average of each two adjacent ordinates, and multiplying this average by the width of each division times 150. The loads so determined are as follows:

$$P_1 = \frac{11.0 + 11.4}{2} \times 3.59 \times 150 = 6000$$

$$P_2 = \frac{9.1 + 11.0}{2} \times 3.59 \times 150 = 5400$$

$$P_3 = \frac{7.7 + 9.1}{2} \times 3.59 \times 150 = 4500$$

$$P_4 = \frac{6.6 + 7.7}{2} \times 3.59 \times 150 = 3800$$

$$P_5 = \frac{5.9 + 6.6}{2} \times 3.59 \times 150 = 3400$$

$$P_6 = \frac{5.4 + 5.9}{2} \times 3.59 \times 150 = 3000$$

$$P_7 = \frac{5.0 + 5.4}{2} \times 3.59 \times 150 = 2800$$

$$P_8 = \frac{4.7 + 5.0}{2} \times 3.59 \times 150 = 2600$$

$$P_9 = \frac{4.5 + 4.7}{2} \times 3.59 \times 150 = 2500$$

$$P_{10} = \frac{4.4 + 4.5}{2} \times 3.59 \times 150 = 2400$$

The center of gravity of each trapezoidal load is determined, and the verticals P_1, P_2, etc., drawn through these centers, which represent the points of application of the loads. The load line is now constructed, any convenient pole O assumed, and the rays of the force polygon drawn. After drawing the corresponding equilibrium polygon AJ, the first and last rays are prolonged to their intersection at K. The vertical through K represents the resultant of all the loads on the half span. To construct an equilibrium polygon passing through both A and C, CK' is drawn horizontally, and LO' parallel to AK'. O' is the pole of the force

polygon required for a corresponding equilibrium polygon passing through both A and C. If this equilibrium polygon fails to coincide with the neutral axis at all sections, the line of thrust for dead load will be eccentric and a bending moment will be produced at such sections. If it is desired to have no bending moments produced by dead load, the shape of the arch axis should be altered to coincide with the equilibrium polygon passing through the crown and the springing. If the difference between the assumed axis and the equilibrium polygon is great, the loads should be revised. In the present case, this is not necessary.

Radii of Neutral Axis. Three-centered curves are to be used for the intrados and the neutral axis, the larger radius in each case being used from crown to quarter point. From the equations of Art. 194, the radii of the neutral axis are computed as follows:

$$R_1 = \frac{\overline{17.95}^2 + \overline{1.73}^2}{2 \times 1.73} = 93.99 \text{ ft.}$$

$$\sin \theta = .191 \qquad \cos \theta = .982$$

$$R_2 = \frac{1}{2} \cdot \frac{\overline{17.95}^2 + \overline{8.04}^2}{8.04 \times .982 - 17.95 \times .191} = 43.30 \text{ ft.}$$

The length of the neutral axis may be computed, scaled, or estimated from the second table of Art. 200. The computations involved are as follows:

$$\theta = 11 \text{ degrees } 2 \text{ minutes}$$

Length crown to quarter point

$$\frac{11\tfrac{1}{30}}{360} \times 93.99 \times 2\pi = 18.10$$

$$\sin \frac{1}{2} \angle \text{ subtending chord of } R_2 = \frac{19.66}{2 \times 43.30} = .227$$

The $\angle$ = 26 degrees 15 minutes

Length quarter point to springing

$$\frac{26\tfrac{1}{4}}{360} \times 43.30 \times 2\pi = 19.84 \text{ ft.}$$

Total length $= 2(18.10 + 19.84) = 75.88$ ft.

By Art. 200 the length $= .529 \times 2 \times 71.8 = 75.96$ ft.

Radii of Intrados. The radial thickness of the arch at the quarter point is, from Art. 200,

$$1.065 \times 22 = 23.4 \text{ in.}$$

In Fig. 119

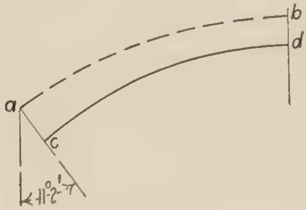


FIG. 119.

The horizontal distance from b to a is 17.95 ft. The horizontal distance from b to c is $17.95 - \frac{23.4}{24} \times .191 = 17.76$ ft. The vertical distance from b to a = 1.73 ft.

The vertical distance d to $c = 1.73 - \frac{11}{12} + \frac{23.4}{24} \times .982 =$ 1.77 ft.

Then

$$R_1 = \frac{\overline{17.76}^2 + \overline{1.77}^2}{2 \times 1.77} = 89.99 \text{ ft.}$$

$$\sin \theta = .197 \qquad \cos \theta = .980$$

and

$$R_2 = \frac{\overline{17.24}^2 + \overline{8.23}^2}{2(8.23 \times .980 - 17.24 \times .197)} = 39.07 \text{ ft.}$$

Location of Axis. Dividing the half of the neutral axis into ten equal divisions, the length of each division is 3.79 ft. Laying off the centers of each one of the divisions on Fig. 120, the coordinates with the origin of coordinates at the crown are scaled and tabulated in the last two columns on page 393. The radial thickness of the arch at the center of each division is also scaled and the moment of inertia computed and tabulated.

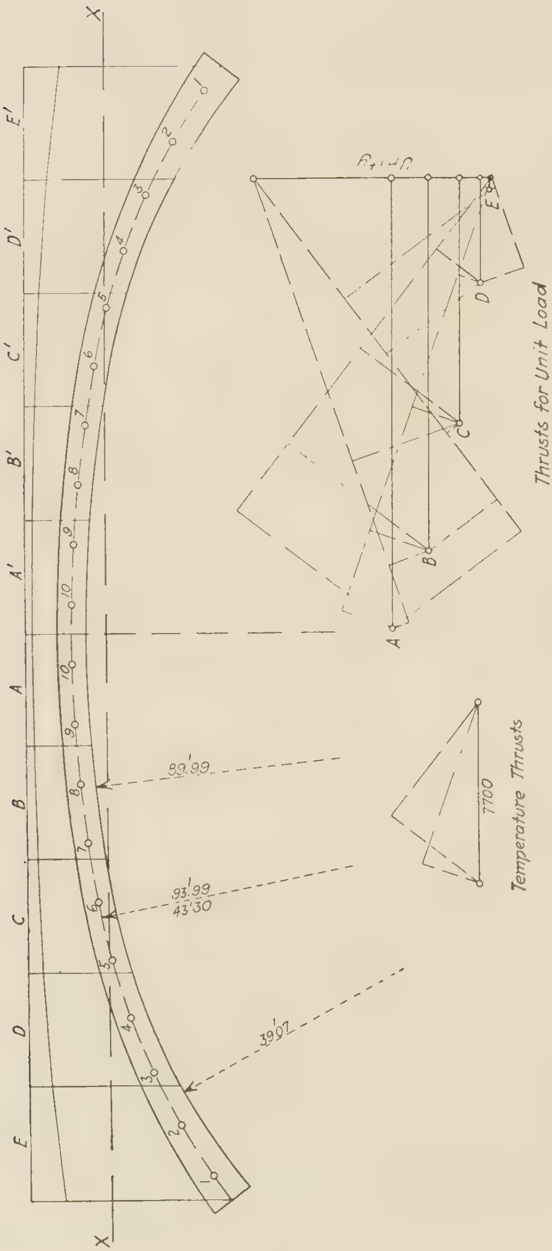

Thrusts for Unit Load
Temperature Thrusts
7700
A
B
C
D
E
A'
B'
C'
D'
E'
X
X

FIG. 120.

Point	a	I_c	$\left(\frac{a}{2} - d'\right)^2$	$14I_s$	I	y_c	$\frac{y_c}{I}$	$\frac{1}{I}$	y	x
10	1.85	.528	0.578	.135	.663	.02	.03	1.51	+2.15	1.9
9	1.87	.545	0.593	.139	.684	.19	.28	1.46	+1.98	5.7
8	1.90	.572	0.608	.142	.714	.50	.70	1.40	+1.67	9.5
7	1.92	.590	0.624	.146	.736	.96	1.30	1.36	+1.21	13.2
6	1.95	.618	0.640	.150	.768	1.57	2.04	1.30	+0.60	17.0
5	1.97	.637	0.672	.157	.794	2.37	2.99	1.26	−0.20	20.7
4	2.02	.687	0.706	.165	.852	3.49	4.10	1.17	−1.32	24.3
3	2.13	.805	0.810	.247	1.052	4.90	4.66	.95	−2.73	27.8
2	2.34	1.068	1.000	.305	1.373	6.61	4.81	.73	−4.44	31.1
1	2.66	1.568	1.346	.411	1.979	8.64	4.37	.51	−6.47	34.4
Crown	1.83	.511	0.563	.132	.643		$\sum\frac{y_c}{I}$	$\sum\frac{1}{I}$		
Springing	2.83	1.889	1.563	.477	2.366		25.28	11.65		

From the above tabulation the axis is located so that

$$\sum\frac{y}{I} = 0, \text{ that is, } y' = \frac{25.28}{11.65} = 2.17 \text{ ft. below the crown.}$$

(See footnote on page 374.)

Determination of Loads. The load is assumed applied to the arch at the equidistant points as indicated on Fig. 120. The dead loads are computed in the same manner as on page 389 and are as follows:

A—4900 lb.
B—5400 lb.
C—6500 lb.
D—8500 lb.
E—11,100 lb.

The live load is 4300 lb. per section.

Moments and Thrusts for Unit Loads. The table on page 395 gives the value of H_o, V_o, and M_o for unit load applied at each of the several load points. The tables on pages 396 to 398 give the moments and thrusts produced at the crown, sixth point, and springing sections respectively by unit load. The sixth point is chosen, not because it is necessarily the point of highest stress, but to show the method of computation for a section other than the crown or springing (see Art. 202).

The value of the thrust N is determined as follows:

In Fig. 121, H_o and V_o represent the thrust and shear, due to a load unity P, applied as shown. These forces produce right and left reactions of R_R and R_L, respectively. At section a, the thrust produced by the load P is equal and opposite to the component of R_L which is parallel to the neutral axis at that point. Similarly, the shear at section a is equal and opposite to

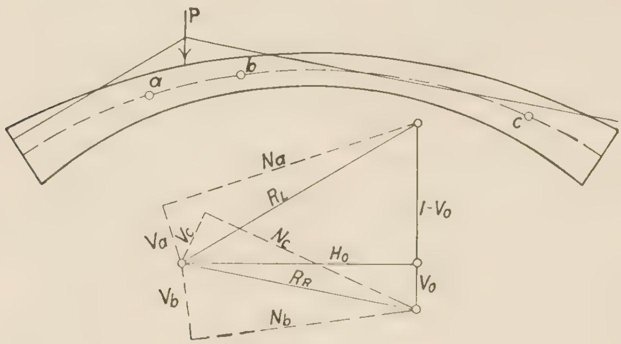


FIG. 121.

the component of R_L perpendicular to the neutral axis. At section b, the components of R_R determine the thrust and shear. For any section on the right half of the arch, section c, the components of R_R determine the thrust and shear. In Fig. 120, the thrusts and shears are laid off as previously computed, and

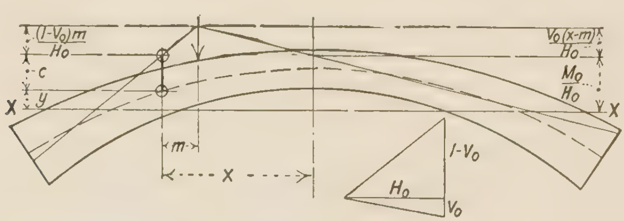


FIG. 122.

the values of the thrusts at the various sections determined as above, by scaling from the diagram.

The moment may be computed as indicated in the tables on pages 397 to 399 or it may be obtained graphically. From Fig. 122 it may be seen that $M = H_oc$, H_o always being negative

and c being positive when measured upward *to* the neutral axis, and negative when measured downward *to* the neutral axis.

Loading for Maximum Stresses. From the values of M and N for unit load as given in the tables it can readily be seen what portions of the arch should be considered loaded in order to obtain the maximum positive and negative moments for the sections investigated. Finally the values of thrusts and moments for the design dead and live loads are obtained by multiplying the values for unit load by the section loads as previously determined. The summations give the resultant maximum moments and thrusts.

s = a constant

$\sum \frac{Y}{I} = 0$

$$H_o = \frac{\sum \left(\frac{my}{I}\right)}{2\sum\left(\frac{y^2}{I}\right)} \qquad V_o = \frac{\sum \left(\frac{mx}{I}\right)}{2\sum\left(\frac{x^2}{I}\right)} \qquad M_o = \frac{\sum \left(\frac{m}{I}\right)}{2\sum\left(\frac{1}{I}\right)}$$

	x	y	$\frac{1}{I}$	x^2	y^2	$\frac{x^2}{I}$	$\frac{y^2}{I}$	Unit load at A				B				C				D				E			
								m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$
A	3.6																										
B	10.8																										
C	18.0																										
D	25.1																										
E	32.3																										
10	1.9	+2.15	1.51	3.6	4.62	5	7.0																				
9	5.7	+1.98	1.46	32.5	3.92	47	5.7	2.1	3.1	18	+ 6.1																
8	9.5	+1.67	1.40	90.3	2.79	126	3.9	5.9	8.3	79	+ 13.9																
7	13.2	+1.21	1.36	174.2	1.46	237	2.0	9.6	13.1	173	+ 15.8	2.4	3.3	44	+ 4.0												
6	17.0	+0.60	1.30	289.0	0.36	376	0.5	13.4	17.4	296	+ 10.4	6.2	8.1	138	+ 4.9												
5	20.7	−0.20	1.26	428.5	0.04	540	0.1	17.1	21.5	445	− 4.3	9.9	12.5	259	− 2.5	2.7	3.4	70	− 0.7								
4	24.3	−1.32	1.17	590.5	1.74	691	2.0	20.7	24.2	588	− 31.9	13.5	15.8	384	− 20.9	6.3	7.4	180	− 9.8								
3	27.8	−2.73	0.95	772.8	7.45	734	7.1	24.2	23.0	640	− 62.8	17.0	16.2	450	− 44.2	9.8	9.3	259	− 25.4	2.7	2.6	72	− 7.1				
2	31.1	−4.44	0.73	967.2	19.71	706	14.4	27.5	20.1	625	− 89.2	20.3	14.8	460	− 65.7	13.1	9.6	298	− 42.6	6.0	4.4	137	−19.5				
1	34.4	−6.47	0.51	1183.4	41.86	604	21.3	30.8	15.7	540	−101.6	23.6	12.0	413	− 77.6	16.4	8.4	289	− 54.4	9.3	4.7	162	−30.4	2.1	1.1	38	−7.1
Σ			11.65			4066	64.0		146.4	3404	−243.6		82.7	2148	−202.0		38.1	1096	−132.9		11.7	371	−57.0		1.1	38	−7.1

	A	B	C	D	E
H_o	$\frac{-243.6}{2 \times 64.0} = -1.903$	$\frac{-202.0}{2\times 64.0} = -1.578$	$\frac{-132.9}{2\times 64.0} = -1.038$	$\frac{-57.0}{2\times 64.0} = -0.445$	$\frac{-7.1}{2\times 64.0} = -0.055$
V_o	$\frac{3404}{2\times 4066} = 0.418$	$\frac{2148}{2\times 4066} = 0.264$	$\frac{1096}{2\times 4066} = 0.135$	$\frac{371}{2\times 4066} = 0.046$	$\frac{38}{2\times 4066} = 0.005$
M_o	$\frac{146.4}{2\times 11.65} = 6.28$	$\frac{82.7}{2\times 11.65} = 3.55$	$\frac{38.1}{2\times 11.65} = 1.64$	$\frac{11.7}{2\times 11.65} = 0.50$	$\frac{1.1}{2\times 11.65} = 0.05$

Crown Section

$x = 0$ $y = +2.17$	$\frac{d'}{a} = 0.09$ $p_o = 0.0091$	$I = 0.643$ $a = 1.83$ $A = 2.07$	Live load 4300 lb. per section	Dead loads, M, and N, in thousands of pounds $M = M_o + H_o y + V_o x - m$

	Section	H_o	$H_o y$	V_o	$V_o x$	$-m$	M_o	M	N	Dead load	Dead load		Live loading for max. comp.			
													Upper Fiber		Lower Fiber	
											M	N	M	N	M	N
Unit Load at	E	−0.055	−0.12				0.05	−0.07	−0.06	11.1	− 0.8	− 0.7			*	*
	D	−0.445	−0.97				0.50	−0.47	−0.45	8.5	− 4.0	− 3.8			*	*
	C	−1.038	−2.25				1.64	−0.61	−1.04	6.5	− 4.0	− 6.8			*	*
	B	−1.578	−3.42				3.55	+0.13	−1.58	5.4	+ 0.7	− 8.5	*	*	*	*
	A	−1.903	−4.13				6.28	+2.15	−1.90	4.9	+10.5	− 9.3	*	*		
	A'												*	*		
	B'												*	*		
	C'															
	D'															
	E'															
	Σ										+ 4.8	−58.2	+19.6	−29.9	− 4.4	−13.5

Sixth Point

$x = 23.9$	$\frac{d'}{a} = 0.08$	$I = 0.845$	Live load	Dead loads, M and N,
		$a = 2.01$	4300 lb.	in thousands of pounds
$y = -1.18$	$p_o = 0.0083$	$A = 2.24$	per section	$M = M_o + H_oy + V_ox - m$

	Section	H_o	H_oy	V_o	V_ox	$-m$	M_o	M	N	Dead load	Dead load M	Dead load N	Live loading for max. comp. Upper Fiber M	Upper Fiber N	Lower Fiber M	Lower Fiber N
Unit Load at	E	−0.055	+0.06	+0.005	+ 0.12		0.05	+0.23	0.05	11.1	+ 2.6	− 0.6	*	*		
	D	−0.445	+0.52	+0.046	+ 1.10		0.50	+2.12	0.41	8.5	+18.0	− 3.5	*	*		
	C	−1.038	+1.22	+0.135	+ 3.23	− 5.90	1.64	+0.19	1.26	6.5	+ 1.2	− 8.2	*	*	*	*
	B	−1.578	+1.86	+0.264	+ 6.31	−13.10	3.55	−1.38	1.73	5.4	− 7.5	− 9.3			*	*
	A	−1.903	+2.24	+0.418	+10.00	−20.30	6.28	−1.78	1.99	4.9	− 8.7	− 9.8			*	*
	A'	−1.903	+2.24	−0.418	−10.00		6.28	−1.48	1.94	4.9	− 7.3	− 9.5			*	*
	B'	−1.578	+1.86	−0.264	− 6.31		3.55	−0.90	1.57	5.4	− 4.9	− 8.5			*	*
	C'	−1.038	+1.22	−0.135	− 3.23		1.64	−0.37	1.03	6.5	− 2.4	− 6.7			*	*
	D'	−0.445	+0.52	−0.046	− 1.10		0.50	−0.08	0.44	8.5	− 0.7	− 3.7			*	*
	E'	−0.055	+0.06	−0.005	− 0.12		0.05	−0.01	0.05	11.1	− 0.1	− 0.6			*	*
	Σ										− 9.8	−60.4	+10.9	−7.4	−25.0	−43.0

SPRINGING

$x = 35.9$, $y = -7.60$; $\frac{d'}{a} = 0.06$, $p_o = 0.0077$; $I = 2.366$, $a = 2.83$, $A = 3.14$; Live load 4300 lb. per section; Dead loads, M and N, in thousands of pounds, $M = M_o + H_oy + V_ox - m$

	Section	H_o	H_oy	V_o	V_ox	$-m$	M_o	M	N	Dead load	Dead load M	Dead load N	Live loading for max. comp. Upper Fiber M	Upper Fiber N	Lower Fiber M	Lower Fiber N
Unit Load at	E	−0.055	+ 0.42	+0.005	+ 0.18	− 3.60	0.05	−2.95	0.64	11.1	−32.7	− 7.1			*	*
	D	−0.445	+ 3.38	+0.046	+ 1.65	−10.80	0.50	−5.27	0.93	8.5	−44.8	− 7.9			*	*
	C	−1.038	+ 7.89	+0.135	+ 4.84	−17.90	1.64	−3.53	1.35	6.5	−22.9	− 8.8			*	*
	B	−1.578	+11.99	+0.264	+ 9.48	−25.10	3.55	−0.08	1.70	5.4	− 4.3	− 9.2	*	*	*	*
	A	−1.903	+14.46	+0.418	+15.02	−32.30	6.28	+3.46	1.87	4.9	+17.0	− 9.2	*	*		
	A'	−1.903	+14.46	−0.418	−15.02		6.28	+5.72	1.77	4.9	+28.0	− 8.7	*	*		
	B'	−1.578	+11.99	−0.264	− 9.48		3.55	+6.06	1.41	5.4	+32.7	− 7.6	*	*		
	C'	−1.038	+ 7.89	−0.135	− 4.84		1.64	+4.69	0.91	6.5	+30.5	− 5.9	*	*		
	D'	−0.445	+ 3.38	−0.046	− 1.65		0.50	+2.23	0.38	8.5	+19.0	− 3.2	*	*		
	E'	−0.055	+ 0.42	−0.005	− 0.18		0.05	+0.29	0.05	11.1	+ 3.2	− 0.6	*	*		
	Σ										+25.7	−68.2	+96.2	−34.8	−50.8	−19.9

Temperature Stresses. The value of H_o for a fall in temperature of 30 degrees is from equation (108)

$$H_o = \frac{288{,}000{,}000 \times 30 \times .000006}{2 \times 64.0} \cdot \frac{71.8}{3.79} = +7700$$

For a rise of 20 degrees.

$$H_o = -5100$$

Stresses Due to Thrust

		Crown	Springing	Average
	Dead load	−28,100	−21,700	−24,900
For live loading producing the maximum compression in the upper fiber at the:—	Crown	−14,400	− 9,200	−11,800
	Sixth point	− 3,200	− 4,000 − 1,800	− 3,100
	Springing	−17,700	−11,100 −13,200	−14,900
For live loading producing the maximum compression in the lower fiber at the:—	Crown	− 6,500	− 6,300 − 3,800	− 5,800
	Sixth point	−19,800	−12,900 −14,500	−16,800
	Springing	− 6,500	− 6,300 − 3,800	− 5,800
Temperature	Fall of 30°F.	+ 3,700	+ 1,900	+ 2,800
	Rise of 20°F.	− 2,500	− 1,300	− 1,900

Since the arch shortening stresses are proportional to those for a fall in temperature, these are more easily obtained in the final summation for maximum moments and thrusts.

Maximum Moments and Thrusts
Crown Section
Maximum Compression in Upper Fiber

	Thrust	Moment	c_a
Dead load	−58,200	+ 4,800	−24,900
Live load	−29,900	+19,600	−11,800
Arch shortening[7]	+ 5,200	+11,200	+ 1,900
	−82,900	+35,600	−34,800
Dead load + live load	−88,100	+24,400	−36,700
Temperature (30° fall)	+ 7,700	+16,700	+ 2,800
Arch shortening	+ 4,800	+10,400	+ 1,700
	−75,600	+51,500	−32,200

Maximum Compression in Lower Fiber

	Thrust	Moment	c_a
Dead load	−58,200	+ 4,800	−24,900
Live load	−13,500	− 4,400	− 5,800
Arch shortening	+ 4,300	+ 9,400	+ 1,600
	−67,400	+ 9,800	−29,100
Dead load + live load	−71,700	+ 400	−30,700
Temperature (20° rise)	− 5,100	−11,100	− 1,900
Arch shortening	+ 4,600	+ 9,900	+ 1,700
	−72,200	− 800	−30,900

[7] For a 30-degree fall in temperature $\omega tE = -51{,}840$. (For a 20-degree rise $+34{,}560$.) c_a for dead and live load $= -36{,}700$. The thrust, moment, and c_a for arch shortening are equal to $\frac{-36{,}700}{-51{,}840}$ times the similar quantities for a 30-degree fall in temperature, or +5500, +11,800, and +2000, respectively. The latter value, when added algebraically to the value of c_a for dead and live load, results in a smaller numerical value for the summation, and consequently the ratio between the arch shortening quantities and the temperature quantities becomes smaller. Hence, the thrust, moment, and c_a as computed above are slightly too large. Assume c_a due to arch shortening = +1900. Then the total $c_a = -34{,}800$ and the thrust, moment, and c_a due to arch shortening are +5200, +11,200, and +1900, respectively. If the latter value does not check the value assumed, another computation must be made.

SIXTH POINT

Maximum Compression in Upper Fiber

	Thrust	Moment	c_o
Dead load	−60,400	− 9,800	−24,900
Live load	− 7,400	+10,900	− 3,100
Arch shortening	+ 3,800	− 4,600	+ 1,500
	−64,400	− 3,500	−26,500
Dead load + live load	−67,800	+ 1,100	−28,000
Temperature (20° rise)	− 4,900	+ 6,000	− 1,900
Arch shortening	+ 4,000	− 4,900	+ 1,600
	−68,700	+ 2,200	−28,300

Maximum Compression in Lower Fiber

Dead load	− 60,400	− 9,800	−24,900
Live load	− 43,000	−25,000	−16,800
Arch shortening	+ 5,600	− 7,000	+ 2,100
	− 97,800	−41,800	−39,600
Dead load + live load	−103,400	−34,800	−41,700
Temperature (30° fall)	+ 7,300	− 9,100	+ 2,800
Arch shortening	+ 5,200	− 6,500	+ 2,000
	− 90,900	−50,400	−36,900

SPRINGING SECTION

Maximum Compression in Upper Fiber

	Thrust	Moment	c_2
Dead load	− 68,200	+ 25,700	−24,900
Live load	− 34,800	+ 96,200	−14,900
Arch shortening	+ 4,400	− 42,500	+ 2,100
	− 98,600	+ 79,400	−37,700
Dead load + live load	−103,000	+121,900	−39,800
Temperature (20° rise)	− 4,000	+ 39,000	− 1,900
Arch shortening	+ 4,600	− 44,600	+ 2,200
	−102,400	+116,300	−39,500

Maximum Compression in Lower Fiber

Dead load	−68,200	+ 25,700	−24,900
Live load	−19,900	− 50,800	− 5,800
Arch shortening	+ 3,400	− 32,900	+ 1,600
	−84,700	− 58,000	−29,100
Dead load + live load	−88,100	− 25,100	−30,700
Temperature (30° fall)	+ 6,000	− 58,500	+ 2,800
Arch shortening	+ 3,100	− 30,400	+ 1,500
	−79,000	−114,000	−26,400

FINAL MAXIMUM UNIT STRESSES

Section	Fiber	Load	M	N	$\frac{e}{a}$	k	$\frac{M}{ba^2f_c}$	f_c	f_s
Crown	Upper	D + L + S	+ 35,600	− 82,900	.235	.88	.117	630	
		D + L + S + T	+ 51,500	− 75,600	.372	.63	.128	820	5,500
	Lower	D + L + S	+ 9,800	− 67,400					
		D + L + S + T	− 800	− 72,200	.005	K = 0.91		250	
Sixth point	Upper	D + L + S	− 3,500	− 64,000					
		D + L + S + T	+ 2,200	− 68,700	.016	K = 0.96		230	
	Lower	D + L + S	− 41,800	− 97,800	.213	.94	.111	650	
		D + L + S + T	− 50,400	− 90,900	.276	.79	.121	720	1,700
Springing	Upper	D + L + S	+ 79,400	− 98,600	.284	.78	.122	560	1,700
		D + L + S + T	+116,300	−102,400	.402	.60	.127	790	6,700
	Lower	D + L + S	− 58,000	− 84,700	.242	.87	.117	430	500
		D + L + S + T	−114,000	− 79,000	.510	.50	.128	770	10,200

204. Design of Abutments. Since a slight settling of its supports will produce large stresses in an arch, it is important that the abutments be so designed that no such settlement occurs. On soft ground it is difficult in the extreme to obtain an abutment large enough to insure stability, without the use of piles. As the size of the abutment increases, its weight and the weight of the filling above it increase so rapidly that in some types of arches an abutment without a pile foundation becomes nearly as large as the arch itself. It is a question whether some other type of structure is not preferable where hardpan or rock is not accessible as a foundation bed, or where a good pile foundation cannot easily be made.

The abutments of a reinforced concrete arch are often designed for full live load and also for live load over one-half the arch. While the first condition of loading may give the maximum total pressure on the abutment, the two most extreme conditions are those loadings which cause maximum compression in the upper and lower fibers of the arch at the springing section. The

moments and thrusts for this section are given on page 403. In a similar manner the shears for unit load are obtained from Fig. 120, and the total dead and live load shears computed and tabulated below. The shear at the springing due to a fall in temperature of 30 degrees Fahrenheit scaled from Fig. 120 is -4700 lb. The shears due to rise of temperature and to arch shortening are proportional.

Load at section	Dead load	Shears			
		For unit load	For dead load	For live loading producing maximum compression in upper fiber	For live loading producing maximum compression in lower fiber
E	11.1	−0.76	−8.4		*
D	8.5	−0.50	−4.3		*
C	6.5	−0.07	−0.5		*
B	5.4	+0.36	+1.9	*	*
A	4.9	+0.68	+3.3	*	
A'	4.9	+0.82	+4.0	*	
B'	5.4	+0.75	+4.1	*	
C'	6.5	+0.52	+3.4	*	
D'	8.5	+0.24	+2.0	*	
E'	11.1	+0.03	+0.3	*	
Σ			+5.8	+14.6	−4.2

SUMMARY OF MOMENTS, THRUSTS, AND SHEARS

FOR MAXIMUM COMPRESSION IN UPPER FIBER AT SPRINGING

	M	N	V
Dead load	+ 25.7	− 68.2	+ 5.8
Live load	+ 96.2	− 34.8	+14.6
Arch shortening	− 42.5	+ 4.4	− 3.4
	+ 79.4	− 98.6	+17.0 $e = +0.81$
Dead load + live load	+121.9	−103.0	+20.4
Temperature	+ 39.0	− 4.0	+ 3.1
Arch shortening	− 44.6	+ 4.6	− 3.5
	+116.3	−102.4	+ 20.0 $e = +1.14$

FOR MAXIMUM COMPRESSION IN LOWER FIBER AT SPRINGING

	M	N	V
Dead load	+ 25.7	−68.2	+5.8
Live load	− 50.8	−19.9	−4.2
Arch shortening	− 32.9	+ 3.4	−2.6
	− 58.0	−84.7	−1.0 $e = -0.69$
Dead load + live load	− 25.1	−88.1	+1.6
Temperature	− 58.5	+ 6.0	−4.7
Arch shortening	− 35.9	+ 3.7	−2.9
	−119.5	−78.4	−6.0 $e = -1.52$

An abutment section $ABCDE$ (Fig. 123) is assumed. From the center of BC, the eccentric distance e (+1.14 ft.) is laid off upward, the value of N (−102,400 lb.) drawn perpendicular to BC, and the resultant of N and V obtained. This resultant must be combined with the forces due to the weight of the earth, filling, and abutment itself.

The forces due to the weight of the filling and the abutment are: (1) the weight of the filling; (2) the weight of the abutment; and

(3) the horizontal pressure due to the weight of the filling. When there is live load over the abutment a fourth force must be considered.

A more simple method giving results nearly the same as the more detailed analysis can be used for all but very large or important structures.

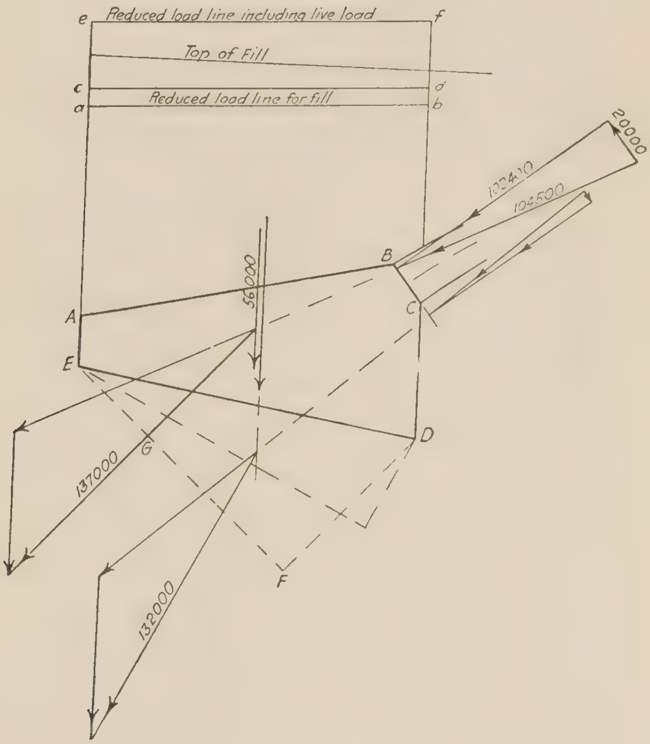


FIG. 123.

The reduced load line for fill at 120 lb. per cu. ft. (ab) is constructed as in the design of the arch. The line cd is drawn representing the top of the ballast as before. The trapezoid $cdDE$ may now be considered as material of the same weight (150 lb. per cu. ft.) and its amount and point of application determined. The horizontal pressure due to the filling (in this case less than 2900 lb.) is neglected. The vertical force and the resultant from the arch are combined graphically, and the total

resultant pressure with its line of action determined. EF is constructed as the projection of the base of the footing on a plane perpendicular to this resultant, and the distances EG and EF scaled.

Then the maximum unit pressure is, from equation (87),

$$\frac{4EF - 6EG}{EF^2} \times 137{,}000 = \frac{4 \times 17.0 - 6 \times 5.8}{17.0^2} \times 137{,}000 =$$

15,700 lb. per sq. ft.

In a similar manner the other condition of loading, that is, that producing maximum compression in the lower fiber at the springing, is analyzed and the maximum unit pressure found to be

$$\frac{4 \times 19.4 - 6 \times 7.8}{19.4^2} \times 132{,}000 = 10{,}500 \text{ lb. per sq. ft.}$$

In determining the resultant pressure for this condition the live load is considered completely to cover the abutment, and the total vertical force is represented by the trapezoid $efDE$.

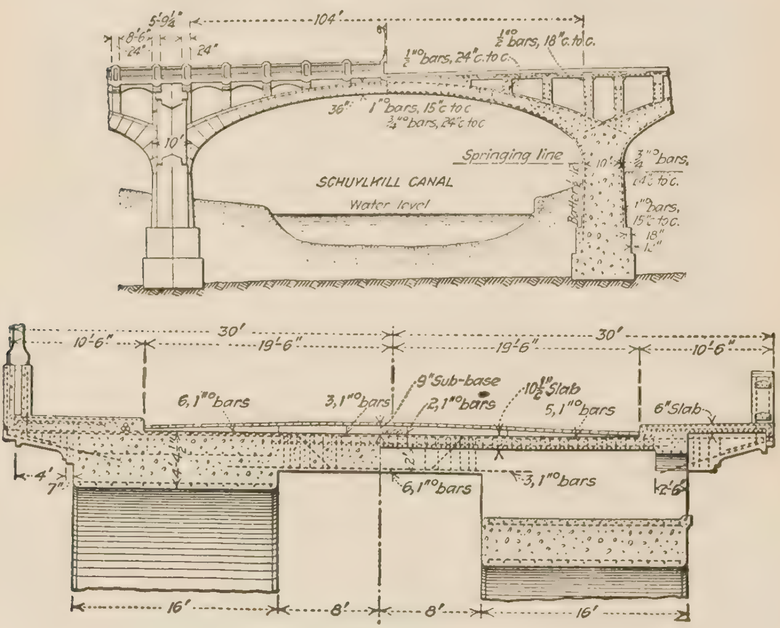


FIG. 124.

FIG. 125.—Fairmount, W. Va. bridge.

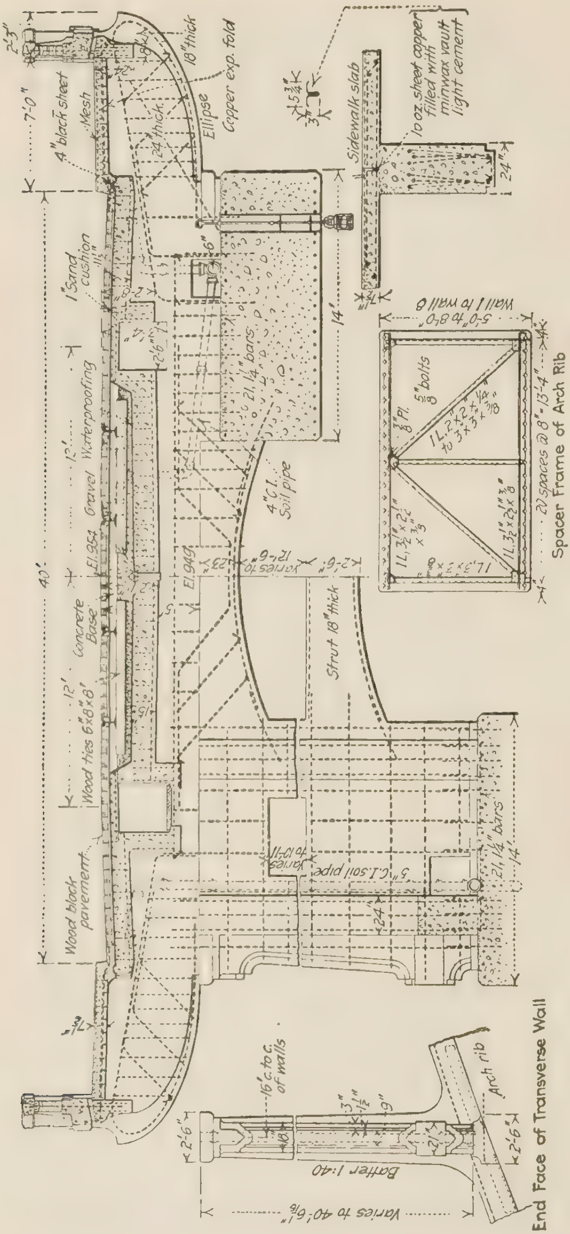


FIG. 126.—Wall and deck construction in main spans.

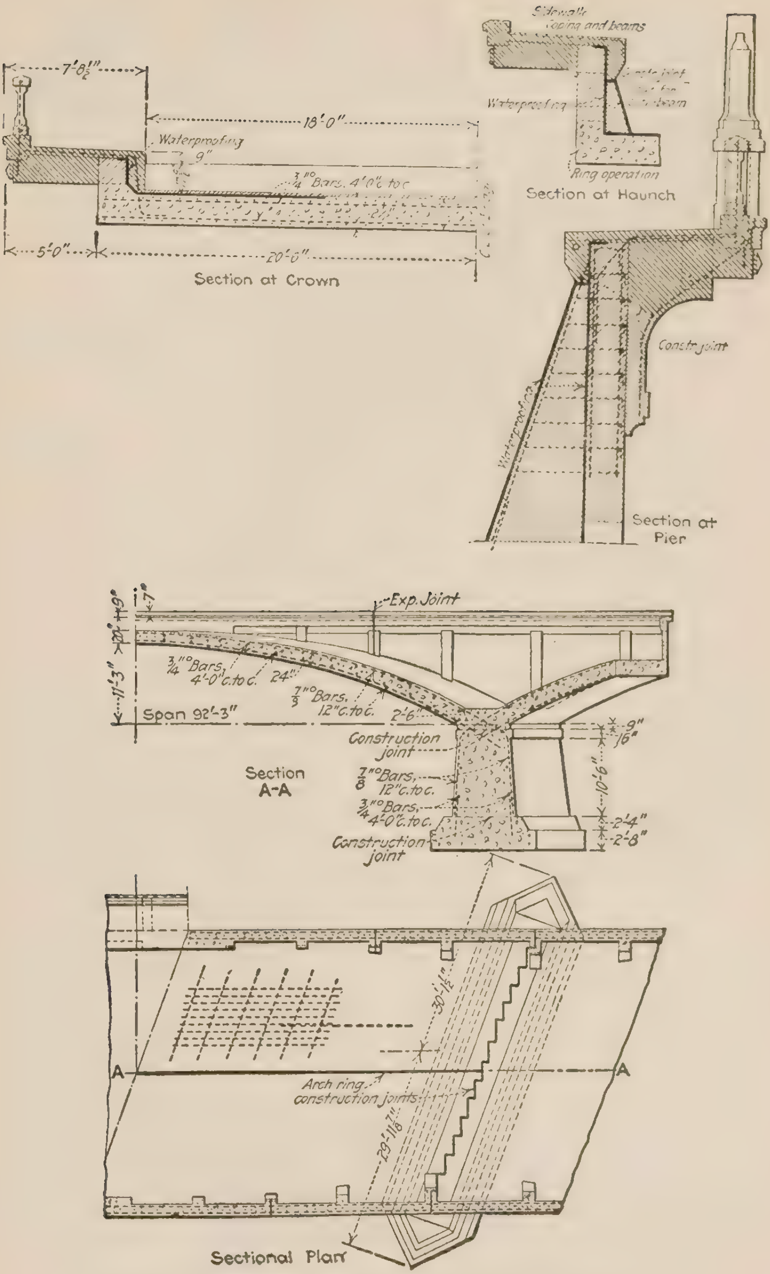


FIG. 127.—Details of Chemung River bridge, Corning, N. Y.

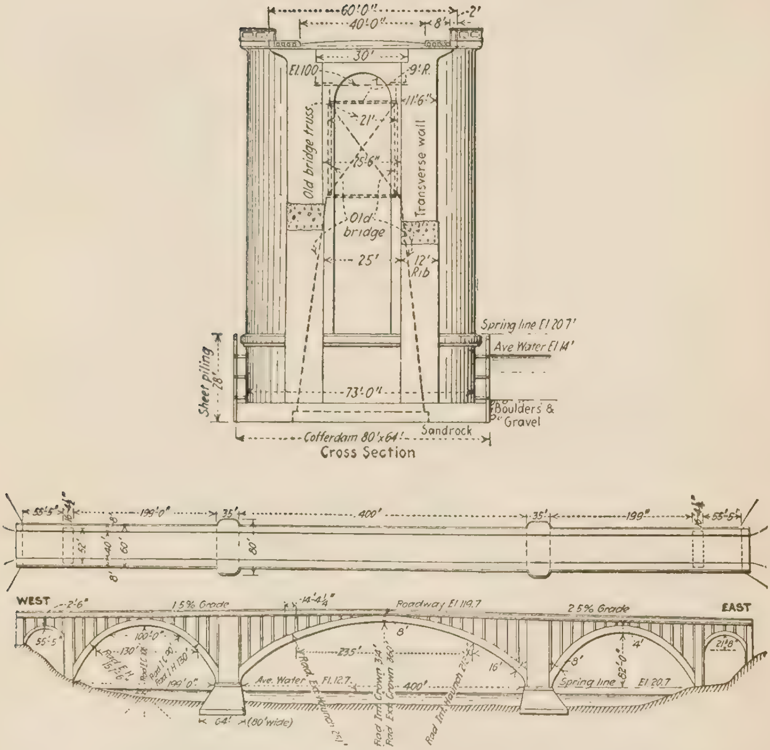


FIG. 128.—Franklin Avenue Concrete Arch bridge, Minneapolis, Minnesota.

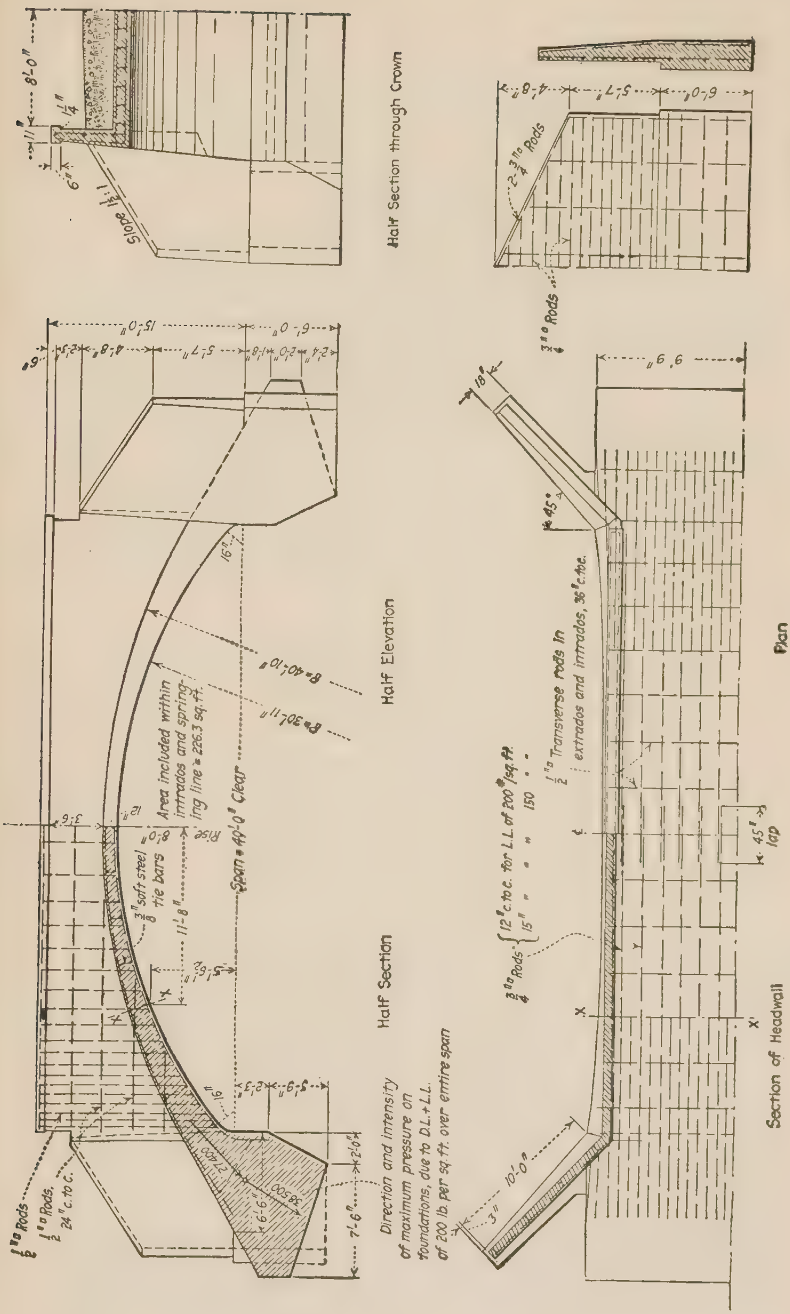


FIG. 129.—Standard 40-ft. Arch, State of Missouri Highway Department.

CHAPTER XI

SLAB, BEAM, AND GIRDER BRIDGES

205. Short span bridges of reinforced concrete are often more economical construction than any other type. They can be treated architecturally so as to give a pleasing appearance, and when well constructed, furnish bridges of permanent character. They may be divided into three groups, as follows: Slab Bridges, Deck Girder Bridges, and Through Girder Bridges.

For highways, slab bridges are adapted to short spans up to about 25 ft. in length. If the head-room is not limited, a beam bridge of the deck girder type is more economical, although not quite so simple in construction.

Deck girder bridges are adapted to spans from 20 to 60 ft. or more, where the head-room is not limited.

Through girder birdges may be used for about the same range in span length, but do not admit of as wide a roadway as is possible with the deck girder bridge. In a few cases longer spans have been built of concrete, where the conditions were ideal, and this type of structure the more economical.

Some concrete viaducts have been built as continuous or cantilever bridges, but the cross-section of the superstructure is not materially different from those of the simple spans.

Railroad bridges have rarely been constructed wholly of reinforced concrete except for short spans, and in such cases the construction has usually been of the slab type.

206. The only feature of the design not covered in Chap. III is the loading and its distribution. The loading to be used in the design of railroad bridges will not be discussed here. Overlying the concrete slab there is a varying depth of ballast which determines the width of distribution of the load that can be assumed.

Highway bridges are usually designed to support a combination of motor trucks and uniform live load. There have been many

tests made to determine the actual distribution of the concentrated load on a concrete slab with more or less varying results. The recommendations of the American Railway Engineering Association which follow are as satisfactory as any, and can be safely used.

Specifications[1]

Live Loads. The live load shall consist of motor trucks followed by a uniform load on the roadways, and a uniform load on the sidewalks, as specified herein. In calculating the stresses produced by trucks, the truck loads shall be considered applied as follows:

Clear Width of Roadway	Lines of Trucks
Less than 18 ft.	1
18 ft. to 30 ft.	2
30 ft. to 40 ft.	3
More than 40 ft.	4

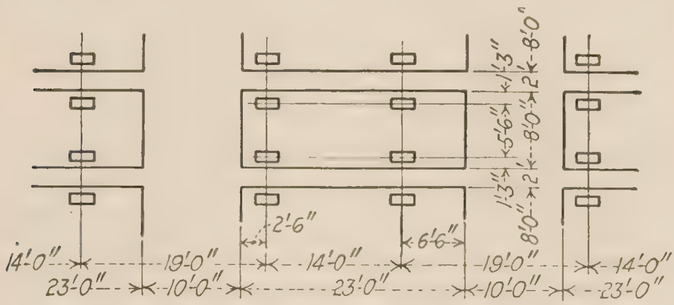


Fig. 130.

Spacing of Trucks. Trucks shall be assumed as spaced relative to one another as shown in Fig. 130. Groups of trucks shall be so arranged relative to the members of the bridge as to produce the maximum stresses in those members.

Distribution of Truck Loads. Truck loads shall be assumed as distributed 80 per cent on the rear axle and 20 per cent on the front axle. The wheel loads shall be assumed as distributed laterally on the roadway surface at 1000 lb. per in.

Multiple Lines of Traffic. In calculating the maximum stresses when two or more lines of traffic are assumed simultaneously on

[1] From *Bulletin*, Amer. Ry. Eng. Ass'n., vol. 24, p. 162.

the bridge, the following percentages of the specified loads shall be used:

On stringers and floor slabs	100 per cent
On floor beams	
For two lines of traffic	100 per cent
For three lines of traffic	90 per cent
For four lines of traffic	80 per cent
On main girders	
For two lines of traffic	90 per cent
For three lines of traffic	80 per cent
For four lines of traffic	75 per cent

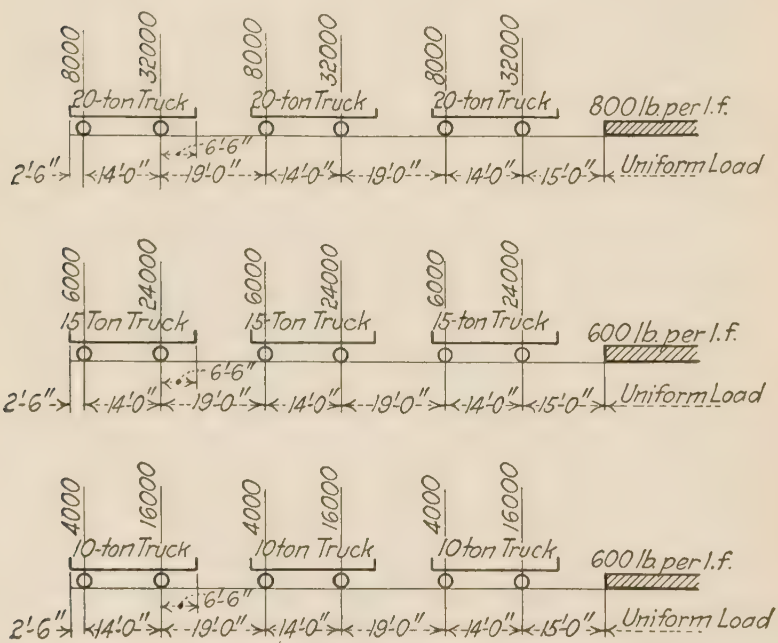


FIG. 131.

Intensity of Loads. Truck loads and uniform loads shall be assumed as follows:

(*a*) For bridges carrying primary highway or electric street railway traffic, three 20-ton trucks followed by a uniform load of 800 lb. per lin. ft. for each line of traffic on the roadways and 80 lb. per sq. ft. on the sidewalks.

(*b*) For bridges carrying secondary highway traffic three 15-ton trucks followed by a uniform load of 600 lb. per lin. ft. for each line of traffic on the roadways, and 60 lb. per sq. ft. on the sidewalks. A single 20-ton truck shall be used as alternative loading on the roadway for bridges of this class.

(*c*) For bridges carrying light country traffic, three 10-ton trucks followed by a uniform load of 600 lb. per lin. ft. for each line of traffic on the roadways, and 60 lb. per sq. ft. on the sidewalks. A single 15-ton truck shall be used as an alternative loading on the roadways for bridges of this class.

(*d*) For foot bridges a uniform load of 80 lb. per sq. ft.

The truck loads shall be as shown in Fig. 131. The truck loads specified shall be the total weight of the trucks loaded.

Street Railway Loads. For bridges carrying street railway tracks on the roadway, the car loads on each track shall be considered as alternative loads displacing an equal length of one line of trucks.

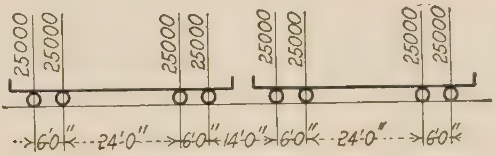


FIG. 132.

The size, weight, and wheel spacing of street railway cars shall be as shown in Fig. 132 unless otherwise specified.

Cantilevered Sidewalks. In bridges with cantilevered sidewalks only one sidewalk shall be considered loaded when calculating the stresses in the truss or girder, and floor beam hangers, adjacent to that sidewalk.

Distribution of Concentrated Loads on Concrete Slabs. The wheel loads shall be assumed as distributed uniformly over a width of slab measured parallel to the supports, as computed by the following formulas, in which

E = effective width of slab, or width over which the load shall be assumed as uniformly distributed.

l = length of span center to center of supports, in feet.

T = width of tire, in feet, taken as 1 in. for each 1000 lb. of wheel load.

For Moment

(*a*) Slab supported by longitudinal stringers with load applied midway between supports:

$$E = \tfrac{2}{3}(l + T), \text{ but not more than 5 ft.-6 in.}$$

(*b*) Slab supported by transverse floor-beams without longitudinal stringers, with load applied midway between supports:

$$E = \frac{2l}{3} + T, \text{ but not more than 5 ft.-6 in.}$$

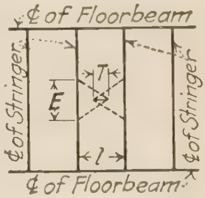


Fig. 133.

For Shear

(*c*) Slab supported by longitudinal stringers with load applied at a distance x from center line of nearest support equal to two and one-half times the effective depth of the slab:

$$E = \tfrac{4}{3}(x + T)$$

(*d*) Slab supported by transverse floor beams without longitudinal stringers with load applied at a distance x from center line of nearest support equal to two and one-half times the effective depth of the slab:

$$E = \frac{4x}{3} + T$$

Distribution of Concentrated Loads to Stringers and Floor Beams. In calculating moments in stringers, the portions of two maximum

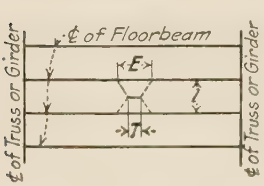


Fig. 134.

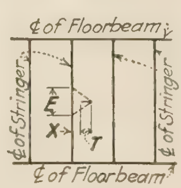


Fig. 135.

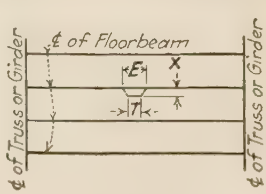


Fig. 136.

truck wheel loads spaced 4 ft.-6 in. center to center (two trucks passing), assumed as carried by one stringer, shall be determined by the following formula:

$$L = \frac{WS}{5}$$

L = load carried by one stringer.
W = concentrated load on one wheel.
S = spacing of stringers, in feet.

In calculating moments in transverse floor beams (without longitudinal stringers), the distribution of each wheel load shall be determined by the following formula:

$$L = \frac{WS}{8}, \text{ but not less than } \frac{W}{2}$$

In calculating the end shears and end reactions of stringers and floor beams, no lateral or longitudinal distribution of concentrated loads shall be assumed.

Impact.[2] An allowance of 30 per cent of the live load stresses shall be made for impact.

207. Application of the Preceding Specifications. A slab bridge with a span of 16 ft.-0 in. is to be designed for light country traffic, with allowable unit stresses of 16,000 lb. per sq. in. in the steel and 650 lb. per sq. in. in the concrete.

The maximum load is that on either one of the rear wheels and is equal to 8000 lb.

$$E = \frac{2}{3}\left(16 + \frac{8}{12}\right) = 11 \text{ ft.-4 in.}$$

But 5 ft.-6 in. is the maximum value allowed in the specifications.

The live load moment is then

$$\frac{1}{4} \times \frac{8000 \times 16}{5\frac{1}{2}} \times 12 = 69{,}800 \text{ in.-lb}$$

The impact is moment 20,900 in.-lb.

Assuming a 13-in. slab, the dead load moment is

$$\frac{1}{8} \times 162 \times \overline{16}^2 \times 12 = 62{,}200$$

The total moment = 152,900

$$\text{Then } d = \sqrt{\frac{152{,}900}{107.7 \times 12}} = 10.9 \quad \text{Use } d = 11 \text{ in.}$$

A deck girder bridge with three girders 7 ft.-0 in. center to center is to be designed to carry secondary highway traffic.

[2] Not a part of Amer. Ry. Eng. Ass'n. recommendations, but a value widely used.

The maximum wheel load is 12,000 lb.

$$E = \tfrac{2}{3}(7 + \tfrac{12}{12}) = 5 \text{ ft.-4 in.}$$

Then the live load moment, considering the slab partially continuous over the girders, is

$$\frac{1}{5} \times \frac{12{,}000}{5\tfrac{1}{3}} \times 7 \times 12 = 37{,}800 \text{ in.-lb.}$$

The impact and dead load moments are obtained as in the previous example.

The distribution of the load to the supporting beams and girders and the moments and shears produced in these members are easily obtained and their design completed according to the principles outlined in Chap. III.

208. Abutments. The principles of Abutment Design are given in the chapter on Retaining Walls. Some details are shown in the following pages.

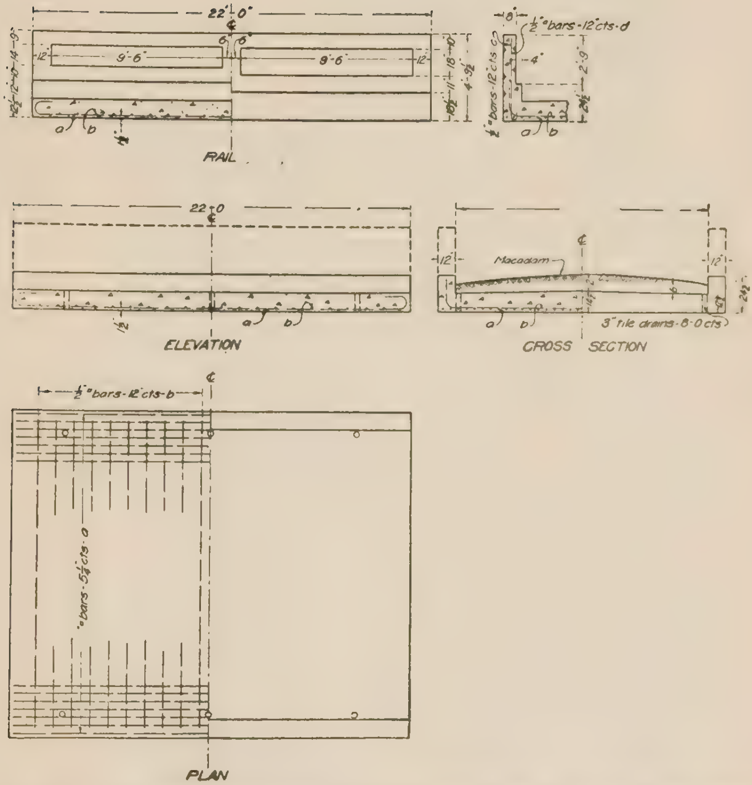


FIG. 137.—Reinforced concrete slab bridge, Illinois State Highway Department.

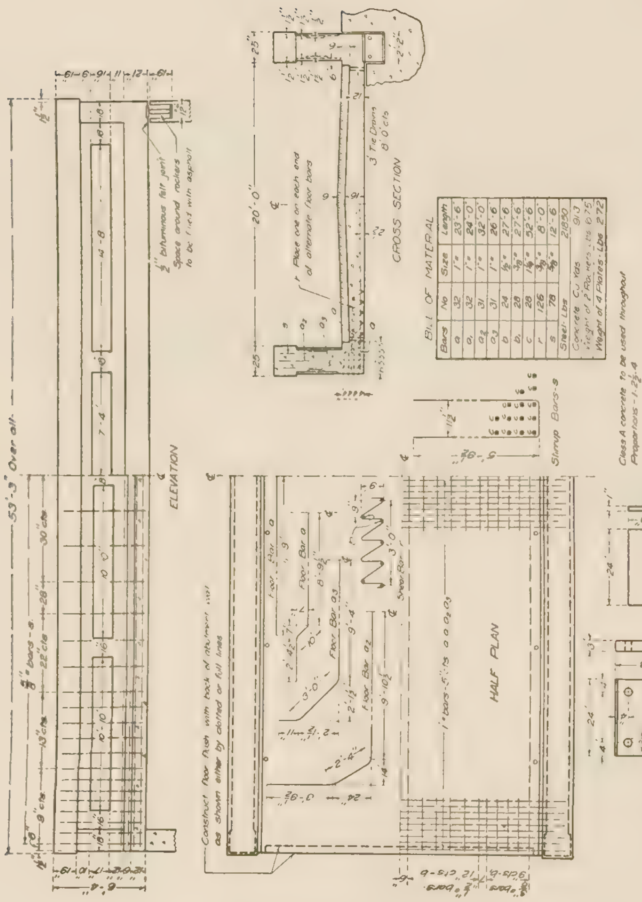


FIG. 138. Through reinforced concrete girder bridge, Illinois State Highway Department.

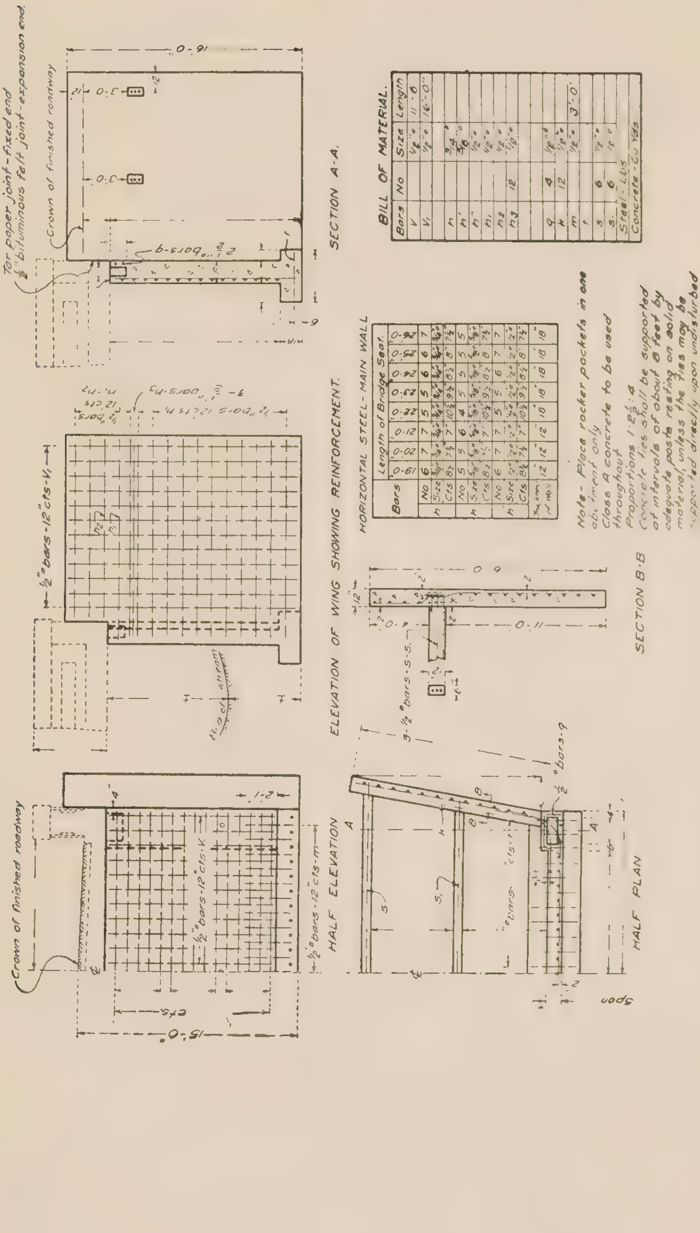


Fig. 139.—Reinforced concrete abutment for girder bridge, Illinois State Highway Department.

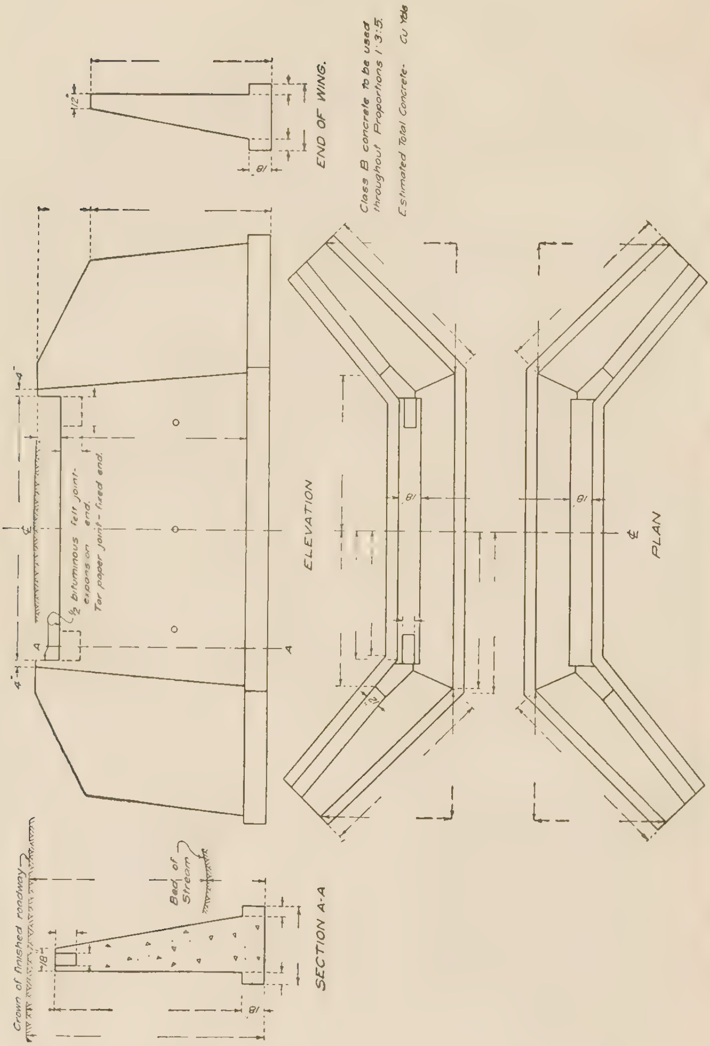


FIG. 140.—Plain concrete abutment for girder bridge, Illinois State Highway Department.

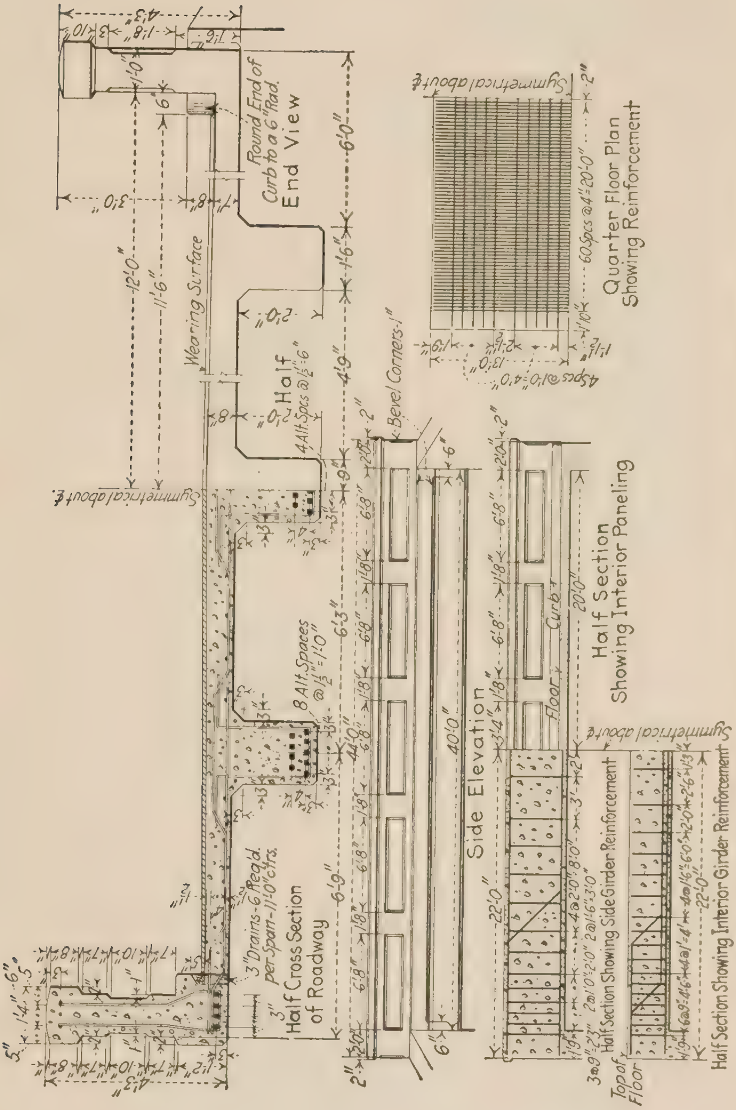


FIG. 141.—Deck reinforced concrete girder bridge, Wisconsin Highway Commission.

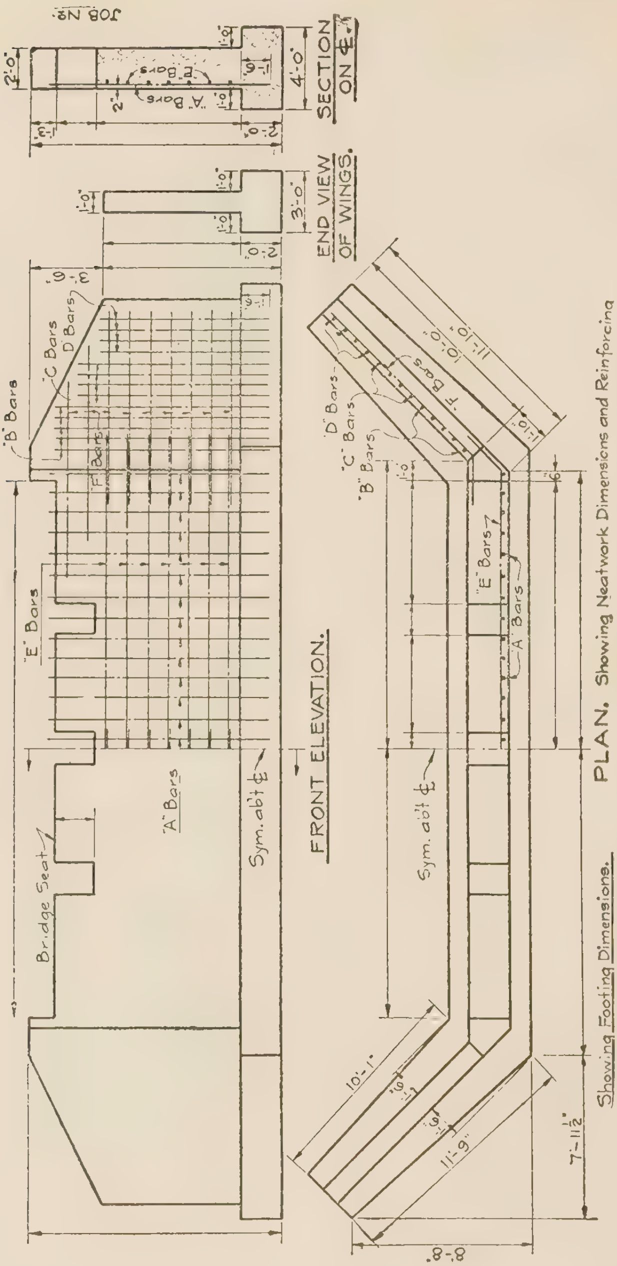


FIG. 142.—Reinforced concrete abutment, Wisconsin Highway Commission.

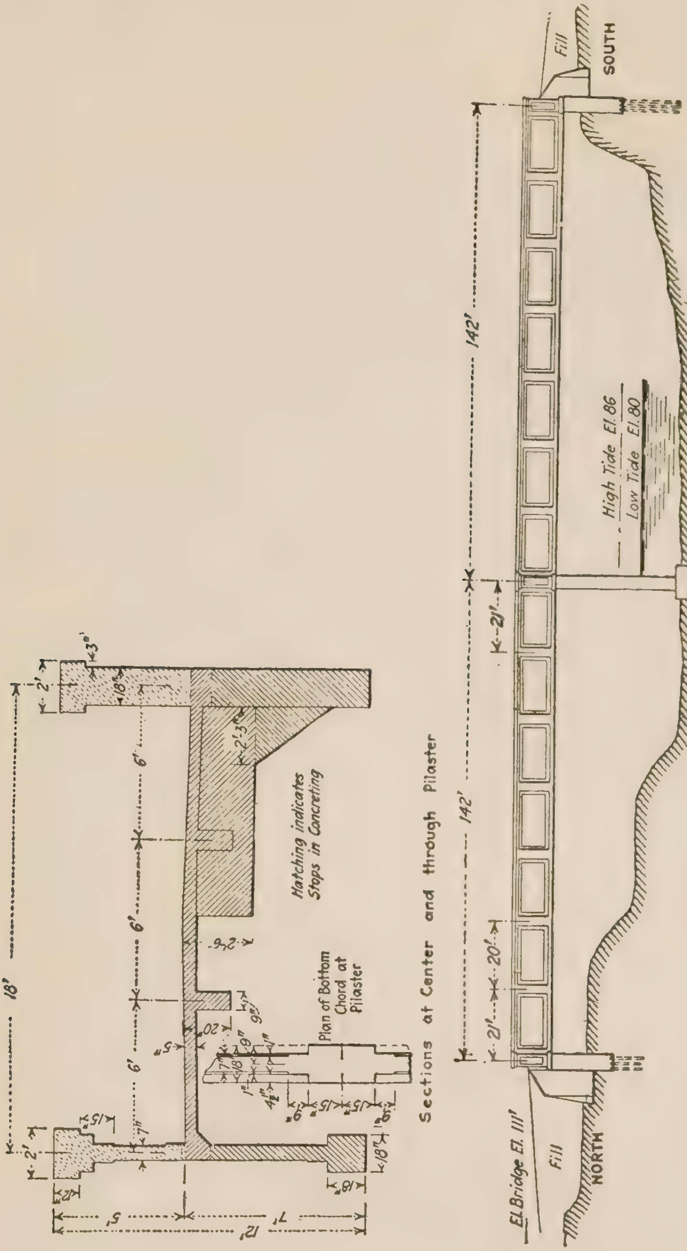


FIG. 143.—Salt River concrete bridge, Humboldt County, California.

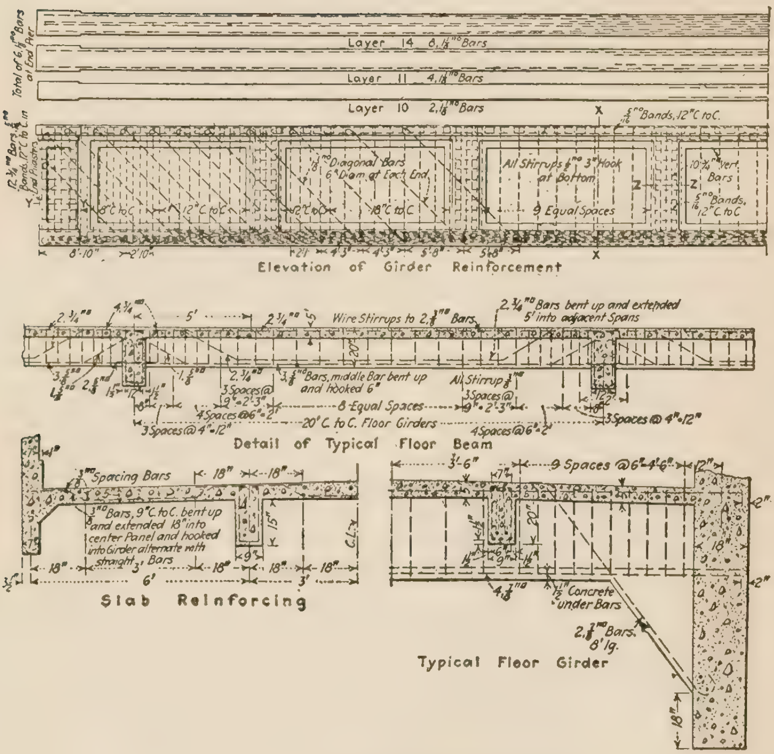


FIG. 144.—Details of Salt River bridge.

CHAPTER XII

FORMS[1]

209. For practically all concrete work of any magnitude forms of one kind or another are required. Their direct and indirect costs therefore have to be considered in estimating or building almost any structure of concrete. The direct cost is of course obvious. It will include the cost of the lumber and the labor to receive it, as well as the cost of fabricating, together with the additional cost of erecting and stripping.

The indirect cost is not so plain. It will include all sorts of expenses caused by lack of sufficient forms, poor design of forms, faulty handling, etc. There will be time for laborers or cement masons cutting and patching to improve defective surfaces on the finished concrete caused by poor form work; removing the fins left where the concrete ran in between loose-fitting boards; straightening up bulged columns and walls and sagged beams. In addition, if the forms are poorly designed, or are heavy and cumbersome to handle, the speed of the entire operation may then be retarded, and to the higher direct cost for labor must be added possible penalties for failure to complete on time, capital tied up, organizations and equipment not available for new work as scheduled, etc.

The form must be: (*a*) light and cheap, but sufficiently strong to insure true lines; and (*b*) so designed that it can be put together and taken apart with the minimum of labor. This latter presupposes that no one piece will be larger than can be handled economically with the organization available. On a large job, where there is a labor gang devoted entirely to the stripping and handling of forms, a piece that it will take eight men to carry may be entirely economical. On a smaller job, where the stripping and handling of forms would not ordinarily warrant

[1] Abstracted by permission of Concrete Cement Age Publishing Co., from a series of articles written by Wm. F. Lockhardt for *Concrete*, 1922.

such a large gang, it might be necessary to borrow men from another gang at the expense of holding up concreting. Such an arrangement would very likely be highly uneconomical.

210. Footings. Practically every structure requires footings of some kind. In building construction there are a few kinds and types of footings that are most generally met with, all very simple to build. As footings are very seldom exposed to view after they have been stripped, old lumber, if available, can often be used in making up the forms with economy. In some few

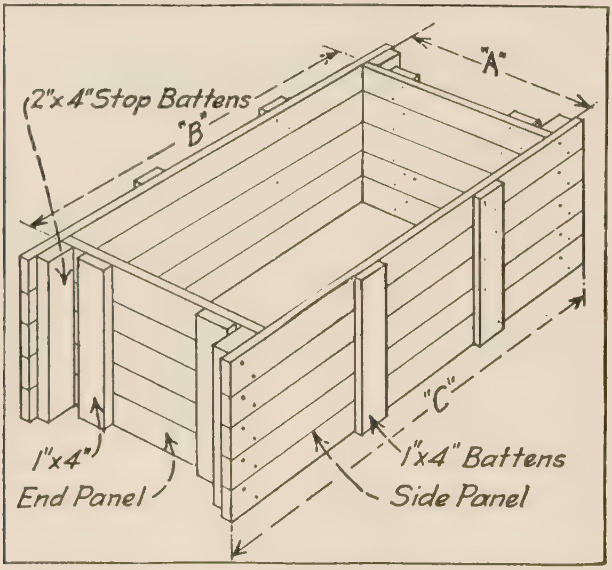


Fig. 145.

cases, though, generally occurring in city construction, part of the footings will appear in the basement or cellar above the floor, in which case it will be better policy to use new material, to avoid the labor required to give the concrete a good surface afterward. Where any one footing is repeated a number of times, it will generally pay to build a good substantial form that will serve for all the footings of that size and type.

Square Box Footing (Fig. 145). This is probably the most common footing type. For this footing it is frequently only necessary to dig a hole of the right dimensions and depth. It is

needless to point out that there is no use in building forms if the excavation is in firm material that will not collapse readily, or get churned up with the concrete. Occasionally, however, it becomes secessary to build a footing of this kind.

In plan the footing may be square or rectangular. The dimension "*A*" for the end panels is the same as the concrete dimension for that side of the footing. For the "*B*" length between the stop battens, add to the concrete dimension twice the thickness of the lumber used for the end panels. (That is, if you have made the end panels up out of 7/8-in. lumber, you will add 1 3/4 in.; if you have used 1 1/8-in. lumber, you will add 2 1/4 in.) For the "*C*" dimension—the over-all length of the long side panels—it is usual to add about 12 in. to the "*B*" figure. This allows for thickness of end panels plus the width of the stop battens, usually 1 × 4 rough lumber.

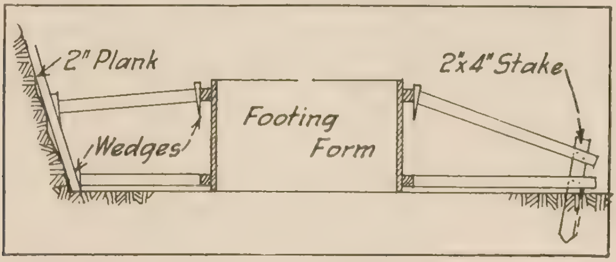


Fig. 146.

If the sides are very long, they should be braced to keep them from bulging. The bulge will not show, but it will take concrete costing so much per yard in place to fill it. The bracing may be done as shown in Fig. 146 which gives two common methods, or by backfilling against the form. The latter is frequently done when it is not intended to use the form again.

Step Footings. The step footing is a common type where the straight box footing would be wasteful of concrete. The steps may be two or three, but are more usually two, as shown in Fig. 147. Both upper and lower forms are made the same as for the box footing shown in Fig. 145. Usually only the form for the first step is set before concreting is started. After the first step has been filled, and while the gang is concreting the next footing

the box for the second step is set in place, and made fast. Filling is resumed after the concrete in the first step has set up enough so that the weight of the concrete in the upper step will not cause it to overflow.

Sometimes top and bottom forms are set at the same time, the form for the upper step being held in place by pieces of 4 × 4 or scantling nailed across the top of the lower box, as shown in the

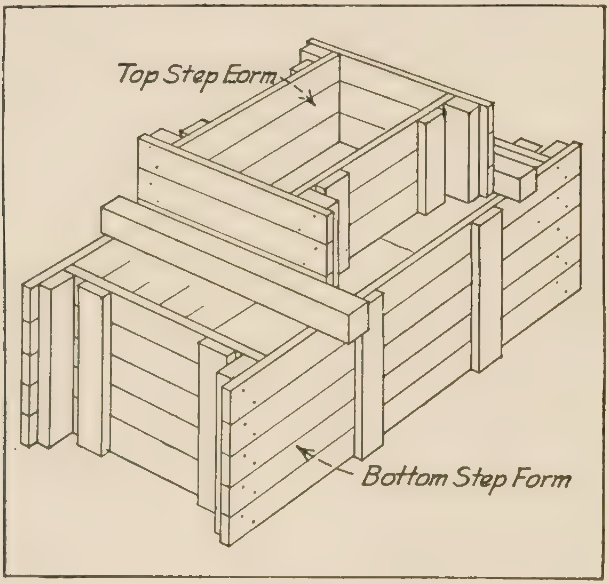


Fig. 147.

drawing. This form has to be filled in two stages just the same, however, to keep the concrete in the lower step from overflowing, just as when the top box is placed afterward. Before building the form up in this way, it should be ascertained whether it will be possible to place the reinforcing steel in the footing on account of the interference of the scantlings.

Both these methods save the expense of building a top form to close the space between steps 1 and 2. If a top form is used it will, of course, then be possible to spout the footing in one operation, but it will be necessary also to weight the footing down very securely to keep it from lifting.

Slope Footings. The slope footing is one of the most common designs found in buildings more than one story in height, or where the soil is soft and of low bearing value. As shown in Fig. 148*a*, a slope footing may be divided into two parts (both poured monolithic, however), the "ring" or box section at the bottom, which rests on the earth, and the sloped upper section, on the flat top of which the column stands. The method of placing

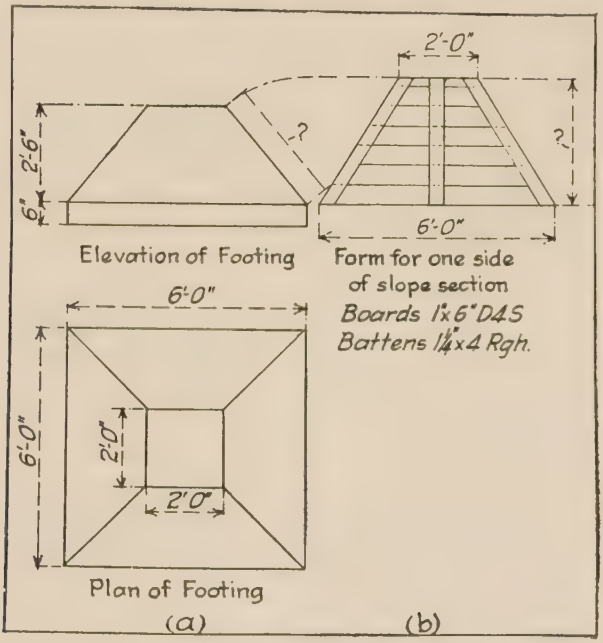


Fig. 148.

concrete and steel in one of these footings has an important bearing on the way the form is built.

Owing to the small size of the opening at the top of the slope section, it is generally impossible to place the long reinforcing bars while this part of the form is in place. As a result, it is customary first to set in place the "ring" and backfill against it to keep it from bulging. Concreting is then started and kept up until all but about 2 in. of the ring has been filled. The steel is then placed, and after all the bars are in, concreting is resumed until the ring is full. While the steel is being placed, the four

panels for the top section are assembled on the bank, and as soon as the ring is full the assembled form is put in place as a unit.

The slope panels for the top section offer very little more trouble in laying out and assembling than the preceding box types. Figure 148a shows a typical drawing of this kind of footing. All the necessary dimensions can be obtained from the footing plan with the exception of the dimension parallel to the slope of the footing, which is needed to give the width of the form panel.

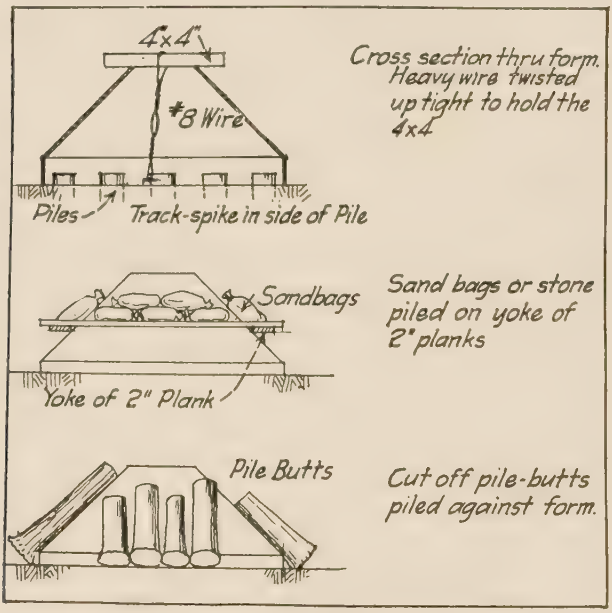


FIG. 149.

This dimension is marked (?) on the drawing, and is found just the same as the length of a rafter when the rise and run are known. The "rise" in this case is the depth of the slope section of the footing—2 ft.-6 in. in the drawing. The run will be one-half the difference between the top and bottom dimensions for that particular side. For the footing given, by subtracting 2 ft. from 6 ft. we get 4 ft., one-half of which, 2 ft., is the run mentioned above.

The panels are generally made of 1 × 6 roofers, with 2- × 4-in. battens along the slanting edges, and with intermediate

battens $1\frac{1}{4}$- × 4-in. rough lumber, the whole panel being put together as shown in Fig. 148*b*. On very large footings, 5 or 6 ft. wide, it is better to make all the battens 2- × 4-in. material for greater stiffness and ease in handling.

With this type of footing it is always necessary to provide means of holding it down until the concrete has set. Several methods are shown in Fig. 149. Where the footing is not on piles, to which it may be wired down, it is necessary to provide sand bags or other suitable material for providing sufficient weight. To carry the sand bags a plank yoke is framed up as shown and put in place about one-third of the way up the slope of the footing, to enable the weight of the bags to be distributed to better advantage. Where the footings are on piles, in addition to wiring the forms down, recourse may be had to the accumulation of pile butts usually to be found on a piling job. These are heavy, fairly compact, and easily handled, and cost nothing.

211. Walls. Wall forms on concrete industrial building construction are frequently not started until the structural skeleton is well under way, as they are not required for the strength of the building. In many of the smaller concrete operations, though, the concrete walls will be started as soon as the footings are poured, and for this reason they will be taken up here, although spandrel curtain walls are not built until after the pouring of the columns between which they stand.

Light cellar and foundation walls do not require anything special in the way of form design. Wall panels are made up usually of 2 × 4's spaced about 16 in. on center, to which the $\frac{7}{8}$-in. sheathing is nailed. These forms are used only once, and generally wrecked as soon as stripped and the lumber used elsewhere, so that it will pay to nail them up very lightly, just enough to hold the boards in place. The 2 × 4's act as standards and run vertically, projecting above the top of the form, according to their lengths, as there is nothing gained by cutting them down. As such walls are always in an excavation, it is a simple matter to brace the 2 × 4's, which would otherwise be too light, to the bank. Often such forms are built in place, at least as far as the outside form is concerned, the standards first being erected and braced, and the sheathing then nailed on.

On industrial buildings the walls are generally of two types: (1) the full story height wall, such as shaft enclosure walls; and (2) the curtain or spandrel wall, 2 or 3 ft. high, capped with a sill and carrying the sash.

Of the former, a typical example is shown in Fig. 150, in which the upper part of the column and wall have been removed. The panels for each side of the wall are made up of $\frac{7}{8}$-in. lumber, the

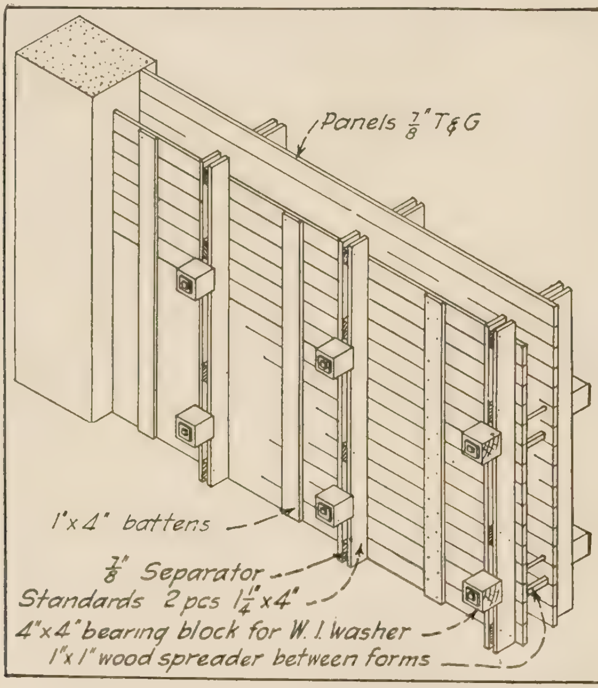


Fig. 150.

boards running horizontally, with 1- × 4-in. or 1- × 6-in. battens on the outside. The panels are held apart by spreaders, which may be any of the types described later. To bring the pressure on the individual boards to the bolts, the bolts are passed through standards, which in Fig. 150 are shown as being built up of two pieces of $1\frac{1}{4}$- × 4-in. lumber, separated by pieces of $\frac{7}{8}$-in. lumber, to provide a slot through which to pass the bolt. In erecting these walls the customary method is to put up the panels for one side, place the steel, tack the spreaders lightly in

place, and then up-end the outside panel against the spreaders and brace it in place temporarily. Bolt holes are then bored through both forms at the same time by the use of a brace with an extension bit, after which the standards are placed in position and the bolts passed through and tightened up. A piece of $\frac{7}{8}$-in. board, or a 4- × 4-in. wood "washer," is often placed under the wrought

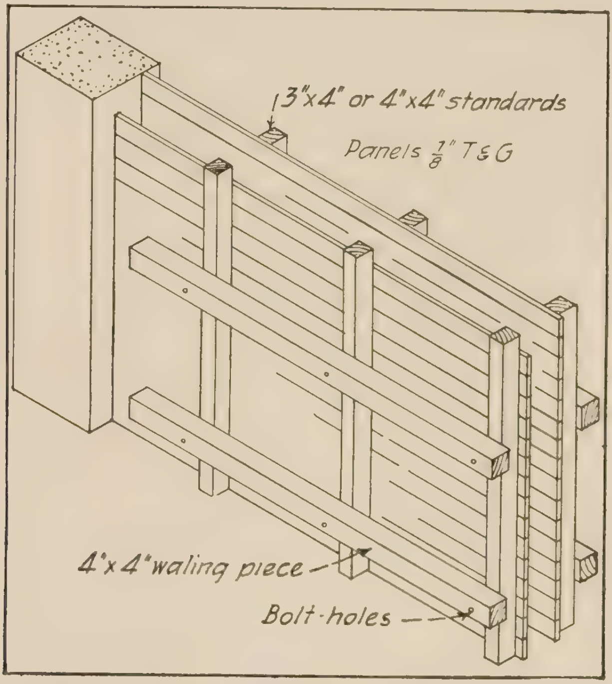


Fig. 151.

iron washer, as shown, both to take up any excess length in the bolt and to give a better bearing on both parts of the standard.

It will readily be seen, however, that it may be difficult to keep a form built in this manner properly lined up, as there is nothing to counteract any tendency the wall may develop to bulge out of line as a result of the filling operation. If it is not convenient to brace each standard back to some fixed object like another wall, it will probably be better to build the wall as shown in Fig. 151.

When it is necessary to leave out an opening for a window, a frame of the proper size is made up of 2-in. plank and set in place against the first panel erected before the wall is closed up. If the window is of the rolled steel type, a beveled strip should be nailed to the outside of the box, so as to form a recess in the concrete jamb of the opening, into which the angle-iron side members of the sash can later be grouted.

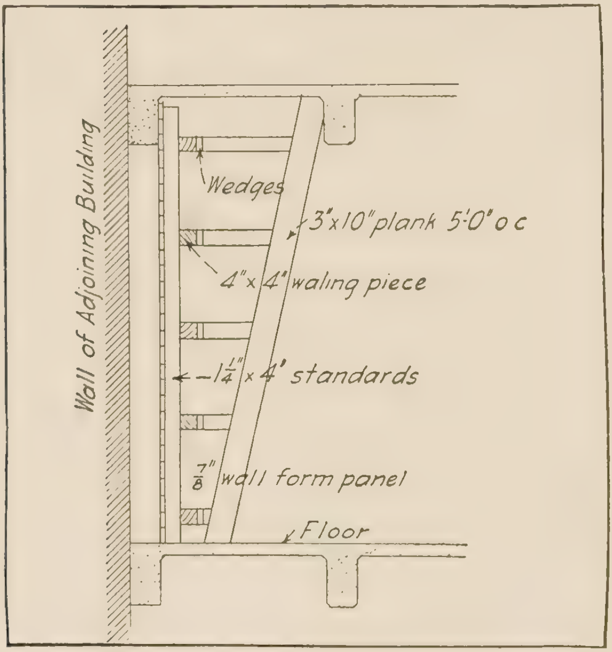


FIG. 152.—Method of bracing wall where through-bolt cannot be used.

When a wall finishes up under a beam, it cannot be conveniently poured directly from above. It is then necessary to build at least three brackets per bay on the outside of the form, extending out and up far enough so that concrete may be poured into them from the floor above. These brackets usually extend out from the beam one board 5½ in., and are about 18 in. long. It is important that the tops of all the brackets be well above the highest point to be concreted in the wall, so that a slight head or pressure may be obtained, which will ensure the wall being filled up under the beam.

Low curtain walls or spandrel walls are a slightly different proposition from the story-height walls we have just been considering. The perspective sketch (Fig. 153) shows how these forms are put together.

While the arrangement will vary in some details from building to building because of differences in the reveal on the column, position with reference to the beam below, etc., the general arrangement shown in the drawing can be followed. There are usually two stud-bolts in each exterior column just below the floor level, which are used to support the exterior column form

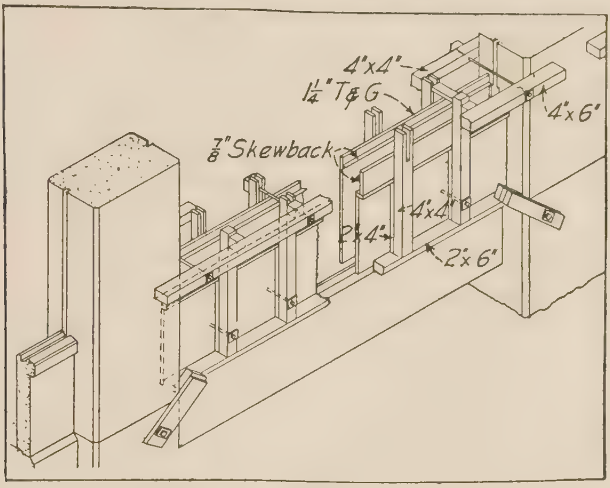


Fig. 153.

for the story above. Cleats slipped over these bolts and tightened up help hold the bottom of the wall form in place. The panel for the inside face of the wall is the same as any other wall form. The exterior panel will frequently carry a step for an overhanging monolithic sill. The overhang of this sill makes it necessary to pack out between the outside of the form and the standard, up to the underside of the sill. The top of the form is held in place by the 4- $\times$ 6-in. timber that spans from column to column, against which it is held in place by bolts to the 4 $\times$ 4 on the inside face of the column. One set of spreaders is placed between the form panels above the lower line of bolts; these can be knocked out easily and removed as the concreting progresses.

The second set of spreaders is placed between the standards just above the sill line.

Seven-eighths inch triangular moldings, called skewbacks, are nailed to the forms at the level of the top of the sills, to avoid sharp edges which are easily damaged. If a drip molding is required on the sill, a piece of skewback may be very lightly tacked to the form for the overhang. This part is important, because if the skewback is nailed so tightly as to come away with the form when the wall is stripped, it will very likely bring along with it the drip molding that it was intended to form.

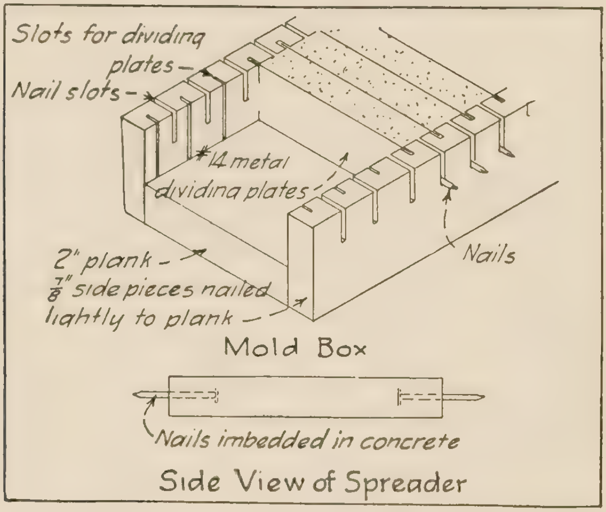


Fig. 154.

By tacking the skewback in place very lightly it will pull away from the form and remain embedded in the concrete, and can be removed later when the outside of the building is being cleaned down, by which time the concrete will be strong enough to allow the strip to be pulled out without damage to the drip.

The most common type of spreader is obtained from a short piece of $\frac{7}{8}$-in. board of the right length, which can be split with a hatchet to supply five or six spreaders about $\frac{7}{8}$ in. square. If the wall is open at the top, these spreaders should be knocked out as the concrete reaches them, and removed. When the wall is closed at the top, as when it reaches to the ceiling, the spreaders

will of course have to be left in. Sometimes by tapering them and greasing them with a very heavy grease, they can be driven out while the concrete is still green, but this does not always work. If the spreaders are left in the wall, they should be cut back with a chisel at least 1 in. before the surface is finally finished.

Various types of concrete spreaders are used. One type is illustrated in Fig. 154.

There are a number of metal wall ties and separators in use, but many of them either have to be ordered of the exact length to fit a wall of a given thickness, thus lacking the flexibility of bolts, or they leave some metal close to the face of the wall where it will rust sooner or later and cause disfigurement. Their economy and usefulness depend to a large degree on the general method of working and the size of the work.

212. Columns. Forms for interior and exterior columns are usually of different construction. Those for exterior columns are usually rectangular in plan, while those for interior columns may be square, octagonal, or round, and if the latter, are frequently of sheet metal instead of wood. For any of these, the construction of the column head or capital will vary according to the type of floor construction, the requirements for beam and girder construction being quite different from those for flat slab.

Sheet metal column forms are usually confined to round columns. As the light metal that is used in these forms is lacking in strength to uphold the floor form system, it is necessary where these forms are used that special provision be made at the column for supporting the floor forms. This will be discussed later.

Wood forms for circular columns are not frequently met with, but under certain conditions they may be entirely satisfactory.

Rectangular and octagonal column forms are usually of wood. Differences in length of sides, size of yokes, etc. have to be taken into account on rectangular or square forms, according to whether they are for interior or exterior columns.

It has been found that the construction is simplest and the board marks least conspicuous when the boards run vertically. For small columns, $\frac{7}{8}$-in. roofers may be used, but the common practice is to use $1\frac{1}{4}$-in. lumber, dressed four sides, tongued and grooved.

The necessity for avoiding sharp edges is well known. The green concrete will get nicked and chipped in the operation of removing the forms, and some method should be used to avoid a sharp arris. The simplest method is by beveling the corners, accomplished by inserting in the corners of the form a triangular wood strip called a "skewback." Sometimes a better appearance is desired, which may be obtained by the use of a rounded corner, or "bullnose," in which case a cove molding must be built into the forms. Figure 155 shows plans of the corner of a

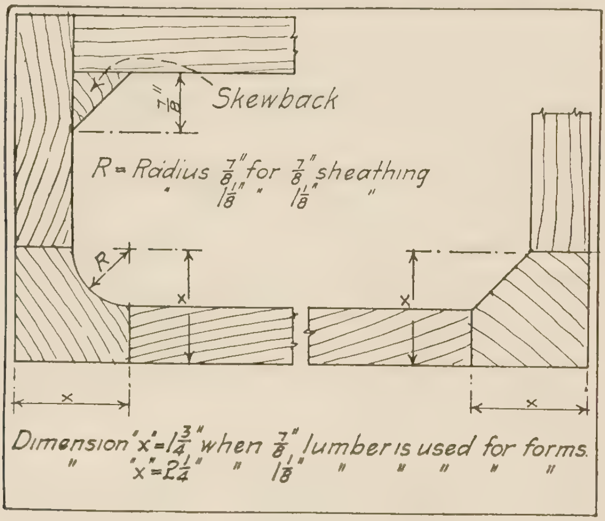


FIG. 155.

column for each of these arrangements, with the usual dimensions. These may of course be varied if it is so desired.

The column sides are stiffened and braced by yokes or battens, according to the system of clamping used. With yokes, bolts or rods, malleable iron clamps are most frequently used. With battens the different types of patented form clamps are adopted. Concrete exerts a pressure on the forms equal to that of a liquid weighing approximately 125 lb. per cu. ft., according to experiments conducted by the Bureau of Standards. As this is twice the weight of water, the necessity for ample strength in yokes and bracing will be seen readily. The pressure varies with the depth, not with the thickness of the concrete in the column.

The yokes are spaced more closely toward the bottom of the column, as this is where the pressure is greatest.

Bolts for column forms are usually $\frac{5}{8}$ in. Half-inch bolts have been used with satisfaction, but for some types of form construction are found to be too light to withstand the strain of wedging against them as will be described under square and rectangular columns. As it is usual to stock only one diameter of bolt on the job, these bolts would also be used for wall construction, and $\frac{1}{2}$-in. bolts bend quite readily in pulling; $\frac{3}{4}$-in. bolts are heavier than necessary in building construction, and have generally been abandoned except for special heavy work.

Cast iron washers may be used, or wrought iron "plate" washers may be bought. These latter are usually 3×3 in. For $\frac{5}{8}$-in. bolts or rods the hole is generally $\frac{3}{4}$ in. Square-head bolts and square nuts are generally used.

Circular Columns

Sheet metal forms for circular columns are ordinarily rented from an agent of the manufacturer, who erects and strips them at a fixed price per column. As these forms are not designed to carry the weight of the floor form construction, provision must be made to support either the drop panel, in flat slab construction, or the beams and girders framing into the column in beam and girder forms. Such a support has been detailed in Fig. 156. This form of bent can be used equally well for either beam and girder or flat slab construction by varying the height dimension as required. The uprights are 4×4's, the top horizontal brace is a 2×10, the lower horizontal brace and the diagonal are 1×6's. The bent is made up of two sides, each of which is permanently assembled; the two sides are set up in place either side of the column center, and the movable horizontal braces dropped into place in sockets on the sides of the bents. If necessary the completed frame is stiffened with temporary diagonal staylathing of 1×6's. In stripping, the horizontal braces are lifted out of the sockets, and after the wedges have been removed from under the posts the two bents may be taken down and hoisted to the next floor complete, ready for reassembling.

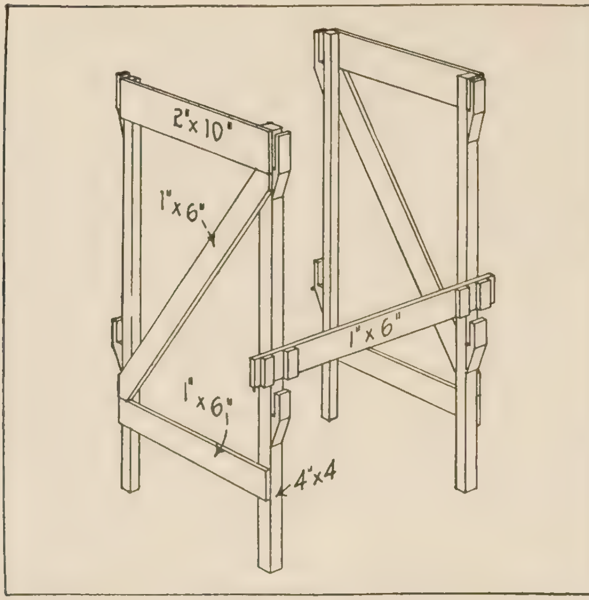


FIG. 156.

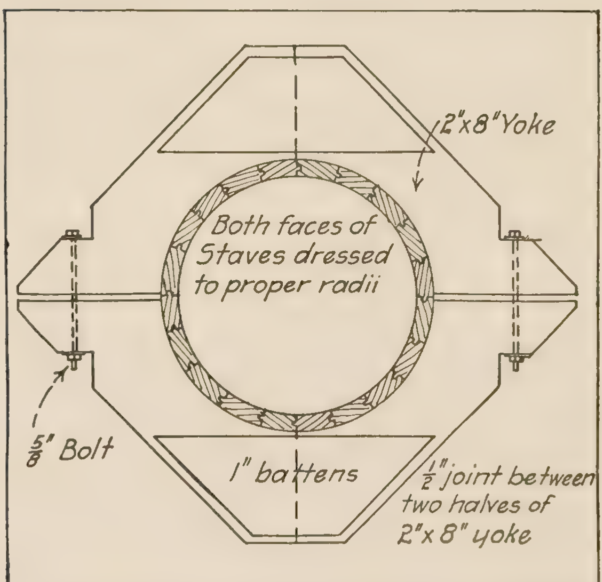


FIG. 157.

Wood forms for circular columns must be made up of staves set in a yoke. The faces of the staves and the yoke must be shaped to circles of the right diameter for the size of the column and the thickness of the staves (see Fig. 157). The radius of the curve on the inner face of the staves will of course be one-half the diameter of the column. The radius of the curves cut in the yoke will be one-half the diameter of the column plus the thickness of the stave.

The form is bolted together through the yokes, which are notched out as shown in the drawing to provide a square seat for the washers on the bolts.

Rectangular Columns

Rectangular and octagonal column forms are usually of wood. The exterior columns of a building usually keep the face that is parallel to the building line unchanged, the reduction

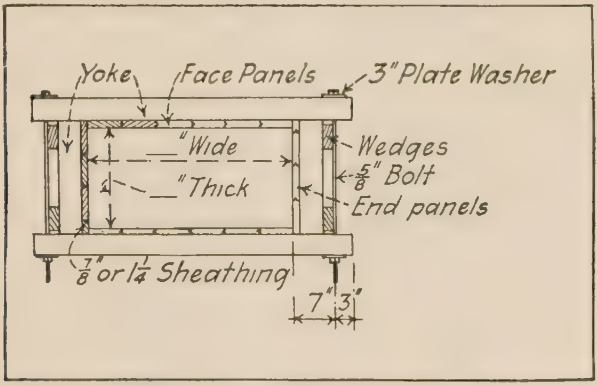


Fig. 158.

being made in the "thickness" (dimension at right angles to the building line). Interior columns are often square, and in this case all four faces will have to be reduced in width. The necessity for being able to reduce the size of a column economically must not be overlooked in planning the forms.

The arrangement of sides shown in Fig. 158 is very largely used and has been found simple to build and reduce and quite generally satisfactory. The drawing shows a rectangular form

for convenience in referring to the different panels. The face panels (as marked on the drawing) are usually called the "sides" and the narrower panels, which fix the thickness of the column, are called the "ends."

A square column would be built just the same as the drawing shows, the dimensions of the different panels alone being changed.

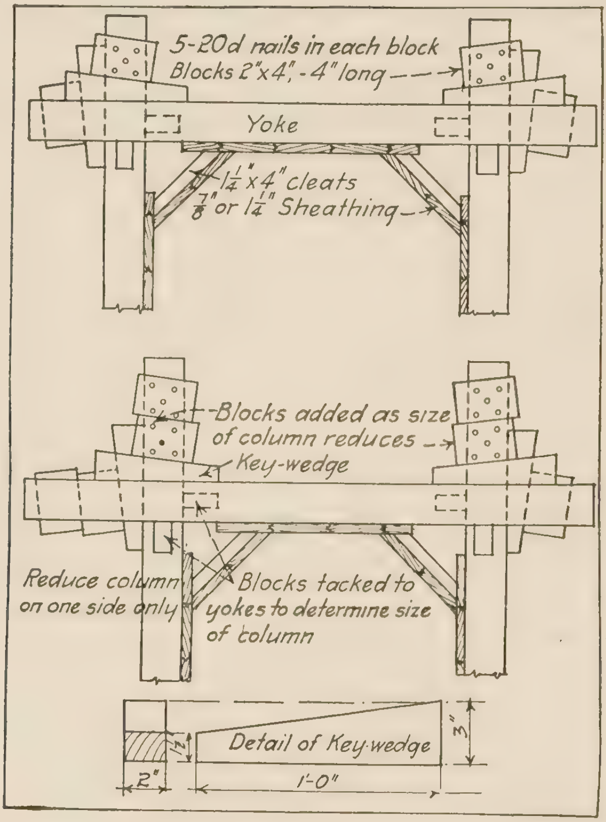


FIG. 159.—Method of yoking or clamping a column without bolts by the use of keys.

The width of the sides is just the same as the concrete dimension. This panel is made up on long yokes, which also serve as battens, which project about 10 in. beyond the panel at each side, to allow space for bolting and wedging. The end panels are wider than the concrete dimensions for the thickness of the column by twice the thickness of the lumber used in the panels.

A glance at the drawing will show why. Necessarily, the yokes on the end panels are cut off flush with the sides of the panels.

For exterior columns, as the upper stories are added to the building it will be found that with each succeeding floor or two the thickness of the column will be reduced, as the load on the

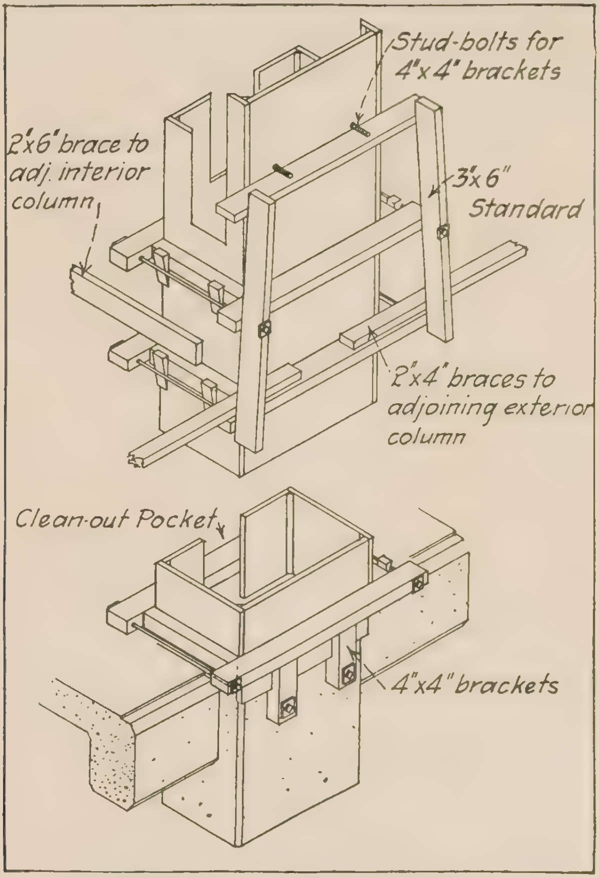


Fig. 160.—Exterior column form set in place.

column reduces, and a smaller column area is required. Architectural appearance demands that the face of the column that shows on the street be the same width from street to cornice, so the smaller area is obtained by reducing the thickness. This merely means that to reduce the column form, a strip the same width as the required reduction in the thickness of the column is

ripped off one side of the end panels, cutting through panel and yokes alike.

After the panels are assembled the bolts are placed and tightened up, and then the wedges shown in the drawings driven down

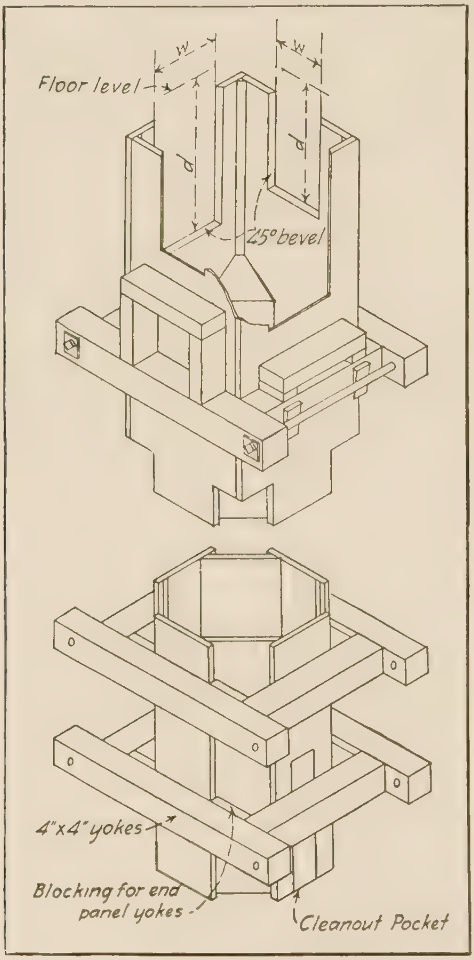


Fig. 161.

between the bolts and the 4- × 4-in. cleat or yoke piece on the ends, to hold the end panels tightly in place against the sides. It is here that a ½-in. bolt frequently proves too light, as it will bend under the wedging or later under the pressure of the concrete.

Column Heads and Capitals

Both square and octagonal columns have square heads; the four corner pieces of the octagonal column are capped with a three-cornered piece to give the effect in the finished column of a chamfer stop. This piece shows in Fig. 161. Openings for beams and girders are cut with a 45-degree bevel as shown. The beams and girders do not actually frame up tight against the column

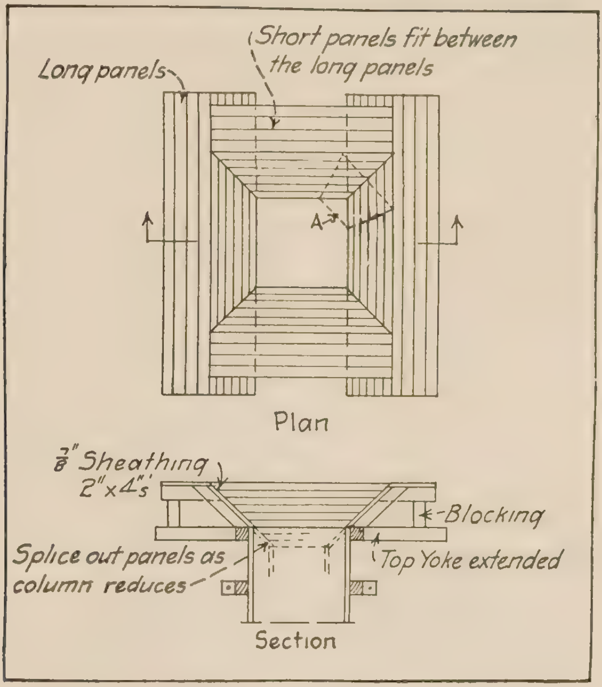


Fig. 162.

form, but are separated from it by removable keys which provide the necessary clearances and make stripping easy.

With flat slab construction a flared capital is necessary on top of the shaft of the column. With circular metal column forms the capital is also of sheet metal, furnished and erected the same as the shaft, and need not be dealt with here. Where wood forms are used for interior columns a square octagonal capital will be required according to the shape of the column.

The square capital is a little simpler than the octagonal. It is generally built up of two long sides and two short sides that fit in between them, as shown in Fig. 162. All the panels are made up for the largest column, and as the size of the column reduces, the panels are pieced out as indicated in the section in broken lines. The octagonal form is somewhat similar, but beveled corner pieces are inserted in the corner of the square capital form, as indicated in broken lines at "*A*" in the plan. These heads are sometimes supported temporarily by making the top yoke of the column long enough to carry them, as shown in the section in Fig. 162, but before any concrete is poured they should be posted up.

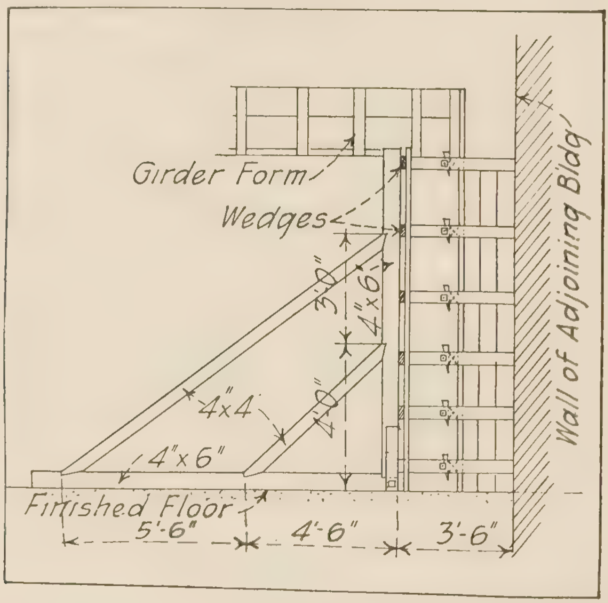


Fig. 163.

Miscellaneous

A minor problem occasionally encountered is that of holding in place a column which adjoins an existing building. An arrangement similar to that shown in Fig. 163 may easily be made up, and in actual service this has proved very satisfactory. The frame of 4 × 4's and 4 × 6's is made up separately and put into

place as a unit, and the wedges between the column yokes and the vertical 4 × 6's are driven afterward to make the whole frame tight.

213. Beams and Girders. An economical system of beam and girder forms for a building job really starts with the lumber list, on which should be shown clearly the manner in which the lumber is to be used. This will avoid confusion in the fabrication of the forms, and if the lumber list shows, as it should, for just which member each particular piece of lumber has been ordered, very little extra material need be bought, and a great deal of waste can be avoided. Before the lumber list can be made up, however, a thorough knowledge is required of just how the forms are put together, so that as the plans are studied the necessary form details for each individual beam and girder can clearly be pictured mentally.

The methods used for stripping and reposting will also affect the quantity of lumber to be bought, so that it will be convenient to consider the several phases of the work in turn, touching on the lumber sizes commonly used, with a few notes on safe carrying capacities, posting, stripping, and reshoring, form details, and the lumber required and the lumber list itself.

The lumber used must be strong enough to contain and support the mass of semiliquid that it encloses, and in the case of beam sides and floor panels be stiff enough to hold its lines after it has been made up into forms, without sagging or bulging, or requiring an undue amount of battening or bracing to keep it true. The sizes given here are in general use, and for ordinary building work can be followed to advantage.

Shores and posts, usually 4 × 4 in. rough. Occasionally 6 × 6 in. for special cases of very heavy loads or stories too high for 4- × 4-in. posts.

Beam and girder bottom, 2 in. × . . . in. Dressed four sides to 1¾ in. × (*required width*) in.

Beam and girder sides, 1½ in. × . . . in. Dressed four sides to 1¼ in. × (*required width*) in.

Cleats or battens 1 × 4 in., 1¼ × 4 in. or 2 × 4 in. rough.

Ledgers 1¼ × 4 in. or 2 × 4 in. rough.

Spreaders 3 × 4 in. rough.

Floor panels 1- × 6-in. roofers, D 4 S to ⅞- × 5½-in. T. & G.

The first step when considering the general layout of the forms, before making up the lumber list, is to figure out the weight to be carried by the forms, and the resultant allowable spacing of the shores.

The formula of the American Railway Engineering and Maintenance of Way Association for unseasoned short-leaf pine and spruce posts is as follows:

$$L = 1100 \times \left(1 - \frac{l}{60d}\right)$$

in which:

L = Safe load in pounds per square inch.
l = Length of post, in inches.
d = Least diameter of post, in inches.
1100 = Extreme fiber safe working stress in bending.

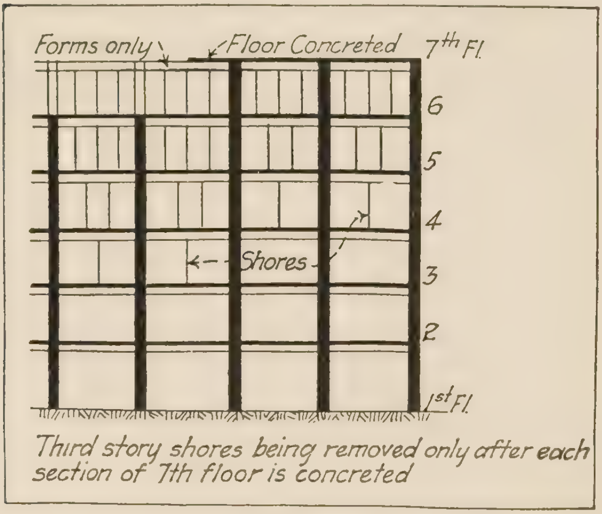


Fig. 164.

The formula is intended for railroad bridges and trestles; for other structures certain increases are allowed in loading, but as the lumber used for form work is often of a poor grade it is advisable to adhere to the values given by the formula.

Because of the speed with which concrete buildings are ordinarily erected, the dead load of the next story is frequently

placed on concrete scarcely a week old, when it has only a part of its ultimate strength. This means that to carry the successive stories as they are erected there should always be at least two stories fully reposted (in addition to the shores under the forms) and one additional story partly reposted. This story should have at least half the reshores in place. This is shown in Fig. 164 which is a vertical section through a typical bay of a building. These rules apply to summer or warm weather work

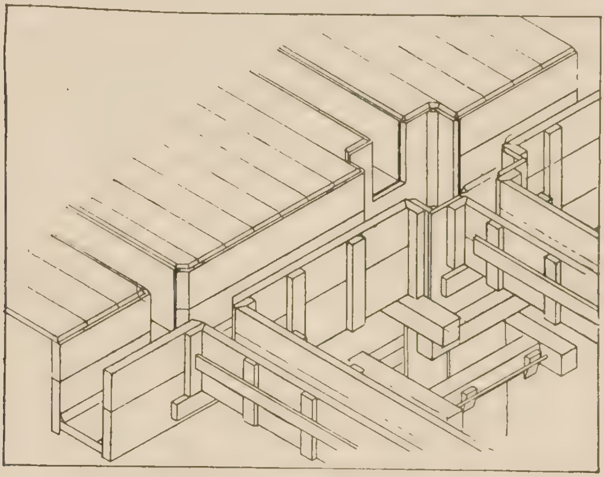


FIG. 165.

carried on at the rate of about a story per week. For winter work or greater speed at least one additional story fully reshored is required, and sometimes more, depending upon the individual job conditions.

A safe stripping schedule is as follows, based on summer conditions:

1. Columns, 24 hours after floor slab is placed.
2. Girders, 60 hours after concreting (third day).
3. Beams, 84 hours after concreting (fourth day).
4. Panels, any time after the beams.

The perspective sketch (Fig. 165) gives a good idea of the general arrangement of beam and girder forms. The girder forms span between the columns, and are generally supported by blocking up from the top column yoke. If metal column forms

are used, the girders are supported by a scaffold or bent similar to that shown. The beam forms are carried by the girder forms, generally by spiking a piece of 2 × 4 to the outside of the girder form just below the beam opening. The floor panels are made up with 3- × 4-in. spreaders on the back in place of battens. These spreaders rest on a 1¼- × 4-in. ledger nailed to the cleats on the outside of the beam form. A 3- × 4-in. spreader will not carry so great a load as a 2 × 6, but the number of spreaders is usually fixed by the allowable span for the ⅞-boards of which the panel is made, and this is 2 ft.-6 in. or less. Under these conditions, for the usual 4- or 4½-in. slab found in building construction, spanning about 6 ft. between beams, a 3- × 4-in. spreader is strong enough, and it has the additional advantage that it does not overturn in handling or under load as readily as a 2- × 6-in.

It should be obvious, then, that all parts of the system must be cut and framed to the right sizes to avoid patching out when the floor forms are being assembled in place; it should be equally obvious that if the various members comprising the form system are to be taken down economically, so that they can be used again, without having to cut and saw, proper clearances must be provided where the different members frame into one another. Some simple means must also be provided for closing up the gaps required for the clearances.

The first member to go into place is of course the column, where wood column forms are used, because the column forms can then be made to carry the girders. Where metal column forms are used, the trestle or bent previously mentioned will be necessary to carry the girders. The required height for the bent is obtained as follows: Make a rough sketch similar to Fig. 166, putting down first the story height and then the distance from the finished upper floor surface to the under side of the girder forms. Note that this is not the underside of the concrete girder, but the underside of the girder forms. This dimension is given for example in the drawing as 2 ft.-2⅞ in., and reference to Fig. 167 will show how this dimension is obtained. If the story height is 14 ft. from finished floor to finished floor, we will have first to subtract the 26⅞ in. and then make some allowance for

wedging under the bent to compensate for uneven spots in the floors. We will allow for example $3\frac{1}{8}$ in., thus making the height of the bent itself over all 11 ft.-6 in. This will be satisfactory where all, or nearly all, of the girders are of the same depth. Where the girders are of varying depth, it would be inconvenient and expensive to make individual bents for each particular case,

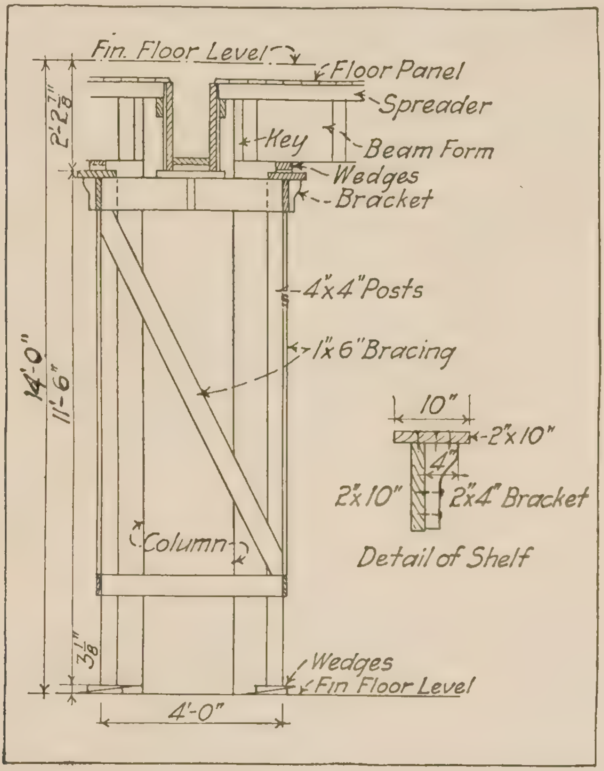


Fig. 166.

so that it is usual to make the bents low enough to take the deepest girders that are typical, and provide a shelf as shown in Fig. 166 upon which to wedge and block the shallow girders to the right elevation.

From this point on it will be an advantage to consider the form details and the lumber list together. The first step in making up the lumber list for the floor forms will be to set down the lumber required for the girders. Refer again to Fig. 167.

At "*A*" is shown the girder section as it will probably appear on the plans. Note that the depth of the girder includes the thickness of the floor slab, in this case 4 in. This is the usual method of giving beam and girder dimensions. At "*B*" we have the superintendent's sketch showing how he intends to build the form. The width of the girder is given as 10 in. For the girder bottom he will order a 2- × 10-in. plank "Dressed 2 Sides" to 1¾ × 10 in. The edges will in this case be left rough, as to dress them down would take ¼ in. from the width of the plank, with nothing gained by so doing, as the joint between the beam bottom and the girder side is covered with a beveled molding

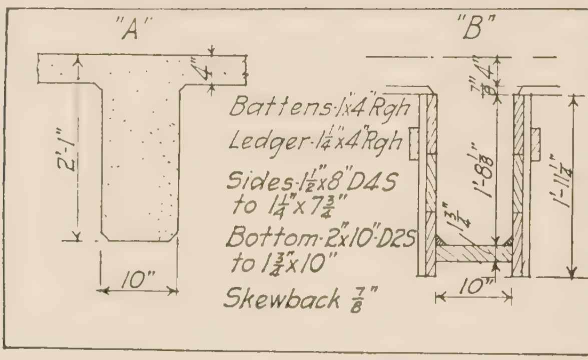


Fig. 167.

called a skewback. The skewback is generally ⅞ in. for beams and girders. Larger skewbacks than ⅞ in. are not often used in building work.

For the sides of the girder form three pieces of 1½- × 8-in. D 4 S to 1¼- × 7¾-in. are called for. These three pieces add up to 23¼ in. while the drawing shows that only 21⅞ in. are required from the bottom of the floor panel to the underside of the beam bottom, but by having the sides project slightly below the bottom piece the shores bear up against the lower edge of the sides instead of against the bottom plank. This makes the form much less likely to sag between the shores and gives better lines. It is important that all beam and girder-side lumber be dressed to exactly the same thickness (1¼ or 1¾ in.) as it will be impossible to get a good-looking job of concrete if some of the planks in

the form are finished $\frac{7}{8}$ in. thick and others $1\frac{1}{8}$ or $1\frac{1}{4}$ in. A note should always be made on the order for the lumber for beams and girders that all the material of a given thickness must be dressed uniformly to that dimension.

The length of the girder form will be somewhat less than the clear span between the concrete columns. In ordering the lumber it must be borne in mind that the girder will first be used between the columns in the basement or first story. Here the columns will have the greatest diameter, and the girder form will consequently be shortest. If the drawing gives the length of the bay as 20 ft. center to center of columns, with the columns assumed as 2 ft. in diameter, the clear span between the columns will be 18 ft. In discussing the length of a beam or girder form

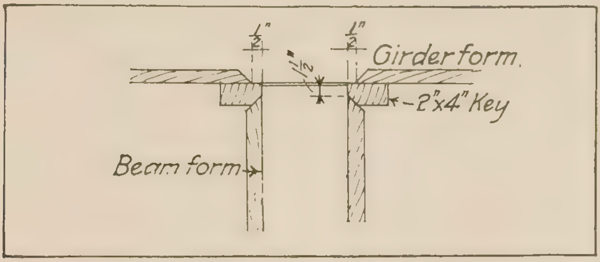


Fig. 168.

it may be well to mention at this point that the "length" of the form will refer to the length of the bottom plank. It is usual to make the sides of the form about $1\frac{1}{2}$ in. shorter at each end than the bottom, to facilitate stripping. When the concrete is poured, the pressure will force all the abutting form surfaces tightly together. If the sides are made full length, the pressure of the column forms against the ends of the girder form will make it next to impossible to get the form down. If the sides are made an inch or more short at each end, a removable "key" piece may be inserted as shown in Fig. 168, which shows the condition at the junction of column and girder forms, or beam and girder forms, the details being similar in each case. The key, being beveled, can usually be removed without difficulty, but if it sticks, it can be chopped out without injury to the main form, thus providing the clearance required. The bottom form may be made practi-

cally full length, as clearance is provided as soon as one end is pried down.

If metal column forms are to be used, the girder form may be made the full 18 ft. long, lacking about 1/4 in. for the thickness of the metal in the column molds, which will be nailed to the ends of the girder forms.

When a wood column form is used, the length is figured in exactly the same way, taking into account, however, the thickness of the wood in the column forms, these being generally built of 7/8-in. lumber for interior columns. For the same case as we have assumed, the length of the girder bottom would be 18 ft. less two thicknesses of 7/8-in. material, or 17 ft.-10 1/4 in. The sides will be 3 in. shorter than the bottom, to allow the 1 1/2 in. at each end for keying, thus making the sides 17 ft.-7 1/4 in. long.

The ends of the girder sides are cut on a 45-degree bevel as shown in Fig. 168. This drawing also shows the opening in column capital or girder, as the case may be, which is cut on a bevel the other way. The opening in the column is made 1 in. greater than the width and 1/2 in. greater than the depth of the beam. When the form is stripped this will show up as a narrow raised molding 1/2 in. wide around the end of the girder which is of value in making less conspicuous any minor irregularities that may occur in fitting the members together. Where wood column forms are used, it is advisable to set the top column yoke at such an elevation that it will carry the girder at the proper height.

As the columns reduce in diameter from story to story, it will be necessary to lengthen out the girder forms. This can easily be done by lightly nailing in place a piece of beam side material of the right width each time the column reduces. If the patch is fastened on with a splice on the back, it can be removed each time the column reduces and a new piece of the required total width substituted. This will mean only one patch at the end of the girder instead of several.

The beam forms are made up in exactly the same manner as the girder forms. The depth of the beam having been taken from the plans, a sketch is made up to show the section and on this section are marked the sizes of the various pieces of lumber required to make up the form. It will always cost slightly more

to make up a form of several pieces of 6- or 8-in. lumber than a lesser number of 10- or 12-in. pieces, but the slight difference in labor cost is more than offset by the difference in the cost of material, the 10- and 12-in. lumber being much more expensive, and therefore to be avoided where possible.

Cleats or battens are required on beams and girders to hold together the various pieces of lumber of which the form is built. These battens are 1 in. × 4 in. or 1¼ in. × 4 in. rough. Spacing is usually 2 ft. to 2 ft.-6 in. center to center. The lumber list for each member should include these miscellaneous items:

Number of feet of batten material required; number of feet of ledger; number of feet of key lumber, and skewback footage. This helps to insure that every item has been given attention.

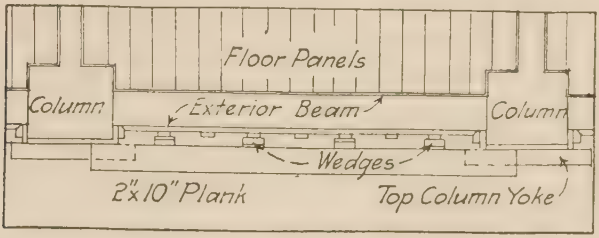


Fig. 169.

Exterior beams present an additional problem in bracing, as there is no panel and spreader construction on the building line side to keep the form from opening up wider at the top than it is at the bottom. Two methods are generally used to keep the outside of the beam in line. The first is to put a large number of shores under the beam and put spur braces at an angle of 45 degrees between the Tee head on the shore and the side of the beam. While this is the method most commonly used it has several disadvantages. It requires an unnecessarily large number of shores, and it also requires that the head of the shore be two or three times as long as would otherwise be necessary. There is the labor of cutting and nailing in place a large number of the small spur braces, and the final disadvantage that the form will often spring anyway, and there is then no practicable method of getting it back into line. This has caused the adoption

of the method shown in Fig. 169. A 2- × 8-in. or 2- × 10-in. plank, depending on the span between columns, is spiked flat to the top yoke of the columns, and wedges are then driven between the plank and the beam side as often as may be necessary to hold the beam side in line. Then, if the form does show signs of working out of line because of spring in the plank, it is a simple matter to drive down such wedges as may be necessary to correct the trouble, while the pouring is going on. By this method no more shores are required than for an interior girder, and the Tee head need be only long enough to give the girder a bearing. This makes the shores much easier to handle. As cutting is rarely required, scaffold plank can be used for this bracing to good advantage without extra expense for special material.

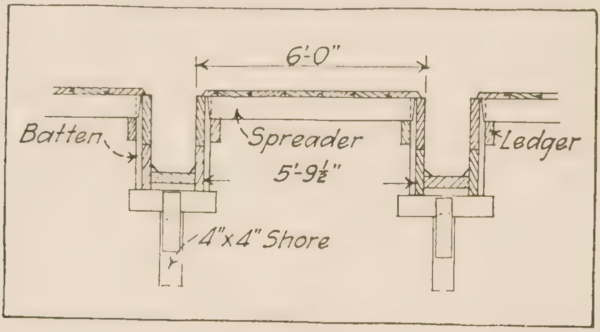


Fig. 170.

The beam forms have a $1\frac{1}{4} \times 4$ or 2- × 4-ledger spiked to the side cleats. This ledger carries the spreaders on which the floor panel is made. Figure 170 gives a cross-section through two beams. The spreaders are 3 × 4 in. rough, sized at the ends to $3\frac{3}{4}$ or $3\frac{1}{2}$ in. This is best done by making a saw cut about 6 in. long at the required length, but before they have been made up into panels. This is just the same as sizing the underside of a floor beam in ordinary brick or frame construction to keep the floor level.

The length of the spreader will be the measurement between the beam forms. As the clear opening between the beams in the drawing is given at 6 ft., the back to back distance between the beveled beam sides would be 5 ft.-$9\frac{1}{2}$ in., which would then be

the length of the spreaders. The ends of the spreaders should be slightly beveled as shown.

The floor panels will have to be patched out each time the columns reduce in diameter. This may be done by simply

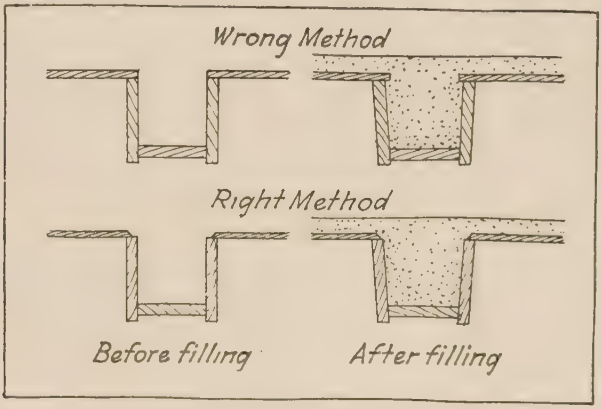


FIG. 171.

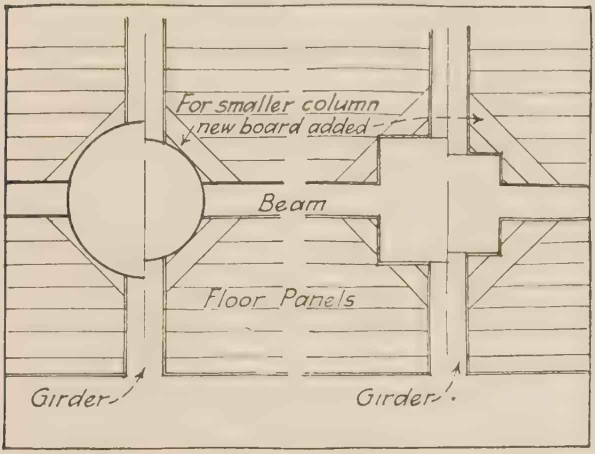


FIG. 172.

adding in a little filler piece each time, which is rarely satisfactory in appearance, or the following method may be used. As shown in Fig. 172, the corners of the panels adjacent to the column are made with the boards running at an angle of 45 degrees. When the column size reduces, the short boards in this corner section are

taken off and new boards substituted. When a circular column is used, the same method is followed, and after the necessary new boards have been put in the panel, a circle of the less required radius is described on the panels and the cut made with a compass saw. This method of building the floor panels and patching them out always leaves a clean, neat corner at the column. When sheet metal column forms are used, the metal form is usually run up first to the underside of the girders, and the space between the girders and beams filled out with four pieces of metal of the proper size curved to the same radius as the column. These pieces usually lap the column form and are bolted to it.

214. Flat Slabs. One of the oldest, simplest, and most common methods of flat slab form construction is shown in Fig. 173. The upper part of the drawing shows the disposition of the panels and the lower part of the drawing the arrangement of 4 × 6's and spreaders.

For the sake of simplicity in this and other drawings, the shores have been omitted. Their spacing under the 4 × 6's is governed largely by the thickness, and consequently the weight of the floor slab, which limits the span of the 4 × 6's.

An interesting system of forms for jobs in which the bays are square and in which the drop panels can be made of the right size, is shown in Fig. 174. These panels are all exactly the same in size and shape and are therefore completely interchangeable. This is a big advantage where time is limited and the job must be pushed. The drop head shown is two-fifths of the length of the bay. As will be seen, three rows of 4 × 6 bents are set up between the lines of columns, the same as in Fig. 173, but between the columns three short bents are used running in the opposite direction. Because of the close spacing of the 3 × 4's or 2 × 4's, which act as battens on the back of the panels, they are usually quite stiff and stand handling well, in addition to being somewhat lighter than the panels used in the first system described.

This system is open to the objection that the board marks do not all run the same way on the ceiling, but, as a rule, because of the rigidity of the panels, a good job of form work is obtained, and when the ceiling is painted, the fact that in some panels the boards

are at right angles to those in the adjoining panels is not specially noticeable or objectionable.

A particularly excellent method of building forms is shown in Fig. 175. Instead of making the panels with longitudinal

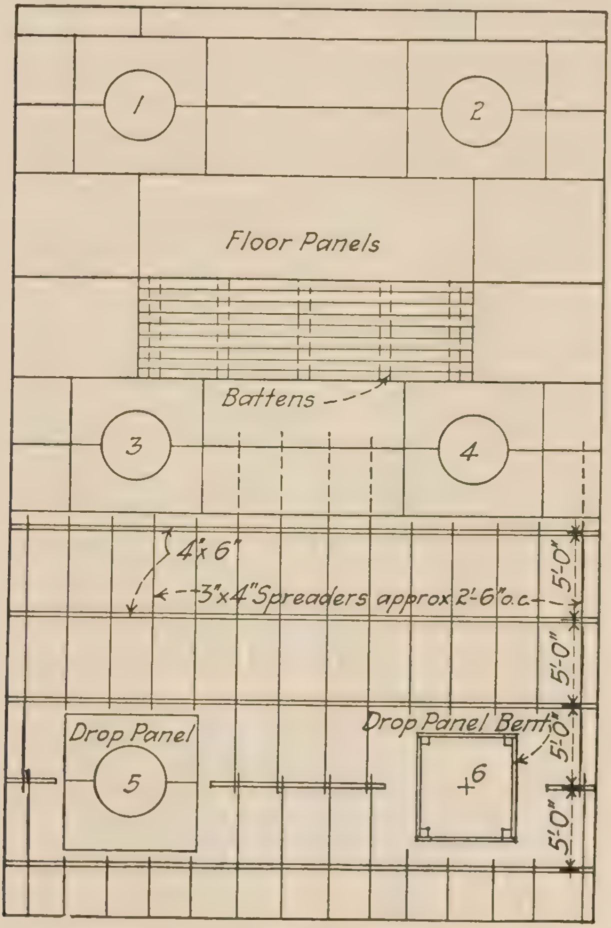


Fig. 173.

spreaders on which the boards are placed crosswise, as shown in Fig. 174, these panels are made up very much like those for beam and girder construction, the boards running the length of the panel, with traverse spreaders on the back spaced according to the load to be carried and the span between the rows of 4 × 6

bents. As the longitudinal joints between the panels come over the center line of the 4 × 6's, accurate spacing of the bents is essential, and is obtained in the following manner, which is

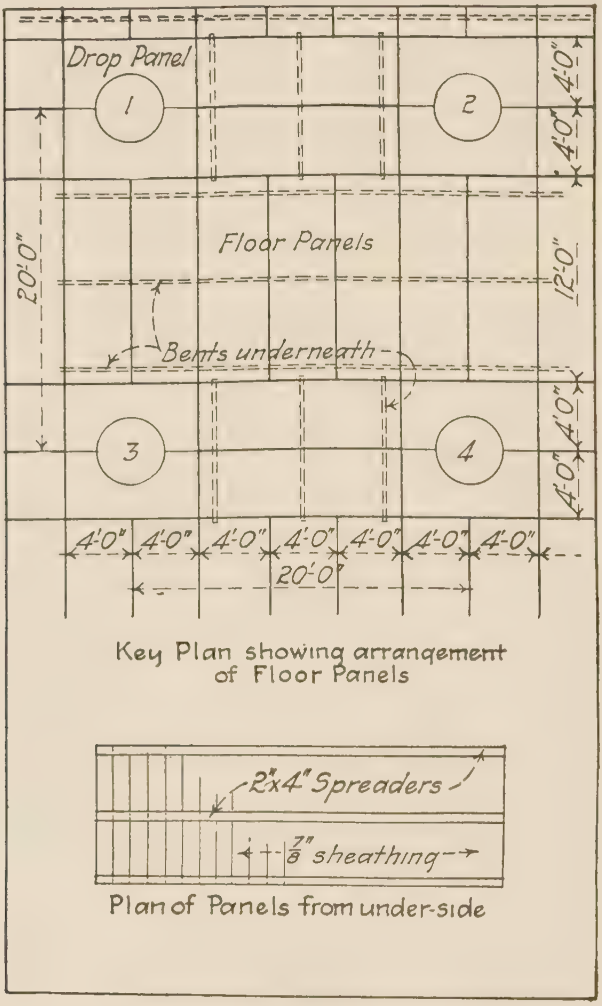


Fig. 174.

also shown later by sketch. A 2 × 6 spacer plank is bolted from one column bent to the next one, at right angles to the line of 4 × 6's. These 2 × 6's are all drilled for the bolts to a standard spacing, so that they automatically furnish a check on the

location of the column bents. Each 2 × 6 carries a set of 1- × 4-in. guide cleats, which project about 4 in. above the top of the plank, and are so located that it is only necessary to drop the 4 × 6's into the space between the guide cleats to ensure their

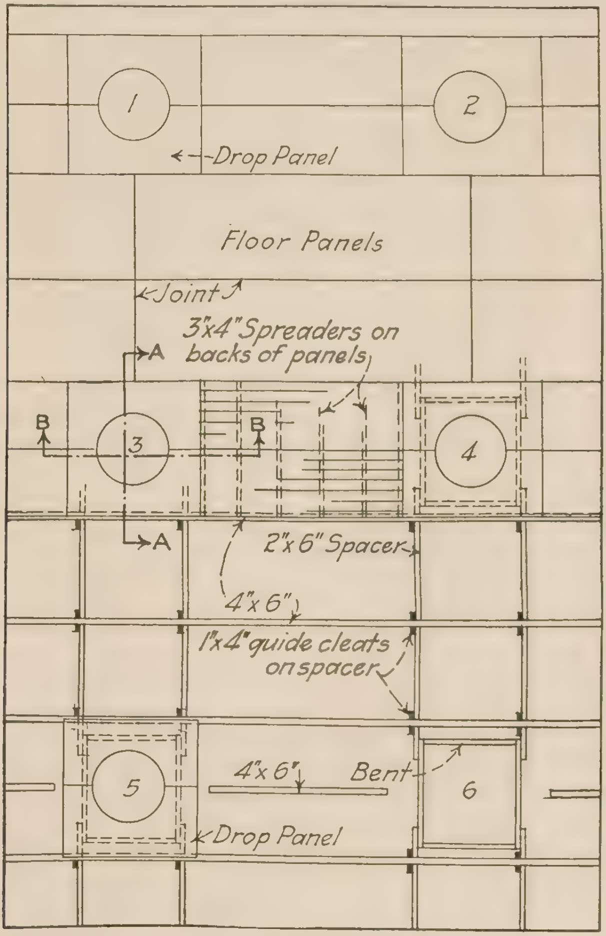


Fig. 175.

being exactly in position to catch the edges of the panels. One 4 × 6 bent is needed between the columns parallel to the rest of the 4 × 6's, to catch the two short panels. This bent can readily be placed after the other bents are up by the use of the staylathing on which the position of the bents has been marked. As the 2 ×

6 spacer plank does not actually carry any of the construction load, it does not have to be bolted up with nut and washer.

The operation of setting up these forms then becomes quite simple. After the first column bent has been located properly with regard to its column center in both directions, it is only

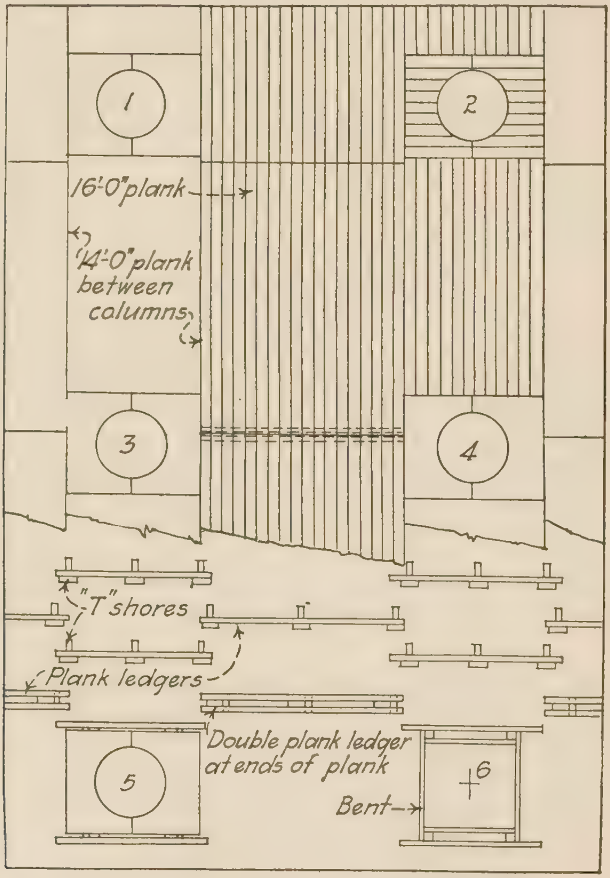


FIG. 176.

necessary to locate the succeeding bent in the same row approximately. The 2 × 6 is lifted into position at the first bent and pinned with a bolt. The second bent is moved backward or forward until the other end of the 2 × 6 can be pinned the same way and the process again repeated. The 4 × 6's are then

dropped into place between the guide cleats and the shores placed under them, after which the panels may be laid down with the assurance that everything will fit as it should. In mill buildings of some types the drop head around the column capital is occasionally eliminated in the design, in order to get a perfectly flat, unbroken ceiling. It then becomes possible to use the 2-in. plank underflooring for form construction, and save a large part of the cost of lumber which would otherwise be wasted. As the planks are used practically loose, there is 100 per cent salvage. The planks are used only once, being left on the floor for use as underflooring after being stripped, and a new set of forms "built" for the next story. The shores, ledgers, etc., of course, are used over again on each floor. Reference to Fig. 176 will show the principal details of the system.

The matter of adequate support for the green concrete in a flat slab floor is one to which a great deal of thought has been given by engineers and contractors. Whatever the system of forms used, this point must be watched very carefully to see that by no chance is the new slab left without proper support, either during the stripping operation, or in the period immediately subsequent to it. Flat slab forms should not be removed earlier than indicated in the schedule below, which is based on summer or warm weather conditions.

Columns—24 hours after slab is concreted.

Depressed heads (drop panels)—36 hours after concreting.

Slab—84 hours after concreting.

Because the construction of flat slab forms is such that it is frequently necessary to remove the shores from a large part of a bay in order to start stripping, it has been found advisable to build in what are known as "permanent" shores. These shores are usually 6 × 6's carrying a "cap" about 12 in. square, which is entirely independent of the adjacent floor panels. This shore is set in the approximate center of the bay at the joint between two panels which are notched out around the "cap." When the panels are stripped, the permanent shore remains in place to afford support to the concrete. The function of the permanent shore cannot be performed by reshoring, for two reasons: first, because of the injurious deflection which will occur if the slab is tem-

porarily left unsupported; second, because in reshoring, the effort to wedge the shore up tightly will often result in putting a heavy pressure on the underside of the slab. The slab is designed and the reinforcement so placed that it is effective only in carrying loads which tend to bend it *downward;* it is entirely unfitted to

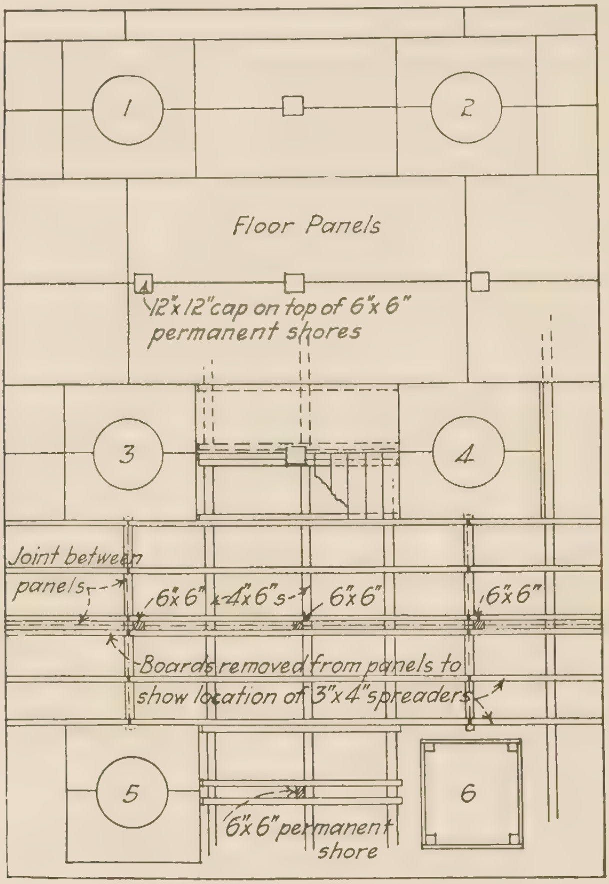


Fig. 177.

resist loads which tend to bend it *upward,* and when it is still green may easily be cracked in this way. The method of reshoring commonly followed involves placing a single 6- × 6-in. post in the center of each band between the columns. This method is open to the same criticism as applied above to the slab, although

there is usually some steel over the bands to resist an upward thrust.

To avoid making the drawing unnecessarily complicated, the permanent shores have been omitted from the center of the bays of the other drawings, but they should always be provided regardless of the system used. If a system of permanent shores is used, as shown in Fig. 177, reshores are needless, but if only the center permanent shore is used, a 6- × 6-in. reshore should be placed under each band mid-way between the columns, in each direction.

Two stories, in addition to the story below the forms, should be left fully reposted at all times, and in the third story below, the

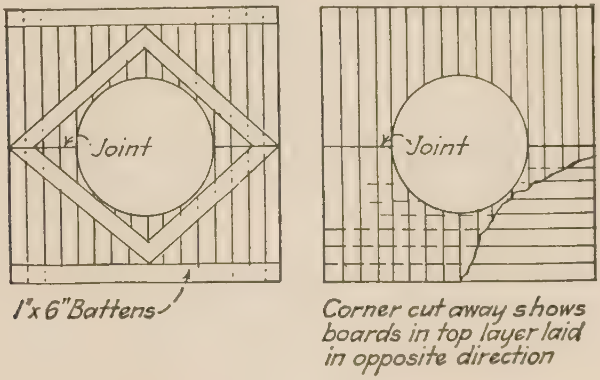


FIG. 178.

shores under the bands may be removed, leaving in place for another week the center shore under the slab.

Because of the nature of flat slab construction it is important that shores be placed one over the other in successive stories, to avoid compelling the floor structure itself to carry the load. Failure to follow this rule may result in bringing a concentrated load to bear on an unsupported section of the slab, with a resulting failure.

The actual sizes of the floor panels, after the general arrangement has been worked out, are dependent upon the size of the depressed section. As the drop panels are always placed before the slab forms, they will be considered first. Figure 178 shows two different methods of building these panels. The type at the

left of the illustration is the more generally used in one form or another, although some carpenter foremen who have used both types claim that the other type is much better. It is a small matter, depending principally upon the individual foreman's way of working. The first type shown is made up of a single thickness of $\frac{7}{8}$ roofers stiffened and held in place by battens or cleats on the back of the panel. The battens as shown in the drawing are 1 × 6's, but the common practice is to use 2 × 4's on

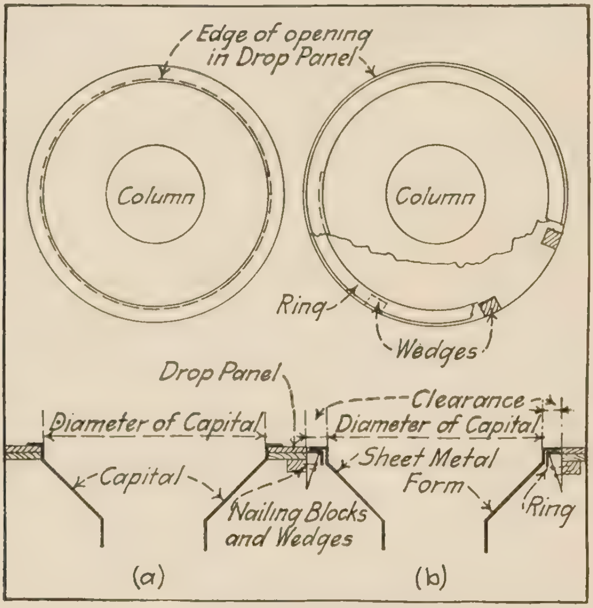


Fig. 179.

edge to make the panel rigid. The other panel is made up of two layers or laminations of roofers laid in opposite directions. This panel has all the advantages of any laminated construction as far as strength is concerned, and will stand more grief than the first type of panel.

When a sheet metal column form is to be used on a job before making up the depressed panels, at least as far as cutting the opening for the column capital is concerned, obtain from the column mold erector the size and details of the opening in the panel required to fit his construction.

Typical details used in two different column mold systems are shown in Fig. 179 for the column capital, where it is connected with the drop panel. In the system marked "*a*" the opening in the drop panel is practically the same diameter as the column capital, which has a lip turned over that rests upon the surface of the panel. This edge of metal leaves a depressed ring in the concrete around the capital, which nearly always requires some pointing to patch up satisfactorily. The arrangement is also open to the objection that the sheet metal capital cannot be taken down until the drop panel has been stripped. The other method, marked "*b*," requires a slightly larger opening in the dropped panel, this being marked "clearance" in the drawing. Nailing blocks and wedges are fastened around the edge of the opening,

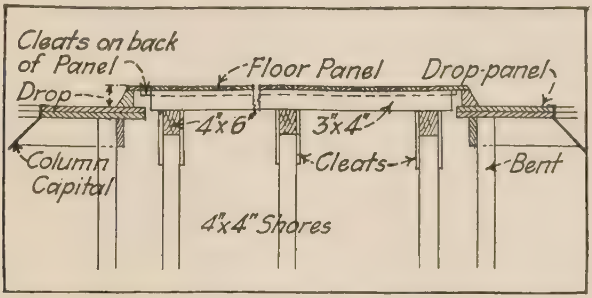


Fig. 180.

and a heavy angle iron ring placed on the blocks to support the light metal of the mold between the blocks. This ring is gauged to come about ⅛ in. below the surface of the drop panel, so that when the capital form is placed, the turnover lip will just come flush with the surface of the drop panel. With this type of construction it is not necessary to wait for the drop panel to be stripped to get the metal mold. By removing the wedges the ring can be dropped, thus freeing the sheet metal form for immediate use elsewhere.

Figure 180 gives two vertical sections through the loose spreader system of forms shown in Fig. 173. These sections show the floor panels made up on 1 × 6 battens or cleats, laid on the loose 3 × 4 spreaders, which are in turn supported by the 4 × 6 ledgers cleated to the top of the shores. A bevel piece is fitted

around the edge of the drop panel and helps to support the extreme ends of the floor panel.

It will readily be seen that where the depressed section around the column capital is sufficiently shallow, it is entirely possible to omit the special bent construction at the columns, and carry the drop panel on the 4 × 6's.

If desired, the system shown in Fig. 175 may be modified by running the panels at right angles to those shown in the plan. The spreaders will then parallel the long axis of the panels, and the boards will run across the panel, similar to the panel shown in Fig. 174. This method has the very decided advantage that the same degree of accuracy in placing the 4 × 6's is not required, except at the ends of the panels.

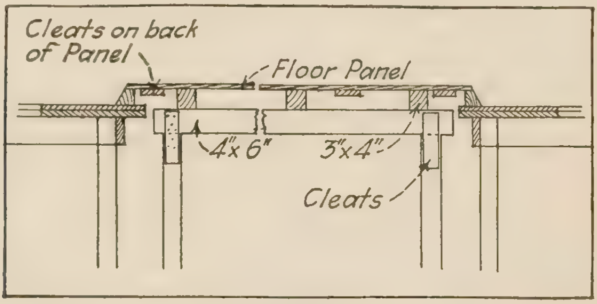


Fig. 180*a*.

The use of loose planks, as shown in Fig. 176, cannot well be extended to columns, curtain walls, parapets, etc. For these and other special purposes, such as stairs, the amount of cutting and fitting required will often render the plank practically valueless for use as underflooring, for which it was bought primarily. For these special uses it is advisable to buy the regular $\frac{7}{8}$-in. sheathing and confine the use of the loose planks to the floor forms alone. The shores can be made up by spiking planks together, but it is frequently just as cheap to buy 4 × 4's, which can afterward be used as standards for walls, etc.

215. Stairs. Stair construction in concrete is not necessarily so complicated as a first view of stair form work in place would seem to indicate it to be. The problem when analyzed simply resolves itself into two or three operations, each easy enough in

itself. Principally, it is required to build one or more landings at locations and elevations shown on the drawings and the placing of a run of steps between these landings and the floors either above or below, as the case may be.

Each run, in brief, has to have so many treads of a certain length to span a fixed horizontal opening, and so many risers of a certain height to span a fixed vertical dimension. A sloping form is needed for the soffit of the stair run and some kind of provision must be made to shape the ends and front of the steps. The whole system, when completed and ready for the concrete, must be adequately supported the same as any other piece of form construction.

That, in short, is a statement of the problem and its solution. The methods employed are just as simple as the problem itself proves to be when separated into its component parts.

The landings will be the first part of the construction to be dealt with, as it is obviously necessary to build them first. They are usually of simple beam and girder construction, sometimes supported by light concrete posts from the floor below and sometimes by recesses cut into the walls of the stair shaft into which the landing beams can be framed. In either case there is nothing complicated about this part of the work and the construction of both columns and beams and girder forms has been fully dealt with in preceding articles.

In Fig. 181 is shown the simplest kind of stair—a short run between two floors. Landings have been omitted in the interest of simplicity and clearness. Where landings are involved the operations are practically the same, except that the forms for the stair run, instead of butting against a beam already concreted, will frame into the beam form at the edge of the landing.

The first thing that will be noted is that the stair opening or hatchway is 8 in. longer than the stair run itself, figuring from face to face of the first and last risers. The exact figure for this excess will vary with different contractors, but 3½ or 4 in. is common. This play room at each end makes it possible to keep the end risers vertically over one another in the different runs regardless of minor variations in the size of the hatchway, which it is usually difficult to hold exactly plumb. The form work is

simpler and a much more workmanlike job of framing is obtained in the completed stairs when viewed from below after the forms have been removed. When this method is used it is not necessary to set the antislip tread for the top step (the floor) when the floor is filled, as the 3½- or 4-in. extension provides the room necessary to set the ordinary non-slip tread.

Where figures are given in this article they will refer only to the stair shown in Fig. 181, which will be taken for an example

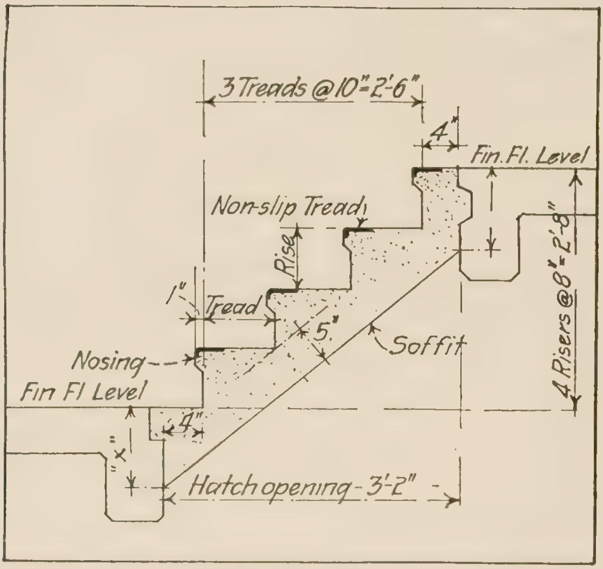


Fig. 181.

throughout. Figures will be used because by so doing the explanations will be made shorter and more easily understandable. The proper figures for rise and tread must be substituted on other jobs for those given here and there will of course be a corresponding variation in the other dimensions.

Referring now to Fig. 181, we find that we have three treads at 10 in. each, with which to bridge the horizontal opening or hatchway. The tread is figured exclusive of the nosing, being the horizontal distance between the faces of two adjoining risers. To this 30 in. must be added the 4 in. at each end for clearance.

The depth of the stair slab is always given on the working drawings. This is the dimension (at right angles to the soffit of

the stair) between the soffit and what is marked as the "slope line" in Fig. 182. For example, it is given as 5 in.

After the landing forms have been built, or if there are no landings as in this case, before any of the other form work can go ahead the panel for the soffit of the stair must be placed. The panel is usually made up of ⅞-in. boards with the boards running parallel to the stair run, mounted on 3- × 4-in. or 4- × 4-in.

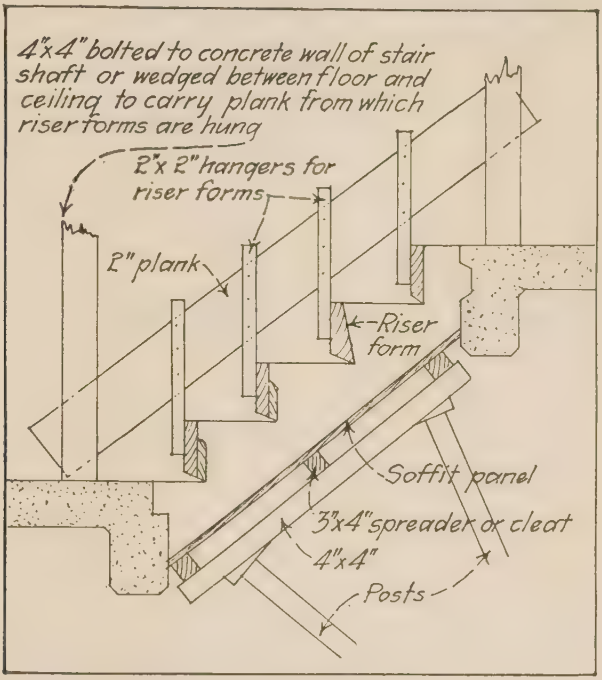


FIG. 182.

spreaders or cleats, spaced about 3 ft. apart. These are in turn upheld by two or more longitudinal stringers under which are the usual posts. Before the panel can be made up and placed, it is necessary to know its length and it must be determined at what elevation it must be placed with reference to the beams against which it rests at each end. As a matter of convenience, a small detail of this part of the work can be laid out on large scale at any convenient point and the necessary dimensions scaled off, close enough for all practical purposes.

Such a sketch is given in Fig. 183. Riser and tread are laid out and connected by a diagonal line such as is marked "line of slope" for convenience in identifying it. This line is of course parallel to the soffit of the stair, which in this case is 5 in. from it (the thickness of the slab). The line for the soffit is next drawn as shown, and the lower end prolonged far enough to intersect a line drawn from a point 4 in. in front of the face of the riser, to indicate the location of the face of the beam.

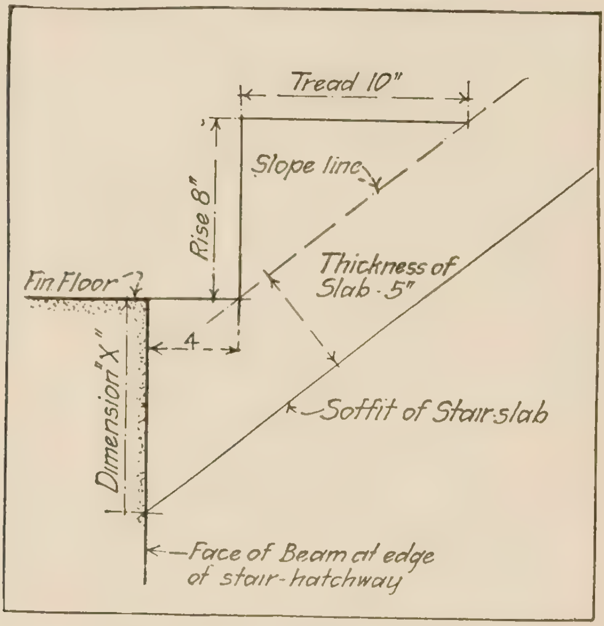


Fig. 183.

The vertical dimension "X" is now measured off from the sketch. The operation is repeated at the other end and the distance between the two points measured, after which the panel is cut and placed.

For the sides or stringers, 2-in. planks dressed two sides to $1\frac{3}{4}$ in. are used. The length of the stair run between girders on the line of the soffit is marked on the plank and each end cut off at the proper angle to make it fit between the beams. This angle is most easily obtained by laying out at the bottom of the plank and starting at one end, one riser and tread, using the lower edge of

the plank as the diagonal. Reference to Fig. 184 will help make this clear, the tentative riser and tread being laid out at the left end of the plank and indicated by dotted lines.

After the end of the plank has been cut off at the proper angle the dimension "X," previously obtained, is marked on the cut edge as shown. The 4-in. clearance space is marked off parallel to the tread. Alternately, then, afterward riser and tread are marked off until the end of the run is reached. By measuring in from the lower edge of the plank 5 in. for the thickness of the slab, the "slope line" may be established, and the correctness of the work checked, as the interior angle between riser and tread should be on this line in every case.

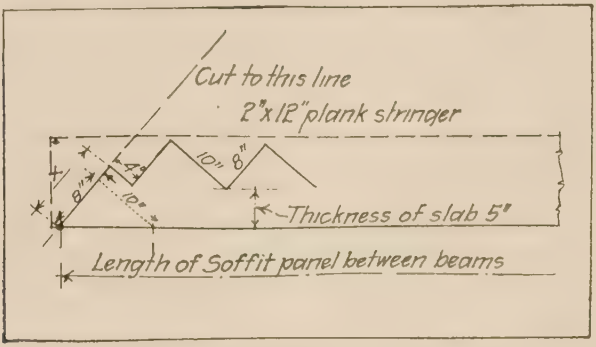


Fig. 184.

It will be found advisable to lay the work out quite lightly at first, as the error resulting from the wide lines drawn with a carpenter's pencil will mount up and the last tread will be shy quite a little. It will then be necessary to go back and lay out the lines with greater accuracy. In this case, light lines which can be worked over easily will be found an advantage.

Different types of risers are shown in profile in Fig. 182. In most cases the riser is vertical with an overhanging nosing, but in some cases the riser itself is battered or sloped outward. In any case the object is the same—to get a wider tread than would otherwise be possible and to improve the appearance of the finished stairs. In laying out such a stair it must be borne in mind that the form work riser will not be the same as the concreted stair riser, unless there is no molding. The form work

riser will naturally be at the edge of the nosing, and the undercut will have to be obtained by means of a panel nailed to the face of the form work riser, as shown in the drawing. This is important in cutting the stringers to receive the risers.

It is customary to bevel the lower edge of the riser form to an edge as shown in Fig. 182 so that in finishing the stairs the mason may reach in with trowel close to the riser.

Where the stair adjoins a concrete wall and no outside stringer is possible, some other means must be found to support the outside ends of the riser forms.

A method probably as simple as any is shown in Fig. 182. A 4- × 4-in. post is wedged up in position on each floor as near to the stair as possible and a heavy plank nailed between the two posts at an angle approximately parallel to the stair run. If the wall is concrete and some of the bolt holes are still available, a 4 × 4 may be bolted to the wall at each end of the stair run. The plank is then nailed to the 4 × 4's. 2- × 2-in. hanger pieces are nailed to the outside of each riser and to the plank to hold up the outside end of the riser forms. If necessary, diagonal spurs from the plank may be used in addition to keep the hanger pieces from swinging out under the pressure of the concrete.

Top forms for the treads are not necessary as the concrete used for stairs is of very stiff consistency and will not overflow the riser forms to any extent. Because the posts under stairs are at an angle from the vertical, they are subjected to heavy bending stresses and should be well staylathed and braced.

Two sets of stair forms are generally used in order that the concrete may get at least a week's set before the forms are removed. Reshores should be immediately put in and kept in place until the stairs are at least 28 days old.

APPENDIX A

NOTATION

The principal symbols used in the previous discussions have been collected here for the convenience of reference.

a = side of column parallel to principal beam, or over-all depth of any member.

α = angle between inclined web bars and longitudinal bars.

A = total net area of column, exclusive of fire-proofing.

$A_c = A(1 - p)$ = net area of concrete core of column (core area minus reinforcement)

A_s = effective cross-sectional area of metal reinforcement in tension in beams.

A'_s = effective cross-sectional area of metal reinforcement in compression in beams or columns.

A_v = total area of web reinforcement in tension within a distance of $s(s_1, s_2, s_3,$ etc.) or the total area of all bars bent up in any one plane.

b = width of rectangular beam or width of flange of T-beam.

b' = width of stem of T-beam.

c = projection of footing from face of column.

C = total compressive stress in concrete.

C' = total compressive stress in reinforcement.

d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement.

d' = depth from compression surface of beam or slab to center of compression reinforcement.

E_c = modulus of elasticity of concrete in compression.

E_s = modulus of elasticity of steel in tension = 30,000,000 lb. per sq. in.

f_c = compressive unit stress in extreme fiber of concrete.

f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6- × 12-in. or 8- × 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete of the American Society for Testing Materials.

f_s = tensile unit stress in longitudinal reinforcement.

f'_s = compressive unit stress in longitudinal reinforcement.

f_v = tensile unit stress in web reinforcement.

h = unsupported length of column.

I = moment of inertia of a section about the neutral axis for bending.

j = ratio of lever arm of resisting couple to depth d.

$jd = d - z$ = arm of resisting couple.

k = ratio of depth of neutral axis to depth d.

l = span length of beam or slab (generally distance from center to center of supports).

M = bending moment or moment of resistance in general.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete.

Σ_o = sum of perimeters of bars in one set.

p = ratio of effective area of tension reinforcement to effective area of concrete in beams $= \frac{A_s}{bd}$.

p' = ratio of effective area of compression reinforcement to effective area of concrete in beams.

p_o = ratio of total effective reinforcement in member subject to compression to effective concrete section.

P = total safe axial load on column whose $\frac{h}{R}$ is less than 40.

P' = total safe axial load on long column.

R = least radius of gyration of a section.

s = spacing of web members, measured at the plane of the lower reinforcement and in the direction of the longitudinal axis of the beam.

t = thickness of flange of T-beam.

T = total tensile stress in longitudinal reinforcement.

u = bond stress per unit of area of surface of bar.

v = shearing unit stress.

V = total shear.

V_c = total shear that can be resisted by the concrete.

V' = total shear on any section after deducting that carried by the concrete, *i.e.*, $V' = V - V_c$.

w = uniformly distributed load per unit of length of beam or slab.

x = length of bar added for anchorage, including the hook, if any.

z = depth from compression surface of beam or slab to resultant of compressive stresses.

APPENDIX B

SUMMARY OF WORKING STRESSES RECOMMENDED BY THE JOINT COMMITTEE

1. Direct Compression
   - (*a*) Bearing on plain concrete piers and pedestals .... $.25\,f'_c$
   - (*b*) Concentric compression, column with longitudinal reinforcement only .... $.20\,f'_c$
   - (*c*) Concentric compression, column with longitudinal reinforcement and spirals .... $300 + (.10 + 4p)f'_c$
2. Compression in Extreme Fiber
   - (*a*) Extreme fiber stress in flexure .... $.40\,f'_c$
   - (*b*) Extreme fiber stress in flexure adjacent to supports of continuous beams .... $.45\,f'_c$
3. Shear
   - (*a*) Beams with no web reinforcement
     1. Longitudinal bars anchored .... $.03\,f'_c$
     2. Longitudinal bars not anchored .... $.02\,f'_c$
   - (*b*) Beams with web reinforcement
     1. Longitudinal bars anchored .... $.12\,f'_c$
     2. Longitudinal bars not anchored .... $.06\,f'_c$
4. Reinforcement

   Tensile or compressive unit stress not to exceed
   - (*a*) Structural steel grade bars .... 16,000 lb.
   - (*b*) Intermediate grade bars .... 18,000 lb.
   - (*c*) Hard grade bars .... 18,000 lb.
5. Bond
   - (*a*) Between concrete and plain reinforcing bars .... $.04\,f'_c$
   - (*b*) Between concrete and approved deformed reinforcing bars .... $.05\,f'_c$
6. Modulus of Elasticity
   - (*a*) For concrete whose ultimate unit compressive strength is between 1500 and 2200 lb .... $\frac{1}{15}\,E_s$
   - (*b*) For concrete whose ultimate unit compressive strength is between 2200 and 2900 lb .... $\frac{1}{12}\,E_s$
   - (*c*) For concrete whose ultimate unit compressive strength is greater than 2900 lb .... $\frac{1}{10}\,E_s$

APPENDIX C

NEW YORK CITY BUILDING CODE REQUIREMENTS FOR FLAT SLAB CONSTRUCTION

Rule 1. *Application.* The rules governing the design of reinforced concrete flat slabs shall apply to such floors and roofs, consisting of three or more rows of slabs, without beams or girders, supported on columns, the construction being continuous over the columns and forming with them a monolithic structure.

Rule 2. *Compliance with Building Code.* In the design of reinforced concrete flat slabs, the provisions of article 16 of the building code shall govern with respect to such matters as are specified therein.

Rule 3. *Assumptions.* In calculations for the strength of reinforced concrete flat slabs, the following assumptions shall be made:

(*a*) A plane section before bending remains plane after bending;

(*b*) The modulus of elasticity of concrete in compression within the allowable working stresses is constant;

(*c*) The adhesion between concrete and reinforcement is perfect;

(*d*) The tensile strength of concrete is nil;

(*e*) Initial stress in the reinforcement due to contraction or expansion in the concrete is negligible.

Rule 4. *Stresses.* (*a*) The allowable unit shear on reinforced concrete flat slabs on the *bd* section around the perimeter of the column capital shall not exceed 120 lb. per sq. in.; and the allowable unit shearing stress on the *bjd* section around the perimeter of the drop shall not exceed 60 lb. per sq. in., provided the reinforcement is so arranged or anchored that the stress may be fully developed for both positive and negative moments.

(*b*) The extreme fiber stress to be used in concrete in compression at the column head section shall not exceed 750 lb. per sq. in.

Rule 5. *Columns.* For columns supporting reinforced concrete flat slabs, the least dimension of any column shall not be less than one-fifteenth of the average span of any slab supported by the columns; but in no case shall such least dimension of any interior column supporting a floor or roof be less than 16 in. when round nor 14 in. when square; nor shall the least dimension of any exterior column be less than 14 in.

Rule 6. *Column Capital.* Every reinforced concrete column supporting a flat slab shall be provided with a capital whose diameter is not less than 0.225 of the average span of any slabs supported by it. Such diameter shall be measured where the vertical thickness of the capital is at least 1½ in.,

and shall be the diameter of the inscribed circle in that horizontal plane. The slope of the capital considered effective below the point where its diameter is measured shall nowhere make an angle with the vertical of more than 45 degrees. In case a cap of less dimensions than hereinafter described as a drop is placed above the column capital, the part of this cap enclosed within the lines of the column capital extended upward to the bottom of the slab or drop at the slope of 45 degrees may be considered as part of the column capital in determining the diameter for design purposes.

Rule 7. *Drop.* When a reinforced concrete flat slab is thicker in that portion adjacent to, or surrounding, the column, the thickened portion shall be known as a drop. The width of such drop when used shall be determined by the shearing stress in the slab around the perimeter of the drop, but in no case shall the width be less than .33 of the average span of any slabs of which it forms a part. In computing the thickness of drop required by the negative moment on the column-head section, the width of the drop only shall be considered as effective in resisting the compressive stress, but in no case shall the thickness of such drops be less than .33 of the thickness of the slab. Where drops are used over interior columns, corresponding drops shall be employed over exterior columns and shall extend to the one-sixth point of the panel from the center of the column.

Rule 8. *Slab Thickness.* The thickness of a reinforced concrete flat slab shall not be less than that derived by the formula $t = .024\, l \sqrt{w} + 1\frac{1}{2}$ for slabs without drops, and $t = .02\, l \sqrt{w} + 1$ for slabs with drops, in which t is the thickness of the slab in inches, l is the average span of the slab in feet, and w is the total live and dead load in pounds per square foot; but in no case shall this thickness be less than one-thirty-second of the average span of the slab for floors, not less than one-fortieth of the average span of the slab for roofs, nor less than 6 in. for floors nor less than 5 in. for roofs.

Rule 9. *Reinforcement.* (*a*) In the calculation of moments at any section, all the reinforcing bars which cross that section may be used, provided such bars extend far enough on each side of such section to develop the full amount of the stress at that section. The effective area of the reinforcement at any moment section shall be the sectional area of the bars crossing such section multiplied by the sine of the angle of such bars with the plane of the section. The distribution of the reinforcement of the several bands shall be arranged to provide fully for the intermediate moments at any section.

(*b*) Splices in bars may be made wherever convenient but preferably at points of minimum stress. The length of any splice shall be not less than 80 bar diameters and in no case less than 2 ft. The splicing of adjacent bars shall be avoided as far as possible. Slab bars which are lapped over the column, the sectional area of both being included in the calculation for negative moment, shall extend to the lines of inflection beyond the column center.

(*c*) When the reinforcement is arranged in bands, at least 50 per cent of the bars in any band shall be of a length not less than the distance center to center of columns measured rectangularly and diagonally; no bars used as positive reinforcement shall be of a length less than one-half the panel length plus 40 bar diameters for cross bands, or less than seven-tenths of the panel length plus 40 bar diameters for diagonal bands, and no bars used as negative reinforcement shall be of a length less than one-half the panel length. All reinforcement framing perpendicular to the wall in exterior panels shall extend to the outer edge of the panel and shall be hooked or otherwise anchored.

(*d*) Adequate means shall be provided for properly maintaining all slab reinforcement in the position as assumed by the computations.

Rule 10. *Line of Inflection.* In the design of reinforced concrete flat slab construction, for the purpose of making calculations of the bending moments at sections other than defined in these rules, the line of inflection shall be considered as being located one-quarter of the distance center to center of columns, rectangularly and diagonally, from center of columns for panels without drops, and three-tenths of such distance for panels with drops.

Rule 11. *Moment Sections.* For the purpose of design of reinforced concrete flat slabs, that portion of the section across the panel, along a line midway between columns, which lies within the middle two-quarters of the width of the panel, shall be known as the inner section, and those portions of the section in the two outer quarters of the width of the panel shall be known as the outer sections. Of the section which follows a panel edge from column to column and which includes the quarter perimeters of the edges of the column capitals, that portion within the middle two-quarters of the panel width shall be known as the mid-section, and the two remaining portions, each having a projected width equal to one-quarter of the panel width, shall be known as the column-head sections.

Rule 12. *Bending Moments.* In the design, the following provisions with respect to bending moments shall be observed. In the moment expressions used:

W is the total dead and live load on the panel under consideration, including the weight of drop, whether a square, rectangle, or parallelogram;

W_1 is the total live load on the panel under consideration;

l is the length of side of a square panel center to center of columns; or the average span of a rectangular panel which is the mean length of the two sides;

n is the ratio of the greater to the less dimension of the panel;

h is the unsupported length of a column in inches; measured from top of slab to base of capital;

I is the moment of inertia of the reinforced concrete column section.

A. Interior Square Panels. The numerical sum of the positive and negative moments shall be not less than $\frac{1}{7}$ Wl. A variation of plus or minus 5 per cent shall be permitted in the expression for the moment on any section,

but in no case shall the sum of the negative moments be less than 66 per cent of the total moment, nor the sum of the positive moments be less than 34 per cent of the total moment for slabs with drops; nor shall the sum of the negative moments be less than 60 per cent of the total moment, nor the sum of the positive moments be less than 40 per cent of the total moment for slabs without drops.

1. *In two-way systems*, for slabs with drops, the negative moment resisted on two column-head sections shall be $-\frac{1}{32}\ Wl$; the negative moment on the mid-section shall be $-\frac{1}{133}\ Wl$; the positive moment on the two outer sections shall be $+\frac{1}{80}\ Wl$; and the positive moment on the inner section shall be $+\frac{1}{133}\ Wl$; and for slabs without drops, the negative moment on the mid-section shall be $-\frac{1}{133}\ Wl$; the positive moment on the two outer sections shall be $+\frac{1}{63}\ Wl$; and the positive moment on the inner section shall be $+\frac{1}{133}\ Wl$.

2. *In four-way systems*, the negative moments shall be as specified for two-way systems; the positive moment on the two outer sections shall be $+\frac{1}{40}\ Wl$, and the positive moment on the inner section shall be $+\frac{1}{100}\ Wl$ for slabs with drops; and the positive moment on the two outer sections shall be $+\frac{1}{74}\ Wl$, and the positive moment on the inner section shall be $+\frac{1}{100}\ Wl$, for slabs without drops.

3. *In three-way systems*, the negative moment on the column head and mid-sections and the positive moment on the two outer sections, shall be as specified for four-way systems. In the expression for the bending moments on the various sections, the length l shall be assumed as the distance center to center of columns, and the load W as the load on the parallelogram panel.

B. Interior Rectangular Panels. 1. When the ratio n does not exceed 1.1, all computations shall be based on a square panel of a length equal to the average span, and the reinforcement shall be equally distributed in the short and long directions according to the bending moment coefficients specified for interior square panels.

2. When the ratio n lies between 1.1 and 1.33, the bending moment coefficients specified for interior square panels shall be applied in the following manner:

(*a*) *In two-way systems*, the negative moments on the two column-head sections and the mid-section and the positive moments on the two outer sections and the inner section at right angles to the long direction shall be determined as for a square panel of a length equal to the greater dimension of the rectangular panel; and the corresponding moments on the sections at right angles to the short direction shall be determined as for a square panel of a length equal to the lesser dimension of the rectangular panel. In no case shall the amount of reinforcement in the short direction be less than two-thirds of that in the long direction. The load W shall be taken as the load on the rectangular panel under consideration.

(*b*) *In four-way systems*, for the rectangular bands, the negative moment on the column-head sections and the positive moment on the outer sections shall be determined in the same manner as indicated for *two-way systems.*

For the diagonal bands, the negative moments on the column-head and the mid-sections and the positive moment on the inner section shall be determined as for a square panel of a length equal to the average span of the rectangle. The load W shall be taken as the load on the rectangular panel under consideration.

(*c*) In *three-way systems*, the negative and positive moments on the bands running parallel to the long direction shall be determined as for a square whose side is equal to the greater dimension; and the moments on the bands running parallel to the short direction shall be determined as for a square whose side is equal to the lesser dimension. The load W shall be taken as the load on the parallelogram panel under consideration.

C. Exterior Panels. The negative moments at the first interior row of columns and the positive moments at the center of the exterior panels on moment sections parallel to the wall shall be increased 20 per cent over those specified above for interior panels. The negative moment on moment sections at the wall and parallel thereto shall be determined by the conditions of restraint, but the negative moment on the mid-section shall never be considered less than 50 per cent and the negative moment on the column-head section never less than 80 per cent of the corresponding moments at the first interior row of columns.

D. Interior columns shall be designed for the bending moment developed by unequally loaded panels, eccentric loading or uneven spacing of columns. The bending moment resulting from unequally loaded panels shall be considered as $\frac{1}{40}\ W_1l$, and shall be resisted by the columns immediately above and below the floor line under consideration in direct proportion to the values of their ratios of $\frac{I}{h}$.

E. Wall columns shall be designed to resist bending in the same manner as interior columns, except that W shall be substituted for W_1 in the formula for the moment. The moment so computed may be reduced by the counter moment of the weight of the structure which projects beyond the center line of the wall columns.

F. Roofing columns shall be designed to resist the total moment resulting from unequally loaded panels, as expressed by the formula in paragraphs *D* and *E* of this rule.

Rule 13. *Walls and Openings.* In the design and construction of reinforced concrete flat slabs, additional slab thickness, girders, or beams shall be provided to carry any walls or concentrated loads in addition to the specified uniform live and dead loads. Such girders or beams shall be assumed to carry 20 per cent of the total live and dead panel load in addition to the wall load. Beams shall also be provided in case openings in the floor reduce the working strength of the slab below the prescribed carrying capacity.

Rule 14. *Special Panels.* For structures having a width of less than three rows of slabs, or in which exterior drops, capitals, or columns are omitted, or in which irregular or special panels are used, and for which the rules relating to the design of reinforced flat slabs do not directly apply, the computations in the analysis of the design of such panels shall, when so required, be filed with the superintendent of buildings.

APPENDIX D

STANDARD METHODS OF TESTING CONCRETE AGGREGATE AS PRESCRIBED BY THE AMERICAN SOCIETY FOR TESTING MATERIALS

Test for Unit Weight of Aggregate for Concrete

1. The unit weight of fine, coarse, or mixed aggregates for concrete shall be determined by the following method:

2. (*a*) The apparatus required consists of a cylindrical metal measure, a tamping rod, and a scale or balance, sensitive to 0.5 per cent of the weight of the sample to be weighed.

(*b*) *Measures.* The measure shall be of metal, preferably machined to accurate dimensions on the inside, cylindrical in form, water-tight, and of sufficient rigidity to retain its form under rough usage, with top and bottom true and even, and preferably provided with handles.

The measure shall be of $\frac{1}{10}$-, $\frac{1}{2}$-, or 1-cu. ft. capacity, depending on the maximum diameter of the coarsest particles in the aggregate, and shall be of the following dimensions:

Capacity, cu. ft.	Inside diameter, in.	Inside height, in.	Minimum thickness of metal, U. S. gauge	Diameter of largest particles of aggregate, in.
$\frac{1}{10}$	6.00	6.10	No. 11	Under $\frac{1}{2}$
$\frac{1}{2}$	10.00	11.00	No. 8	Under $1\frac{1}{2}$
1	14.00	11.23	No. 5	Over $1\frac{1}{2}$

(*c*) *Tamping Rod.* The tamping rod shall be a straight metal rod $\frac{3}{4}$ in. in diameter and 18 in. long with one end tapered for a distance of 1 in. to a blunt bullet-shaped point.

3. The measure shall be calibrated by accurately determining the weight of water at 16.7 deg. Centigrade (62 deg. Fahrenheit) required to fill it. The factor for any unit shall be obtained by dividing the unit weight of water at 16.7 deg. Centigrade (62 deg. Fahrenheit)[1] by the weight of water at 16.7 deg. Centigrade (62 deg. Fahrenheit) required to fill the measure.

4. The sample of aggregate shall be room dry and thoroughly mixed.

5. (*a*) The measure shall be filled one-third full and the top leveled off with the fingers. The mass shall be tamped with the pointed end of the

[1] The unit weight of water at 16.7 deg. Centigrade (62 deg. Fahrenheit) is 62.355 lb. per cu. ft.

tamping rod twenty-five times, evenly distributed over the surface. The measure shall be filled two-thirds full and again tamped twenty-five times as before. The measure shall then be filled to overflowing, tamped twenty-five times, and the surplus aggregate struck off, using the tamping rod as a straight edge.

In tamping the first layer the rod should not be permitted forcibly to strike the bottom of the measure. In tamping the second and final layers, only enough force to cause the tamping rod to penetrate the last layer of aggregate placed in the measure should be used. No effort should be made to fill holes left by the rod when the aggregate is damp.

(*b*) The net weight of the aggregate in the measure shall be determined. The unit weight of the aggregate shall then be obtained by multiplying the net weight of the aggregate by the factor found as described in Section 3.

6. Results with the same sample should check within 1 per cent.

Test for Impurities in Sands for Concrete

1. The test herein specified is an approximate test for the presence of injurious organic compounds in natural sands for cement mortar or concrete. The principal value of the test is in furnishing a warning that further tests of the sands are necessary before they be used in concrete. Sands which produce a color in the sodium hydroxide solution darker than the standard color should be subjected to strength tests in mortar or concrete before use.

2. (*a*) A representative test sample of sand of about 1 lb. shall be obtained by quartering or by the use of a sampler.

(*b*) A 12-oz. graduated glass prescription bottle shall be filled to the $4\frac{1}{2}$-oz. mark with the sand to be tested.

(*c*) A 3 per cent solution of sodium hydroxide (NaOH) in water shall be added until the volume of sand and liquid after shaking gives a total value of 7 liquid ounces.

(*d*) The bottle shall be stoppered and shaken thoroughly and then allowed to stand for 24 hours.

(*e*) A standard color solution shall be prepared by adding 2.5 c.c. of a 2 per cent solution of tannic acid in 10 per cent alcohol to 22.5 c.c. of a 3 per cent sodium hydroxide solution. This shall be placed in a 12-oz. prescription bottle, stoppered, and allowed to stand for 24 hours, and then 25 c.c. of water added.

(*f*) The color of the clear liquid above the sand shall be compared with the standard color solution prepared as in paragraph (*e*) or with a glass of color similar to the standard solution.

3. Solutions darker in color than the standard color have a "color value" higher than 250 parts per million in terms of tannic acid.

Test for Sieve Analysis of Aggregates for Concrete

1. A representative test sample of the aggregate shall be selected by quartering or by use of a sampler, which after drying will give not less than the following:

(*a*) Fine aggregate, 500 g.

(*b*) Coarse aggregate, or a mixture of fine and coarse aggregates, weight in grams, 3000 times size of largest sieve required, measured in inches.

TABLE I

Sieve number[2] or size in inches	Sieve opening		Wire diameter		Tolerance, per cent			
	Mm.	In.	Mm.	In.	Average opening	Wire diameter		Maximum opening
						Under	Over	
No. 100	0.149	0.0059	0.102	0.0040	6	15	35	40
No. 50	0.297	0.0117	0.188	0.0074	6	15	35	40
No. 30	0.59	0.0232	0.33	0.0130	5	15	30	25
No. 16	1.19	0.0469	0.54	0.0213	3	15	30	10
No. 8	2.38	0.0937	0.84	0.0331	3	15	30	10
No. 4	4.76	0.187	1.27	0.050	3	15	30	10
3/8 in.	9.5	0.375	2.33	0.092	3	10	10	10
3/4 in.	19.0	0.75	3.42	0.135	3	10	10	10
1 in.	25.4	1.00	4.12	0.162	3	10	10	10
1½ in.	38.0	1.50	4.50	0.177	3	10	10	10
2 in.	50.8	2.00	4.88	0.192	3	10	10	10
3 in.	76.0	3.00	6.3	0.25	3	10	10	10

2. The sample shall be dried at not over 110 deg. Centigrade (230 deg. Fahrenheit) to constant weight.

3. (*a*) The sieves shall be of square-mesh wire cloth and shall be mounted on substantial frames constructed in a manner that will prevent loss of material during sifting.

(*b*) The size of wire and sieve openings shall be as given in Table I.

4. (*a*) The sample shall be separated into a series of sizes by means of the sieves specified in Section 3. Sifting shall be continued until not more than 1 per cent by weight of the sample passes any sieve during 1 min.

(*b*) Each size shall be weighed on a balance or scale which is sensitive to 1/1000 of the weight of the test sample.

(*c*) The percentage by weight of the total sample which is finer *than each* of the sieves shall be computed.

5. (*a*) The percentages in sieve analysis shall be reported to the nearest whole number.

(*b*) If more than 15 per cent of a fine aggregate is coarser than the No. 4 sieve, or more than 15 per cent of a coarse aggregate is finer than the No. 4 sieve, the sieve analysis of the portions finer and coarser than this sieve shall be reported separately.

[2] The requirements for sieves No. 100 to No. 4 conform to the requirements of the U. S. Standard Sieve Series as given in U. S. Bureau of Standards *Letter Circular*, 74. The liberal tolerances will permit the use of certain sieves which do not exactly correspond to the numbers given in the table.

Test for Quantity of Clay and Silt in Gravel for Highway Construction

1. This method of test covers the determination of the total quantity of silt, loam, clay, etc., in sand and other fine aggregates.[3]

2. The pan or vessel to be used in the determination shall be approximately 9 in. (230 mm.) in diameter and not less than 4 in. (102 mm.) in depth.

3. The sample must contain sufficient moisture to prevent segregation and shall be thoroughly mixed. A representative portion of the sample sufficient to yield approximately 500 g. of dried material shall then be dried to a constant weight at a temperature not exceeding 110 deg. Centigrade (230 deg. Fahrenheit).

4. The dried material shall be placed in the pan and sufficient water added to cover the sample (about 225 c.c.). The contents of the pan shall be agitated vigorously for 15 sec., and then shall be allowed to settle for 15 sec., after which the water shall be poured off, care being taken not to pour off any sand. This operation shall be repeated until the wash water is clear. As a precaution, the wash water shall be poured through a 200-mesh sieve and any material retained thereon returned to the washed sample. The washed sand shall be dried to a constant weight at a temperature not exceeding 110 deg. Centigrade (230 deg. Fahrenheit) and weighed.

5. The results shall be calculated from the formula:

Percentage of silt, loam, clay, etc. =

$$\frac{\text{original dry weight} - \text{weight after washing}}{\text{original dry weight}} \times 100.$$

6. When check determinations are desired, the wash water shall be evaporated to dryness, the residue weighed, and the percentage calculated from the formula:

$$\text{Percentage of silt, loam, clay, etc.} = \frac{\text{weight of residue}}{\text{original dry weight}} \times 100.$$

[3] This determination of the percentage of silt, loam, clay, etc., will include all water-soluble material present, the percentage of which may be determined separately if desired.

APPENDIX E

PROPORTIONS FOR CONCRETE OF GIVEN COMPRESSIVE STRENGTH AT 28 DAYS

The following tables give the proportions in which Portland cement and a wide range in sizes of fine and coarse aggregates should be mixed to obtain concrete of compressive strengths ranging from 1500 to 3000 lb. per sq. in. at 28 days. Proportions are given for concrete of four different consistencies.

The purpose of the tables is twofold:

1. To furnish a guide in the selection of mixtures to be used in preliminary investigations of the strength of concrete from given materials.

2. To indicate proportions which may be expected to produce concrete of a given strength under average conditions where control tests are not made.

If the proportions to be used in the work are selected from the table without preliminary tests of the materials, and control tests are not made during the progress of the work, the mixtures in bold-face type shall be used.

The use of these tables as a guide in the selection of concrete mixtures is based on the following:

1. Concretes shall be plastic;
2. Aggregates shall be clean and structurally sound;
3. Aggregates shall be graded between the sizes indicated.

Apply the following rules in determining the size assigned to a given aggregate:

1. Not less than 15 per cent shall be retained between the sieve which is considered the maximum size and the next smaller sieve.

2. Not more than 15 per cent of a coarse aggregate shall be finer than the sieve considered as the minimum size.

Proportions may be interpolated for concrete strengths, aggregate sizes, and consistencies not covered by the table or determined by test.

PROPORTIONS FOR 1500-LB. PER SQ. IN. CONCRETE

Size of coarse aggregate	Slump, in inches	Size of fine aggregate				
		0–No. 30	0–No. 16	0–No. 8	0–No. 4	0–⅜ in.
None	½ to 1	1:2.8	1:3.2	1:3.8	1:4.4	1:5.1
	3 to 4	1:2.4	1:2.8	1:3.3	1:3.8	1:4.5
	6 to 7	**1:1.9**	**1:2.2**	**1:2.6**	**1:3.0**	**1:3.6**
	8 to 10	1:1.4	1:1.6	1:1.8	1:2.1	1:2.5
No. 4 to ¾ in.	½ to 1	1:2.6:4.6	1:2.9:4.3	1:3.4:4.1	1:3.9:3.6	1:4.6:3.1
	3 to 4	1:2.3:4.0	1:2.6:3.8	1:2.9:3.6	1:3.4:3.2	1:4.1:2.8
	6 to 7	**1:1.8:3.4**	**1:2.0:3.2**	**1:2.3:3.1**	**1:2.6:2.8**	**1:3.1:2.5**
	8 to 10	1:1.1:2.5	1:1.3:2.4	1:1.5:2.4	1:1.7:2.2	1:2.1:2.0
No. 4 to 1 in.	½ to 1	1:2.4:5.3	1:2.7:5.2	1:3.1:5.0	1:3.5:4.7	1:4.3:4.3
	3 to 4	1:2.1:4.7	1:2.4:4.5	1:2.7:4.4	1:3.1:4.1	1:4.7:3.7
	6 to 7	**1:1.6:3.9**	**1:1.8:3.8**	**1:2.1:3.7**	**1:2.4:3.5**	**1:2.9:3.3**
	8 to 10	1:1.1:2.9	1:1.2:2.8	1:1.4:2.8	1:1.6:2.7	1:1.9:2.5
No. 4 to 1½ in.	½ to 1	1:2.4:6.0	1:2.7:5.9	1:3.1:5.8	1:3.5:5.4	1:4.1:5.1
	3 to 4	1:2.0:5.4	1:2.3:5.3	1:2.7:5.2	1:3.0:5.0	1:3.5:4.6
	6 to 7	**1:1.6:4.4**	**1:1.8:4.3**	**1:2.0:4.3**	**1:2.3:4.1**	**1:2.7:3.9**
	8 to 10	1:1.0:3.3	1:1.1:3.2	1:1.3:3.2	1:1.5:3.1	1:1.8:2.9
No. 4 to 2 in.	½ to 1	1:2.2:6.9	1:2.4:6.8	1:2.8:6.8	1:3.1:6.6	1:3.7:6.4
	3 to 4	1:1.8:6.2	1:2.0:6.1	1:2.4:6.1	1:2.7:6.0	1:3.1:5.7
	6 to 7	**1:1.4:5.1**	**1:1.6:5.0**	**1:1.8:5.0**	**1:2.0:5.0**	**1:2.4:4.8**
	8 to 10	1:0.9:3.8	1:1.0:3.8	1:1.1:3.8	1:1.3:3.8	1:1.5:3.7
⅜ to 1 in.	½ to 1	1:2.8:5.2	1:3.1:5.1	1:3.6:4.8	1:4.2:4.6	1:4.8:4.1
	3 to 4	1:2.4:4.5	1:2.6:4.5	1:3.1:4.3	1:3.6:4.0	1:4.1:3.6
	6 to 7	**1:1.9:3.9**	**1:2.1:3.7**	**1:2.4:3.6**	**1:2.8:3.4**	**1:3.2:3.1**
	8 to 10	1:1.3:2.8	1:1.4:2.8	1:1.6:2.7	1:1.9:2.6	1:2.2:2.4
⅜ to 1½ in.	½ to 1	1:2.8:5.8	1:3.1:5.7	1:3.5:5.5	1:4.1:5.3	1:4.7:4.9
	3 to 4	1:2.4:5.2	1:2.7:5.1	1:3.1:5.0	1:3.5:4.8	1:4.1:4.4
	6 to 7	**1:1.9:4.3**	**1:2.1:4.2**	**1:2.4:4.2**	**1:2.7:4.0**	**1:3.1:3.7**
	8 to 10	1:1.2:3.2	1:1.4:3.2	1:1.6:3.1	1:1.8:3.0	1:2.1:2.9
⅜ to 2 in.	½ to 1	1:2.7:6.6	1:3.0:6.6	1:3.4:6.5	1:3.9:6.4	1:4.4:6.0
	3 to 4	1:2.3:5.9	1:2.6:5.9	1:2.9:5.8	1:3.3:5.6	1:3.7:5.5
	6 to 7	**1:1.8:4.9**	**1:2.0:4.8**	**1:2.2:4.8**	**1:2.6:4.8**	**1:3.0:4.5**
	8 to 10	1:1.2:3.7	1:1.3:3.7	1:1.5:3.7	1:1.7:3.6	1:1.9:3.5
¾ to 1½ in.	½ to 1	1:3.2:5.4	1:3.6:5.3	1:4.1:5.1	1:4.7:4.8	1:5.3:4.4
	3 to 4	1:2.8:4.8	1:3.2:4.8	1:3.6:4.6	1:4.0:4.4	1:4.6:4.0
	6 to 7	**1:2.1:4.0**	**1:2.5:4.0**	**1:2.8:3.9**	**1:3.2:3.7**	**1:3.5:3.4**
	8 to 10	1:1.5:3.0	1:1.7:3.0	1:1.9:2.9	1:2.2:2.8	1:2.5:2.7
¾ to 2 in.	½ to 1	1:3.2:6.2	1:3.6:6.1	1:4.0:6.0	1:4.6:5.8	1:5.2:5.4
	3 to 4	1:2.8:5.5	1:3.1:5.5	1:3.5:5.4	1:3.9:5.2	1:4.5:4.9
	6 to 7	**1:2.1:4.5**	**1:2.4:4.6**	**1:2.7:4.5**	**1:3.1:4.4**	**1:3.5:4.1**
	8 to 10	1:1.4:3.4	1:1.6:3.4	1:1.8:3.4	1:2.1:3.4	1:2.4:3.3
¾ to 3 in.	½ to 1	1:3.2:7.1	1:3.6:7.1	1:4.0:7.0	1:4.6:6.9	1:5.2:6.6
	3 to 4	1:2.7:6.3	1:3.0:6.3	1:3.4:6.3	1:4.0:6.2	1:4.5:5.9
	6 to 7	**1:2.1:5.1**	**1:2.4:5.2**	**1:2.7:5.2**	**1:3.1:5.1**	**1:3.5:4.9**
	8 to 10	1:1.4:3.8	1:1.6:3.9	1:1.8:3.9	1:2.1:3.9	1:2.4:3.8

Proportions for 2000-Lb. per Sq. In. Concrete

Size of coarse aggregate	Slump, in inches	Size of fine aggregate				
		0–No. 30	0–No. 16	0–No. 8	0–No. 4	0–⅜ in.
None	½ to 1	1:2.2	1:2.6	1:3.0	1:3.5	1:4.1
	3 to 4	1:1.9	1:2.2	1:2.6	1:3.0	1:3.5
	6 to 7	**1:1.5**	**1:1.7**	**1:2.0**	**1:2.3**	**1:2.7**
	8 to 10	**1:1.0**	**1:1.1**	**1:1.3**	1:1.6	1:1.8
No. 4 to ¾ in.	½ to 1	1:2.1:3.8	1:2.3:3.7	1:2.6:3.5	1:3.0:3.1	1:3.6:2.8
	3 to 4	1:1.7:3.3	1:1.9:3.2	1:2.2:3.1	1:2.6:2.8	1:3.0:2.4
	6 to 7	**1:1.3:2.7**	**1:1.4:2.6**	**1:1.7:2.5**	**1:1.9:2.3**	**1:2.3:2.1**
	8 to 10	1:0.8:1.9	1:0.9:1.9	1:1.0:1.8	1:1.2:1.7	1:1.5:1.6
No. 4 to 1 in.	½ to 1	1:1.9:4.5	1:2.2:4.3	1:2.5:4.2	1:2.8:3.9	1:3.4:3.6
	3 to 4	1:1.6:3.9	1:1.8:3.8	1:2.1:3.7	1:2.4:3.5	1:2.8:3.2
	6 to 7	**1:1.2:3.1**	**1:1.3:3.1**	**1:1.5:3.0**	**1:1.8:2.9**	**1:2.1:2.7**
	8 to 10	1:0.7:2.2	1:0.8:2.2	1:1.0:2.3	1:1.1:2.1	1:1.3:2.0
No. 4 to 1½ in.	½ to 1	1:1.9:5.0	1:2.1:4.9	1:2.4:4.9	1:2.7:4.6	1:3.2:4.4
	3 to 4	1:1.6:4.4	1:1.7:4.3	1:2.0:4.2	1:2.4:4.0	1:2.7:3.8
	6 to 7	**1:1.1:3.5**	**1:1.3:3.5**	**1:1.4:3.5**	**1:1.7:3.4**	**1:2.0:3.2**
	8 to 10	1:0.7:2.5	1:0.8:2.5	1:0.9:2.5	1:1.0:2.4	1:1.2:2.3
No. 4 to 2 in.	½ to 1	1:1.7:5.8	1:1.9:5.7	1:2.1:5.8	1:2.4:5.6	1:2.8:5.5
	3 to 4	1:1.4:5.0	1:1.5:5.0	1:1.8:5.0	1:2.0:4.9	1:2.3:4.7
	6 to 7	**1:1.0:4.1**	**1:1.1:4.1**	**1:1.2:4.1**	**1:1.4:4.1**	**1:1.7:3.9**
	8 to 10	1:0.6:2.9	1:0.7:2.9	1:0.7:3.0	1:0.8:2.9	1:1.0:2.9
⅜ to 1 in.	½ to 1	1:2.2:4.4	1:2.5:4.2	1:2.8:4.1	1:3.3:3.8	1:3.8:3.4
	3 to 4	1:1.9:3.8	1:2.1:3.7	1:2.4:3.6	1:2.8:3.4	1:3.2:3.1
	6 to 7	**1:1.4:3.1**	**1:1.5:3.0**	**1:1.8:3.0**	**1:2.1:2.8**	**1:2.4:2.5**
	8 to 10	1:0.9:2.2	1:1.0:2.2	1:1.1:2.2	1:1.3:2.0	1:1.5:1.9
⅜ to 1½ in.	½ to 1	1:2.2:4.9	1:2.5:4.8	1:2.8:4.7	1:3.2:4.6	1:3.7:4.2
	3 to 4	1:1.9:4.3	1:2.1:4.2	1:2.4:4.1	1:2.7:4.0	1:3.1:3.7
	6 to 7	**1:1.4:3.5**	**1:1.5:3.4**	**1:1.7:3.4**	**1:2.0:3.3**	**1:2.3:3.1**
	8 to 10	1:0.9:2.5	1:1.0:2.5	1:1.1:2.4	1:1.3:2.4	1:1.5:2.3
⅜ to 2 in.	½ to 1	1:2.1:5.6	1:2.3:5.5	1:2.6:5.5	1:3.0:5.4	1:3.5:5.1
	3 to 4	1:1.7:4.8	1:2.0:4.8	1:2.2:4.8	1:2.5:4.7	1:2.9:4.4
	6 to 7	**1:1.3:4.0**	**1:1.4:3.9**	**1:1.6:3.9**	**1:1.8:3.9**	**1:2.1:3.8**
	8 to 10	1:0.8:2.9	1:0.9:2.9	1:1.0:2.9	1:1.2:2.9	1:1.3:2.8
¾ to 1½ in.	½ to 1	1:2.6:4.5	1:2.9:4.5	1:3.3:4.4	1:3.8:4.2	1:4.3:3.9
	3 to 4	1:2.2:3.9	1:2.5:3.9	1:2.8:3.8	1:3.2:3.6	1:3.6:3.3
	6 to 7	**1:1.6:3.2**	**1:1.8:3.2**	**1:2.1:3.1**	**1:2.4:3.0**	**1:2.7:2.8**
	8 to 10	1:1.0:2.3	1:1.2:2.3	1:1.4:2.2	1:1.6:2.2	1:1.8:2.1
¾ to 2 in.	½ to 1	1:2.5:5.2	1:2.8:5.2	1:3.2:5.1	1:3.6:5.0	1:4.1:4.7
	3 to 4	1:2.1:4.5	1:2.4:4.5	1:2.7:4.4	1:3.1:4.3	1:3.5:4.0
	6 to 7	**1:1.6:3.7**	**1:1.8:3.7**	**1:2.0:3.7**	**1:2.3:3.6**	**1:2.6:3.5**
	8 to 10	1:1.0:2.6	1:1.1:2.7	1:1.3:2.6	1:1.5:2.7	1:1.7:2.6
¾ to 3 in.	½ to 1	1:2.5:6.0	1:2.9:5.9	1:3.2:5.9	1:3.6:5.8	1:4.1:5.6
	3 to 4	1:2.1:5.1	1:2.4:5.2	1:2.7:5.2	1:3.1:5.1	1:3.5:4.9
	6 to 7	**1:1.5:4.1**	**1:1.7:4.2**	**1:2.0:4.2**	**1:2.3:4.2**	**1:2.5:4.0**
	8 to 10	1:1.0:2.9	1:1.1:3.0	1:1.3:3.0	1:1.5:3.0	1:1.7:3.0

Proportions for 2500-Lb. per Sq. In. Concrete

Size of coarse aggregate	Slump, in inches	Size of fine aggregate				
		0–No. 30	0–No. 16	0–No. 8	0–No. 4	0–3/8 in.
None	1/2 to 1	1:1.8	1:2.1	1:2.4	1:2.9	1:3.3
	3 to 4	1:1.5	1:1.8	1:2.1	1:2.4	1:2.8
	6 to 7	**1:1.1**	**1:1.3**	**1:1.6**	**1:1.8**	**1:2.1**
	8 to 10	1:0.7	1:0.8	1:0.9	1:1.1	1:1.3
No. 4 to 3/4 in.	1/2 to 1	1:1.6:3.2	1:1.8:3.1	1:2.1:3.0	1:2.4:2.7	1:2.9:2.4
	3 to 4	1:1.3:2.8	1:1.5:2.7	1:1.7:2.6	1:2.0:2.4	1:2.4:2.2
	6 to 7	**1:1.0:2.2**	**1:1.1:2.2**	**1:1.3:2.1**	**1:1.5:2.0**	**1:1.8:1.8**
	8 to 10	1:0.5:1.4	1:0.6:1.4	1:0.7:1.4	1:0.8:1.4	1:1.0:1.3
No. 4 to 1 in.	1/2 to 1	1:1.5:3.7	1:1.7:3.7	1:2.0:3.5	1:2.2:3.4	1:2.7:3.1
	3 to 4	1:1.2:3.3	1:1.4:3.2	1:1.6:3.1	1:1.9:3.0	1:2.2:2.7
	6 to 7	**1:0.9:2.6**	**1:1.0:2.5**	**1:1.1:2.5**	**1:1.3:2.4**	**1:1.6:2.3**
	8 to 10	1:0.5:1.7	1:0.6:1.7	1:0.6:1.7	1:0.7:1.6	1:0.9:1.5
No. 4 to 1 1/2 in.	1/2 to 1	1:1.4:4.2	1:1.6:4.1	1:1.9:4.1	1:2.2:4.0	1:2.5:3.8
	3 to 4	1:1.2:3.7	1:1.3:3.6	1:1.5:3.6	1:1.8:3 5	1:2.1:3.3
	6 to 7	**1:0.9:2.9**	**1:0.9:2.8**	**1:1.1:2.8**	**1:1.3:2.8**	**1:1.5:2.6**
	8 to 10	1:0.5:1.9	1:0.5:1.9	1:0.6:1.9	1:0.7:1.8	1:0.8:1.8
No. 4 to 2 in.	1/2 to 1	1:1.3:4.9	1:1.4:4.8	1:1.6:4.9	1:1.9:4.8	1:2.2:4.7
	3 to 4	1:1.1:4.3	1:1.2:4.2	1:1.3:4.2	1:1.6:4.2	1:1.8:4.1
	6 to 7	**1:0.7:3.3**	**1:0.8:3.3**	**1:0.9:3.4**	**1:1.1:3.3**	**1:1.2:3.3**
	8 to 10	1:0.4:2.2	1:2.4:2.2	1:0.5:2.2	1:0.6:2.2	1:0.6:2.2
3/8 to 1 in.	1/2 to 1	1:1.8:3.7	1:2.0:3.6	1:2.3:3.5	1:2.6:3.3	1:3.0:2.9
	3 to 4	1:1.4:3.2	1:1.6:3.1	1:1.9:2.9	1:2.2:2.9	1:2.5:2.6
	6 to 7	**1:1.0:2.5**	**1:1.2:2.5**	**1:1.3:2.4**	**1:1.6:2.3**	**1:1.8:2.2**
	8 to 10	1:0.6:1.6	1:0.7:1.6	1:0.8:1.6	1:0.9:1.6	1:1.0:1.5
3/8 to 1 1/2 in.	1/2 to 1	1:1.7:4.1	1:1.9:4.1	1:2.2:4.0	1:2.5:3.9	1:2.9:3.6
	3 to 4	1:1.5:3.6	1:1.6:3.6	1:1.8:3.5	1:2.1:3.4	1:2.3:3.2
	6 to 7	**1:1.0:2.9**	**1:1.2:2.8**	**1:1.3:2.8**	**1:1.5:2.7**	**1:1.8:2.6**
	8 to 10	1:0.6:1.9	1:0.6:1.9	1:0.8:1.8	1:0.9:1.8	1:1.0:1.8
3/8 to 2 in.	1/2 to 1	1:1.7:4.7	1:1.8:4.7	1:2.1:4.7	1:2.4:4.6	1:2.7:4.4
	3 to 4	1:1.4:4.1	1:1.5:4.1	1:1.7:4.1	1:2.0:4.0	1:2.3:3.9
	6 to 7	**1:1.0:3.2**	**1:1.1:3.2**	**1:1.2:3.2**	**1:1.4:3.2**	**1:1.6:3.1**
	8 to 10	1:0.5:2.1	1:0.6:2.1	1:0.7:2.2	1:0.8:2.2	1:0.9:2.1
3/4 to 1 1/2 in.	1/2 to 1	1:2.0:3.8	1:2.3:3.8	1:2.6:3.7	1:3.0:3.6	1:3.4:3.3
	3 to 4	1:1.7:3.3	1:2.0:3.3	1:2.2:3.2	1:2.5:3.2	1:2.9:2.9
	6 to 7	**1:1.2:2.6**	**1:1.4:2.6**	**1:1.6:2.6**	**1:1.9:2.5**	**1:2.1:2.3**
	8 to 10	1:0.7:1.7	1:0.8:1.7	1:0.9:1.7	1:1.1:1.7	1:1.2:1.6
3/4 to 2 in.	1/2 to 1	1:2.0:4.4	1:2.2:4.4	1:2.5:4.3	1:2.9:4.3	1:3.3:4.1
	3 to 4	1:1.7:3.8	1:1.9:3.8	1:2.1:3.8	1:2.5:3.7	1:2.8:3.6
	6 to 7	**1:1.2:3.0**	**1:1.4:3.0**	**1:1.5:3.0**	**1:1.8:3.0**	**1:2.0:2.8**
	8 to 10	1:0.7:2.0	1:0.8:2.0	1:0.9:2.0	1:1.0:2.0	1:1.2:2.0
3/4 to 3 in.	1/2 to 1	1:2.0:5.0	1:2.2:5.0	1:2.5:5.0	1:2.7:5.0	1:3.2:4.7
	3 to 4	1:1.7:4.3	1:1.9:4.3	1:2.1:4.3	1:2.4:4.3	1:2.7:4.1
	6 to 7	**1:1.2:3.3**	**1:1.4:3.4**	**1:1.5:3.4**	**1:1.8:3.4**	**1:2.0:3.3**
	8 to 10	1:0.7:2.2	1:0.8:2.2	1:0.9:2.2	1:1.0:2.3	1:1.2:2.3

PROPORTIONS FOR 3000-LB. PER SQ. IN. CONCRETE

Size of coarse aggregate	Slump, in inches	Size of fine aggregate 0–No. 30	0–No. 16	0–No. 8	0–No. 4	0–3/8 in.
None	½ to 1	1:1.5	1:1.7	1:2.0	1:2.3	1:2.7
	3 to 4	1:1.2	1:1.4	1:1.7	1:1.9	1:2.3
	6 to 7	**1:0.9**	**1:1.0**	**1:1.2**	**1:1.4**	**1:1.6**
	8 to 10	1:0.5	1:0.6	1:0.7	1:0.8	1:0.9
No. 4 to ¾ in.	½ to 1	1:1.3:2.7	1:1.5:2.6	1:1.7:2.5	1:1.9:2.4	1:2.3:2.1
	3 to 4	1:1.0:2.3	1:1.2:2.2	1:1.4:2.2	1:1.6:2.0	1:1.9:1.8
	6 to 7	**1:0.7:1.7**	**1:0.8:1.7**	**1:0.9:1.7**	**1:1.1:1.6**	**1:1.3:1.4**
	8 to 10	1:0.3:1.0	1:0.4:1.0	1:0.5:1.0	1:0.5:1.0	1:0.6:0.9
No. 4 to 1 in.	½ to 1	1:1.2:3.1	1:1.3:3.1	1:1.5:3.0	1:1.8:2.9	1:2.1:2.7
	3 to 4	1:0.9:2.7	1:1.1:2.6	1:1.2:2.6	1:1.4:2.5	1:1.7:2.3
	6 to 7	**1:0.6:2.0**	**1:0.7:2.0**	**1:0.8:2.0**	**1:0.9:1.9**	**1:1.1:1.8**
	8 to 10	1:0.3:1.2	1:0.3:1.2	1:0.4:1.2	1:0.5:1.2	1:0.6:1.2
No. 4 to 1½ in.	½ to 1	1:1.1:3.6	1:1.2:3.5	1:1.5:3.5	1:1.7:3.4	1:2.0:3.2
	3 to 4	1:0.9:3.0	1:1.0:2.9	1:1.2:2.9	1:1.4:2.9	1:1.6:2.7
	6 to 7	**1:0.6:2.2**	**1:0.7:2.2**	**1:0.8:2.2**	**1:0.9:2.2**	**1:1.1:2.1**
	8 to 10	1:0.3:1.4	1:0.3:1.3	1:0.4:1.4	1:0.5:1.4	1:0.5:1.3
No. 4 to 2 in.	½ to 1	1:1.0:4.1	1:1.1:4.1	1:1.2:4.1	1:1.4:4.1	1:1.6:4.0
	3 to 4	1:0.8:3.4	1:0.9:3.4	1:1.0:3.5	1:1.1:3.4	1:1.3:3.4
	6 to 7	**1:0.5:2.6**	**1:0.6:2.6**	**1:0.6:2.7**	**1:0.7:2.6**	**1:0.9:2.6**
	8 to 10	1:0.2:1.6	1:0.3:1.6	1:0.3:1.7	1:0.4:1.7	1:0.4:1.7
3/8 to 1 in.	½ to 1	1:1.4:3.1	1:1.5:3.0	1:1.8:2.9	1:2.1:2.8	1:2.4:2.6
	3 to 4	1:1.1:2.6	1:1.3:2.6	1:1.5:2.5	1:1.7:2.4	1:2.0:2.2
	6 to 7	**1:0.8:2.0**	**1:0.8:2.0**	**1:1.0:1.9**	**1:1.1:1.9**	**1:1.3:1.8**
	8 to 10	1:0.4:1.2	1:0.4:1.2	1:0.5:1.2	1:0.6:1.2	1:0.7:1.1
3/8 to 1½ in.	½ to 1	1:1.4:3.5	1:1.5:3.4	1:1.7:3.4	1:2.0:3.3	1:2.3:3.1
	3 to 4	1:1.1:3.0	1:1.2:2.9	1:1.4:2.9	1:1.6:2.8	1:1.9:2.6
	6 to 7	**1:0.6:2.2**	**1:0.8:2.2**	**1:1.0:2.2**	**1:1.1:2.1**	**1:1.3:2.0**
	8 to 10	1:0.4:1.4	1:.04:1.4	1:0.5:1.4	1:0.6:1.3	1:0.7:1.3
3/8 to 2 in.	½ to 1	1:1.3:4.0	1:1.4:4.0	1:1.6:4.0	1:1.9:3.9	1:2.1:3.8
	3 to 4	1:1.0:3.4	1:1.2:3.4	1:1.3:3.3	1:1.5:3.3	1:1.7:3.2
	6 to 7	**1:0.7:2.6**	**1:0.8:2.5**	**1:0.9:2.6**	**1:1.0:2.6**	**1:1.1:2.5**
	8 to 10	1:0.4:1.6	1:0.4:1.6	1:0.5:1.6	1:0.5:1.6	1:0.6:1.6
¾ to 1½ in.	½ to 1	1:1.6:3.2	1:1.8:3.2	1:2.1:3.2	1:2.4:3.1	1:2.7:2.9
	3 to 4	1:1.3:2.7	1:1.5:2.7	1:1.7:2.7	1:2.0:2.6	1:2.3:2.5
	6 to 7	**1:0.9:2.0**	**1:1.0:2.1**	**1:1.2:2.0**	**1:1.4:2.0**	**1:1.5:1.8**
	8 to 10	1:0.5:1.2	1:0.5:1.3	1:0.6:1.3	1:0.7:1.3	1:0.8:1.2
¾ to 2 in.	½ to 1	1:1.6:3.7	1:1.8:3.7	1:2.0:3.7	1:2.4:3.6	1:2.6:3.5
	3 to 4	1:1.3:3.1	1:1.5:3.1	1:1.6:3.1	1:1.9:3.1	1:2.2:3.0
	6 to 7	**1:0.9:2.4**	**1:1.1:2.4**	**1:1.1:2.4**	**1:1.3:2.4**	**1:1.5:2.3**
	8 to 10	1:0.5:1.5	1:0.5:1.5	1:0.6:1.5	1:0.7:1.5	1:0.8:1.5
¾ to 3 in.	½ to 1	1:1.6:4.2	1:1.8:4.2	1:2.0:4.2	1:2.3:4.1	1:2.6:4.0
	3 to 4	1:1.3:3.5	1:1.5:3.6	1:1.6:3.6	1:1.9:3.6	1:2.1:3.5
	6 to 7	**1:0.9:2.6**	**1:1.0:2.6**	**1:1.1:2.6**	**1:1.3:2.6**	**1:1.4:2.6**
	8 to 10	1:0.5:1.6	1:0.5:1.6	1:0.6:1.7	1:0.7:1.7	1:0.8:1.7

INDEX

Date Due

AUG 2 1947			